The Disturbances Approach to the Demand for International Reserves

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I. THE DISTURBANCES APPROACH IN THE LITERATURE ON THE DEMAND FOR INTERNATIONAL RESERVES

Governments accumulate and hold stocks of international reserves primarily for the purpose of financing external disequilibria that otherwise would disrupt the domestic economy. These external disequilibria usually arise from two sources: First, fluctuations will occur more or less regularly in the balance of payments owing to seasonal or temporal differences in the business cycle in different nations, and, second, persistent deficit or surplus positions will develop in certain countries as a result of diverging rates of price inflation, income growth, and economic development in different nations.

Broadly speaking, two approaches to measuring the size of external disequilibria have appeared in the literature on the demand for international reserves. They may be called the *transactions approach*, where the size of disequilibria is measured in terms of the mean level of some class of external transactions (usually import expenditures), and the *disturbances approach*, where the size of disequilibria is measured in terms of the variations in the level of some class of external transactions. No one measure has won general acceptance under the disturbances approach similar to the acceptance given import expenditures under the transactions approach.

This paper is a study of the demand for international reserves utilizing the disturbances approach to measuring the size of external disequilibria.[1] Chapter I is a review of the theoretical and empirical work of several authors that has led to the present study.[2] In Chapter II a general model of reserve demand is developed based upon the major points discussed in Chapter I. And in Chapter III the study is concluded with several regressions run upon a sample of sixty-one nations for the period 1960–65. The results of these regressions are very encouraging.

[2]The reader is referred to Grubel (1971) and Williamson (1973) for surveys of the literature on the demand for international reserves. Williamson's comments on pp. 688–697 are especially pertinent to this study.

For the purposes of this study, "international reserves" are defined as the "official reserves" reported by the International Monetary Fund. Such reserves include a government's holdings of gold and foreign exchange plus three lines of credit with the Fund — the super gold tranche position, the gold tranche position, and special drawing rights.

Major studies of the demand for reserves using the transactions approach have been those by the International Monetary Fund (1953, 1958), Triffin (1959), Brown (1964), Machlup (1966), Thorn (1967), Courchene and Youssef (1967), Hawkins and Rangarajan (1970), Clark (1970b), and Flanders (1971). The early studies by the Fund, Triffin, Brown, and Machlup stated reserves as a ratio to some transactions variable, usually imports, in an attempt to see if the ratios displayed sufficient clustering to be interpreted as a "demand" for reserves. The consensus, as expressed by Machlup, was that no such clustering could be observed. The later studies all employed regression analysis. Thorn, and Courchene and Youssef, regressed reserves against imports and found a statistically significant relationship. Hawkins and Rangarajan regressed IMF quotas against the sum of export and import transactions (and other variables) with little success. Clark took the ratio of reserves to the sum of export and import transactions and with little success regressed this ratio against several explanatory variables. Flanders regressed reserve/import ratios against several explanatory variables but also found few relationships of significance.

Major studies of the demand for reserves using the disturbances approach have been those by Nagabhushanam and Sastry (1962), Heller (1964, 1966), Brown (1964), Kenen and Yudin (1965), Machlup (1966), Hodjera (1969), Clark (1970a, 1970b), Kelly (1970), Flanders (1971), and Archibald and Richmond (1971). The studies by Brown, Machlup, Clark, and Flanders used both approaches. Brown and Machlup computed the ratio of reserves to some measure of disturbances but found no clustering in these ratios. Nagabhushanam and Sastry, Heller (1964), Kenen and Yudin, Hodjera, and Archibald and Richmond were primarily interested in the stochastic relationship of the reserve stock to some measure of disturbances and were very successful in refining this approach. Heller (1966), Clark, and Kelly, while also considering the stochastic relationships, were interested in linking reserve demand to a broad range of explanatory variables. All three of the last-named studies yielded mixed results. Flanders included a measure of disturbances among the explanatory variables in her study on reserve/import ratios.

In the remainder of this chapter, the studies by Nagabhushanam and Sastry, Kenen and Yudin, Heller (1966), Kelly, Clark, Archibald and Richmond, and some others will be discussed in more detail. A common system of notation has been developed and will be introduced as the discussion proceeds.

Nagabhushanam and Sastry, "A Stochastic Model for Foreign-Exchange Reserves"

Two Indian economists, K. Nagabhushanam and M. Perayya Sastry (1962), in an innovative but overlooked study, first used the disturbances approach to estimate a demand function for reserves. The demand for reserves, they pointed out, is analogous to the demand for water in a reservoir. There is a branch of inventory analysis on water storage called "dam theory," which they applied to reserve demand. If the inflow of water to a reservoir is stable in a stochastic sense, then it may be described by a probability frequency distribution. The operator of the dam determines the stock of water in the reservoir by releasing water from the reservoir. It is assumed that the operator is rational, releasing water according to some "release rule" that reflects his demand function for the stock of water in the reservoir. Given data on inflows, outflows, and stock levels, the release rule can be estimated.

Nagabhushanam and Sastry assumed that India's receipts of foreign exchange were exogenously determined and stochastically stable over time, and that the Indian government completely controlled foreign-exchange expenditures. Thus the source of disturbances to be covered by reserves was the variation in total external receipts. The release rule was assumed to be a variable function of the inherited reserve stock and current foreign-exchange earnings. Based upon these assumptions, they estimated the demand function for international reserves of the Indian government using quarterly data for the years 1940–60. The importance of this study, aside from being the first of its type, is that it anticipates by several years the methodology used by Kelly (1970) in his very important study of reserve demand.

Kenen and Yudin, "The Demand for International Reserves"

Peter B. Kenen and Elinor B. Yudin (1965) conducted the first multinational study of reserve demand under the disturbances approach. They

began by treating changes in reserves R as a stochastic process of the Markov type:

$$(R_t - R_{t-1}) = p(R_{t-1} - R_{t-2}) + Q_t$$
$$0 < p < 1 \qquad Q_t = N[E(Q), V(Q)]. \tag{1.1}$$

The change in reserves in period t is $(R_t - R_{t-1})$. This change is composed of two elements: the first is a "carry-forward" element $p(R_{t-1} - R_{t-2})$ from the change in reserves in period $(t-1)$; and the second is a random disturbance Q_t (for disequilibrium) generated by a normal distribution with a nonzero mean $E(Q)$ and an independent variance $V(Q)$. Kenen and Yudin point out that the balance of payments is a sum of many separate transactions. While each transaction may not be drawn from a normal population, the cental-limit theorem suggests that the sum of the transactions will have a normal distribution. Since changes in reserves are an approximation of the balance of payments, they assumed that Q_t was normally distributed.

It is important to note that under (1.1) the variance of reserves $V(R)$ is never stable at the limit.[3] This apparently disturbed Kenen and Yudin, and they went on to note (p. 247) that governments could stabilize the reserve stock by changing the value of $E(Q)$. In that case, $E(Q)$ is no longer a truly stochastic element in reserve changes. This difficulty in the formulation of the stochastic process was remedied by Clark (1970a) when he stated reserve changes as a lagged-adjustment process. It should also be noted that Kenen and Yudin treated reserve changes as a flow-stochastic equation (R_{t-1} is to the left of the equality sign), while later writers use a stock-stochastic equation (R_{t-1} is to the right of the equality sign). While this leads to obvious differences in empirical work (the dependent variable is changes in reserves as opposed to reserve levels), the estimated values of coefficients should not be affected by this difference.

Equation (1.1) may be restated as

$$(R_t^* - R_{t-1}) = E(Q) + p(R_{t-1} - R_{t-2}) + [Q_t - E(Q)]$$
$$0 < p < 1 \qquad [Q_t - E(Q)] = N[0, V(Q)]. \tag{1.2}$$

Note that the mean is zero for the $[Q_t - E(Q)]$ distribution. Equation (1.2) can then be written as

$$(R_t - R_{t-1}) = a_0 + a_1 (R_{t-1} - R_{t-2}) + e_t. \tag{1.3}$$

Using (1.3) as a regression equation, Kenen and Yudin estimated the

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[3]This point is discussed further by Archibald and Richmond (1971, pp. 246–247).

values of the a_0 and a_1 coefficients and the parameters of the error distribution for fourteen trading nations, using monthly reserve data for the period 1958–62. From a_0 came estimates of $E(Q)$; from a_1 came estimates of p; from $V(e)$ came estimates of $V(Q)$; and from $S(e)$, the "standard error" of (1.3), came estimates of $S(Q)$.

Kenen and Yudin then assumed that a nation's reserve-demand function was of the form

$$R = b_0 - b_1 E(Q) + b_2 p + b_3 S(Q) . \tag{1.4}$$

The size of disturbances is represented by the standard deviation of disturbances $S(Q)$. Using the values of $E(Q)$, p, and $S(Q)$ for each nation calculated by (1.3), they performed cross-sectional regressions for the beginning and end of their sample period 1958–62. Since b_1, the coefficient for $E(Q)$, came out with the wrong (positive) sign, it was deleted and a second set of regressions performed. The only variable to show any significance in the second set of regressions, and the only one to show significance (at the 5 per cent level) in all four regressions, was (SQ). Kenen and Yudin then added explanatory variables to represent holding cost H (per capita income) and "liquid liabilities," but only $S(Q)$ continued to show any explanatory significance.

The contributions of this paper to the literature are extremely important. First, a technique was developed for estimating the variance of disturbances $V(Q)$. Note that $V(Q)$ is the same parameter under the assumption of a zero mean for disturbances in (1.2) and under the assumption of a nonzero mean for disturbances in (1.1). By equating the nonzero mean for disturbances with the trend component in reserve changes, Clark (1970b) and Archibald and Richmond (1971) built their own analyses around the Kenen and Yudin technique. Second, the significance of the $V(Q)$ parameter was amply demonstrated by Kenen and Yudin in their regressions.

Heller, "Optimal International Reserves"

H. Robert Heller (1966) performed the first formal cost-benefit study of reserve demand. He wanted to estimate optimal reserves for sixty member nations of the IMF, those for which he could assemble data. In scope, his study was the broadest since the 1958 IMF study of reserve adequacy. Heller defined optimal reserves $E(R)^*$ as that average stock of reserves which will maximize national income over time. To optimize reserves, the marginal benefit of each reserve unit is compared with the marginal cost.

Reserve units should be accumulated until the marginal benefit equals the marginal cost.

The marginal cost of holding reserves H is the rate of return r foregone by not transforming reserves into real physical capital. The higher this cost of holding reserves, the smaller the optimal reserve stock $E(R)^*$. To measure holding costs, Heller assumed heroically that r was a uniform 5 per cent for all nations. The marginal benefit of holding reserves is the avoided economic cost of adjusting to a deficit not covered by reserves. (The cost of adjusting to a surplus was not considered in the literature until later.) The marginal benefit of holding reserves takes the form (cA), where c is the probability of having to use the marginal reserve unit,[4]

$$c = P[R < 0 \mid E(R), V(R)] \qquad c > 0, \tag{1.5}$$

and A is the change in income Y necessary to change temporarily the balance of payments by one unit, thereby altering the reserve stock by one unit. To measure adjustment cost A for a particular country, Heller assumed a purely Keynesian posture. A deficit not covered by reserves would be handled by reducing income to reduce imports sufficiently to eliminate the deficit. Under this assumption, A is the reciprocal of the marginal propensity to import, or $(1/MPM)$. In lieu of good estimates of MPM, Heller used the average propensity to import in 1963. To derive the probability c of using the marginal reserve unit, Heller used a continuous approximation to a binary treatment of the reserve variations experienced by each nation over the period 1949–63. Thus his measure of disturbances describes the fluctuations in the reserve stock. The last unit of reserves in the optimal stock would equate marginal cost and marginal benefit $(r = c/MPM)$.

Heller went on to estimate optimal reserves for each of his sixty nations for the end of 1963 and to test regional and world reserve adequacy by aggregating his results. He found that world reserves were adequate by his criterion but badly distributed. The advanced nations possessed excessive reserves, while the developing nations suffered shortages. Heller's results have been criticized by Clower and Lipsey (1968, p. 591) as overstating the size of optimal reserves. They point out that, under Heller's formulation, the probability of using the last reserve units in the current year is quite low, and that these units should not be accumulated and

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[4]To attach a certain probability of usage to each reserve unit as Heller has done, one must suppose that reserve units are accumulated and used on a first-in, first-out (FIFO) basis. As reserve units are accumulated, the value of $E(R)$ will increase and the probability c of using the last acquired unit of reserves will fall.

held. For our purposes, the main contribution of Heller's study is his methodology, which provides an element missing in the models of Kelly (1970) and Clark (1970a).

Kelly, "The Demand for International Reserves"

Michael G. Kelly (1970) began his study on the demand for international reserves by setting out a utility-maximization model. The model is based upon the explanatory variables of disturbances Q, holding cost H, and adjustment cost A. Kelly assumed that disturbances are neutralized by a combination of reserve variability and income variability. The division of this task is shown by a policy parameter k (for Kelly), where a zero value places the whole burden upon reserves while unity places the whole burden upon income.[5] Paralleling Nagabhushanam and Sastry (1962), Kelly envisioned a current-account world where the level of exports is exogenously determined and can be expressed as a frequency function, and the government determines reserve holdings by its control over imports. The source of disturbances Q is export variability, as shown by $S(X)$ or $V(X)$; the cost of adjustment A is the reciprocal of the marginal propensity to import $(1/MPM)$. The variances of reserves and income are

$$V(R) = (1 - k)^2 V(X) \qquad 0 < k < 1 \tag{1.6}$$

$$V(Y) = (k/MPM)^2 V(X) . \tag{1.7}$$

To link mean reserves $E(R)$ with reserve variance $V(R)$, Kelly assumed that governments wish to maintain some low probability of reserve exhaustion c^*, contained in the distribution (1.5). He chooses the algebraic form

$$c^* = fV(R)/E(R)^2 \qquad f > 0 . \tag{1.8}$$

Substituting (1.6) into (1.8) yields

$$E(R)^2 = (f/c^*)(1 - k)^2 V(X) . \tag{1.9}$$

Solving (1.7) for k^2, substituting into (1.9), and simplifying,

$$E(R) = (f/c^*)^{1/2}[S(X) - (MPM)S(Y)] . \tag{1.10}$$

The cost of holding reserves H is the reduction in mean income $E(Y)$ incurred by holding part of the nation's assets as reserves instead of productive capital:

$$rE(R) = E(Y)_m - E(Y) , \tag{1.11}$$

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[5]Kelly's treatment of the policy parameter has been simplified here.

where r is the foregone rate of return and $E(Y)_m$ is the maximum possible income.

Kelly employed a quadratic utility function showing increasing marginal disutility to both income reductions and income variations:

$$U = -a[E(Y)_m - E(Y)]^2 - b[(Y) - E(Y)]^2 . \tag{1.12}$$

Substituting (1.11) into (1.12) yields

$$U = -ar^2E(R)^2 - bV(Y) . \tag{1.13}$$

The general solution to the model is obtained by using (1.7) to replace $V(Y)$ in (1.13), then maximizing (1.13) subject to (1.10) with respect to $E(R)$ and $S(Y)$. The optimal level of reserves $E(R)^*$ is

$$E(R)^* = S(X)/[(c^*/f)^{1/2} + (f/c^*)^{1/2}(MPM)^2r^2(a/b)] . \tag{1.14}$$

Optimal reserves vary directly with the measure of disturbances $S(X)$, directly with the cost of adjustment $(1/MPM)$, and inversely with holding cost r. Note that the solution is achieved only by specifying exogenously the desired probability of reserve exhaustion c^*.

Kelly turned next to a statistical test of his model. He assembled annual data for forty-six nations for the years 1953–65. He used as his measure of disturbances for each nation in each year the standard deviation of its exports $S(X)$ calculated from the preceding five-year period. He tried two separate indirect measures of holding costs based on capital scarcity: per capita national income and external dividend and interest payments and receipts. His rationale for using per capita income was that high per capita income should indicate plentiful capital (low marginal productivity). Income from external investment should also measure capital scarcity, because a nation rich in capital should be an exporter of capital. To measure adjustment cost, he used the average propensity to import APM. Finally, for reserves he used "official reserves" as defined by the IMF. The reserve-stock observation, for each country in each year, was a mean value calculated from the preceding period of one to six years. The length of the averaging period varied inversely with the nation's average propensity to import, to take into account lags in adjustment.

Kelly attempted regressions in both linear and logarithmic form. The logarithmic form yielded the best fit, and these are the results he reported. His first regressions used the entire forty-six-country sample over the thirteen annual cross-sections, and all coefficients were significant at the 5 per cent level, but the cost of adjustment (average propensity to import)

carried the wrong (positive) sign.[6] The sample nations were then split into (1) advanced countries and less developed countries and (2) relatively open countries and relatively closed countries. When the regressions were repeated on these subsamples, the cost of adjustment now acquired the correct (negative) sign; however, two measures of holding cost (per capita income and external dividend and interest payments) now carried incorrect signs. Only a few coefficients were nonsignificant. Regression runs on the annual cross-sections of the data yielded similar results. Kelly's study presents the most successful empirical results so far in the literature on the demand for reserves. He can be criticized primarily for omitting wealth as an explanatory variable, and for defining his source of disturbances as fluctuations in current-account expenditures. Even so, like Kenen and Yudin (1965), Kelly's measure of disturbances is the most successful variable in his regressions.

Clark, "Optimum International Reserves and the Speed of Adjustment" and "Demand for International Reserves: A Cross-Country Analysis"

Peter B. Clark (1970a) began his analysis of reserve demand by setting out a utility-maximization model based upon the explanatory variables of disturbances Q, holding cost H, adjustment cost A, and wealth W. He structured his model around the Kenen and Yudin (1965) procedure, decomposing the change in reserves for any period into two components: (1) a change in reserves due to a current random disturbance and (2) a change in reserves stemming from disturbances in previous periods. Clark identified the first component with the capital account and the second component with the current account. In respect to the current account, he assumed that the level of exports is exogenously determined and fixed in amount and that the government controls imports by varying national income. Thus the cost of adjustment A is again the reciprocal of the marginal propensity to import ($1/MPM$). The difference between export revenues and import expenditures shows the change in reserves for each period attributable to action by the government to compensate for disturbances in the capital account in previous periods. Clark represents this by a Koyck distributed-lag response equation, where the reserve stock R_t at the end of period t is equal to the inherited reserve stock R_{t-1} plus (1)

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[6]Kelly did not use ($1/APM$) in his regressions, but just APM; thus the proper sign is negative. However, it will be assumed in Chapter III that the propensity to import is not a measure of the cost of adjustment but a measure of "external vulnerability." On this assumption, the coefficient carries the correct (positive) sign.

the current random disturbance Q_t and (2) the proportion L of the difference between inherited reserves R_{t-1} and optimal reserves $E(R)*$:[7]

$$R_t = L[E(R)* - R_{t-1}] + R_{t-1} + Q_t$$
$$0 < L < 1 \qquad Q_t = Q[0, V(Q)] . \tag{1.15}$$

Clark assumed that disturbances were generated by a distribution with a zero mean and constant variance $V(Q)$. The relationship of $V(Q)$ to the variance of reserves $V(R)$ and the variance of income $V(Y)$ is

$$V(R) = V(Q)/L(2 - L) \tag{1.16}$$

$$V(Y) = LV(Q)/(2 - L)(MPM)^2 . \tag{1.17}$$

The cost of adjustment A enters equation (1.17) in the form $(1/MPM)^2$. When the speed of adjustment L is unity, the variance of income is maximized and income receives most of the burden in handling disturbances. When L is close to zero, the variance of income is very small, and reserves receive most of the burden in handling disturbances.[8] To provide a linkage between mean reserves $E(R)$ and reserve variance $V(R)$, Clark next defined c, the probability of reserve exhaustion, from (1.5). Clark does not follow Kelly in assuming c to be an exogenous parameter $c*$. To give an algebraic expression to (1.5), Clark used the Chebychev inequality as an equality:

$$c = V(R)/2E(R)^2 . \tag{1.18}$$

Substituting (1.16) into (1.18) yields

$$c = V(Q)/L(2 - L) 2E(R)^2 . \tag{1.19}$$

Clark defined the cost of holding reserves as

$$E(Y) = E(Y)_m - rE(R) , \tag{1.20}$$

where $E(Y)_m$ is maximum income, which depends upon the amount of wealth W, and r is the foregone rate of return on reserves.[9]

Clark next introduced a linear utility function of the form

$$U = a_0 + a_1E(Y) - a_2S(Y) - a_3c , \tag{1.21}$$

[7]Clark, like Kelly (1970) and Archibald and Richmond (1971), expresses reserve changes as a stock-stochastic equation rather than follow Kenen and Yudin (1965) and use a flow-stochastic equation. In equation (1.15), transfer R_{t-1} to the left of the equality sign and compare with (1.1).

[8]Compare Clark's policy parameter L in (1.16) and (1.17) with Kelly's policy parameter k in (1.6) and (1.7).

[9]Compare (1.18) with (1.8), (1.19) with (1.9), and (1.20) with (1.11).

which shows utility for increased income $E(Y)$ and disutility for income variations $S(Y)$ and higher levels of probability c for reserve exhaustion.[10] By entering c into his utility function, Clark did not have to assume, like Kelly, that c is an exogenous parameter. Substituting (1.17), (1.19), and (1.20) into (1.21), and maximizing with respect to $E(R)$ and L, yields the general solution of the model. Unfortunately, the solution is too complex to be reduced to a single equation for optimal mean reserves $E(R)^*$ like Kelly's (1.14). Instead, by total differentation Clark established that the proper relationships exist between optimal reserves and the independent variables. He also went on to establish the causal relationships between the speed of adjustment L and the independent variables:

$$E(R)^* = R(Q, H, A, W)$$
$$R_Q > 0 \; ; R_H < 0 \; ; R_A > 0 \; ; R_W > 0 \, . \tag{1.22}$$

$$L = (Q, H, A, W)$$
$$L_Q < 0 \; ; L_H > 0 \; ; L_A < 0 \; ; L_W < 0 \, . \tag{1.23}$$

Clark (1970b) then turned to an empirical test of his lagged-adjustment model. Since his solution was in terms of two variables, $E(R)$ and L, he needed to explain observed values for both. He used a sample of thirty-eight nations for the period 1958–67. Clark first defined his reserve variable as "official reserves" plus the IMF credit tranches. Using monthly data on "reserves," he calculated the linear trend in reserves for each sample nation. Defining $E(R)^*$ in (1.15) as the trend value, he then estimated the value of L for each nation. In one nation, the estimate of L was negative, and in seventeen it was not significantly different from zero at the 5 per cent significance level (indicating no adjustment whatsoever to disturbances). This procedure also provided estimates of the measure of disturbances $S(Q)$ for each nation.[11] The cost of adjustment was represented by the marginal propensity to import MPM, which was also estimated by a set of regressions. Wealth was represented by per capita income in 1963, and holding cost was deleted from the regressions. Finally, optimal reserves $E(R)^*$ was the mean reserve level for the period 1958–67.

Thus, Clark ran two sets of regressions, one for $E(R)^*$ and one for L. In the regression for optimal reserves, the cost of adjustment MPM took the

[10]Compare (1.21) with Kelly's (1.12). Note that Clark uses a linear utility function, while Kelly uses a quadratic utility function. The quadratic form is to be preferred, since it can show increasing or decreasing marginal utility.

[11]The procedure used by Clark is detailed below in the section on "Disturbances (Q)" in Chapter III.

wrong (positive) sign and only the disturbances term $S(Q)$ was significant at the 5 per cent level. In the regression for the lagged-adjustment parameter L, only MPM was significant, while $S(Q)$ took the wrong sign. Clark then scaled optimal reserves $E(R)^*$ and disturbances $S(Q)$ by the sum of exports and imports in 1963, so that the dependent variable became a type of reserve/trade ratio and, again, only the disturbances term was significant and MPM took the wrong sign. He split his sample into developed and less developed nations, reran the regressions, and had similar results. Attempts to manipulate the regressions on L were equally disappointing.

Clark's two studies present the most refined model of reserve demand in the literature, together with some successful empirical results. He can be criticized only for omitting holding cost from his regressions and for using per capita income to represent wealth W. Kenen and Yudin (1965) and Kelly (1970) had used per capita income as a measure of holding cost. Actually, it is a measure of the level of development and should not be used to represent wealth (an aggregate concept) or holding cost. It should be noted that Clark's most successful variable in his regressions was his measure of disturbances, which he borrowed from Kenen and Yudin.

Archibald and Richmond, "On the Theory of Foreign Exchange Reserve Requirements"

G. C. Archibald and J. Richmond (1971) followed the Kenen and Yudin (1965) approach, considering reserve changes to be a purely stochastic process of the form

$$R_t = a_0 + a_1 t + U_t \tag{1.24}$$

$$U_t = p U_{t-1} + Q_t$$
$$0 < p < 1 \qquad Q_t = N[0, V(Q)] , \tag{1.25}$$

where a_1 is the linear trend coefficient for reserves, and disturbances Q_t are normally distributed with a zero mean and a constant variance $V(Q)$. Under (1.24) and (1.25), the variance of reserves $V(R)$ is never stable at the limit, as is also true, it will be recalled, of the Kenen and Yudin formulation. Archibald and Richmond were interested in a purely stochastic treatment of reserve time-series data and therefore could deal with questions of reserve exhaustion only within some arbitrary time horizon. With a sample of fourteen trading nations over the period 1961–67, they determined that for the short term (twelve months) most of these nations had little risk of reserve exhaustion. Their results show that, in the absence of a lagged-adjustment process such as that used by Clark (1970a),

the probability of reserve exhaustion can never be stabilized for the long term.

Other Studies

Three additional studies need to be singled out of the literature because of their importance to the present study. The first is by H. Robert Heller, "Wealth and International Reserves" (1970). Heller suggested that the aggregrate wealth W of a nation could be a main determinant of reserve demand. As a nation acquires greater aggregate wealth, its demand for all types of assets should increase. However, as data on aggregate wealth is scarce, Heller employed national income as a proxy variable. He regressed reserve holdings for several nations against national income, with statistically significant results. His estimated coefficients conformed with the unitary elasticity-of-demand hypothesis for wealth effects.

The second study is by Robert G. Hawkins and C. Rangarajan, "On the Distribution of New International Reserves" (1970). The authors sought to explain IMF quotas by a multiple regression upon a number of explanatory variables. They identified the three major determinants of reserve demand as being wealth W, cost of adjustment A, and external disequilibria Q. As measures for these explanatory variables, they used population and per capita income for wealth, degree of industrialization and openness of the economy and degree of export concentration for the cost of adjustment, and the sum of current-account receipts and expenditures for the measure of disequilibria (a transactions approach). Hawkins and Rangarajan then regressed IMF quotas upon their variables but found that their coefficient estimates were statistically nonsignificant. From this finding, they went on to argue that the IMF quotas do not properly reflect the need for reserves by member nations. What makes this study important from our viewpoint is the selection of various measures for the explanatory variables.

The third and last study is by June Flanders, *The Demand for International Reserves* (1971). Flanders sought to explain observed differences in reserve/import ratios by regressing these ratios on ten explanatory variables: the variability of exports, the level of near-reserves, the holding cost for reserves, the rate of return on invested reserves, the variability of reserves, the willingness to adjust by altering the exchange rate, the cost of adjustment by other methods, the level of inventories of traded goods, the cost of borrowing, and the level of income. The results of the various regressions were disappointing. Nevertheless, Flanders still asserted that

nations do hold the reserves they desire in the long run. This study is valuable for its suggestions of additional explanatory variables and the statistical series to represent explanatory variables.

II. A GENERAL MODEL OF RESERVE DEMAND

The Explanatory Variables

This chapter will develop a formal model of a nation's demand for international reserves. Based upon the theoretical discussion in the literature, the demand for reserves may be considered to be a function of four explanatory variables: (1) a measure Q of disturbances, (2) a measure H of the cost of holding reserves, (3) a measure A of the cost of adjustment to disturbances, and (4) a measure W of the nation's aggregate wealth. The level of reserves is expected to vary directly with disturbances Q, inversely with the cost of holding reserves H, directly with the cost of adjustment A, and directly with aggregate wealth W. The demand for reserves, as shown by the mean reserve stock $E(R)$, would be

$$E(R) = R(Q, H, A, W)$$
$$R_Q > 0, R_H < 0, R_A > 0, R_W > 0. \tag{2.1}$$

The model of reserve demand will be developed in the following stages: First, the lagged-adjustment process to disturbances will be described. Then, with the speed of adjustment exogenously given, the level of reserves that maximizes utility will be found. Finally, the speed of adjustment will be made part of the general solution of the model.

The Lagged-Adjustment Process to Disturbances

In each period t there will be some disturbance Q_t generated by a normal distribution with a zero mean and a constant and independent variance $V(Q)$ over time:

$$Q_t = N[0, V(Q)]. \tag{2.2}$$

The mean is set to zero so that the process described below will be purely random in a stochastic sense. A nonzero mean for Q_t would result in a secular change in the reserve stock, and such changes are best treated as a trend component. The presumption that Q is normally distributed is based upon the point made by Kenen and Yudin (1965, pp. 243–244) that Q may be considered to be a sum of separate transactions, where each transaction is subject to a stochastic disturbance. While each transaction may not be drawn from a normal population, the central limit theorem

suggests that the sum of the transactions will have a normal distribution. Archibald and Richmond (1971, pp. 246–247) have also made this point.

Assume that the reserve stock is at the optimal level $E(R)^*$ at the beginning of period 1, and during period 1 a disturbance of size Q occurs. At the end of period 1 the reserve stock will be $[E(R)^* + Q]$. In period 2, the government will undertake to adjust the inherited reserve stock back toward $E(R)^*$. It may attempt to do this in period 2 alone, or more likely it will spread the adjustment across several periods. In each period, it will be necessary to vary income by a multiple A of the desired effect upon the balance of payments (and the reserve stock) in that period. The cost of adjustment A may be defined as the change in income Y necessary to change the balance of payments (and the reserve stock) by one monetary unit. If several periods are taken to return R to the $E(R)^*$ level, it will not be necessary to adjust income by the full amount (AQ), since there is also some stochastic tendency in later disturbances for reserves to return to the mean level.

Thus we can envision a disturbance occurring in one period followed by adjustment spread out over subsequent periods. Following Clark (1970a, pp. 358–361), this can be shown by a Koyck lagged-adjustment process:

$$R_t' = L[E(R)^* - R_{t-1}] + R_{t-1} \qquad 0 < L < 1 , \tag{2.3}$$

where R_t' is the target reserve level for the end of period t, R_{t-1} is the inherited reserve stock, $E(R)^*$ is the optimal mean level of reserves, and L is the lagged-adjustment coefficient. When $L = 1$, the entire adjustment to the disturbance is accomplished in one period. When $0 < L < 1$, the adjustment will be spread over several periods, and when $L = 0$, there is no adjustment whatsoever. Reserves at the end of t, however, will not be at the target level of R_t' owing to the need to neutralize the disturbance that will occur in t. Actual reserves at the end of t will be R_t:

$$R_t = R_t' + Q_t . \tag{2.4}$$

Combining (2.3) and (2.4) yields the basic lagged-adjustment equation for the model:[1]

$$R_t = L[E(R)^* - R_{t-1}] + R_{t-1} + Q_t . \tag{2.5}$$

Following Clark (1970a, pp. 361, 365), the variance of reserves $V(R)$ and

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[1]Equation (2.5) is the same as (1.15). Several of the equations from Chapter I are used in this chapter as well but are given new numbers. This is done so that the model may be considered as a separate whole.

the variance of income $V(Y)_1$ necessary to establish and maintain $V(R)$ are derived from (2.5):[2]

$$V(R) = V(Q)/L(2 - L) \tag{2.6}$$

$$V(Y)_1 = A^2 L V(Q)/(2 - L) \qquad A > 0 . \tag{2.7}$$

Figures 1(d) and 1(e) below show the relationship between the lagged-adjustment parameter L and $V(Y)_1$ and $V(R)$. When $L = 1$, all the adjustment to a disturbance occurs in the following period. Reserve variance is minimized at the level of $V(R) = V(Q)$, while income variance is maximized at the level of $V(Y)_1 = A^2V(Q)$. When $L = 0$, there is no adjustment by varying income, and so $V(Y)_1 = 0$ while $V(R)$ is undefined. Therefore, to establish some defined level of $V(R)$, it is necessary to have some variation in income as shown by $V(Y)_1$.

Within the framework of this lagged-adjustment process, a government will want to amass that reserve stock which will maximize social welfare. Following Kelly (1970, p. 659), social welfare can be shown by the quadractic utility function[3]

$$U = - a[E(Y)_m - E(Y)]^2 - b[(Y) - E(Y)]^2 , \tag{2.8}$$

where social utility U is a decreasing function a of reductions in mean income $E(Y)$ below maximum income $E(Y)_m$ and a decreasing function b of variations in income (Y) about the mean level $E(Y)$. Equation (2.8) can be restated as

$$U = - a[E(Y)_m - E(Y)]^2 - bV(Y) , \tag{2.9}$$

which will be the utility function used in this model. The community indifference curves generated by (2.9) are shown in Figures 1(a) and 2(a) below as curves U_1, U_2, and U_3, where $U_1 > U_2 > U_3$.[4]

The maximization of social welfare under the lagged-adjustment process to disturbances is best approached in two stages. First, treating the

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[2]Equation (2.6) is the same as (1.16). Equation (2.7) should be compared with Clark's (1.17). Note that the cost of adjustment is shown in the abstract as A^2 in (2.7), in contrast to the specific form of $(1/MPM)^2$ in (1.17). Also note that the income variance in (2.7) is subscripted as $V(Y)_1$ to identify its relationship with reserve variance $V(R)$ in equation (2.6). Equation (2.15) below will introduce the second form of income variance, $V(Y)_2$.

[3]Equation (2.8) is the same as (1.12).

[4]The origin for the community indifference curves is the northwest corner of the graph at $E(Y)_m$. By portraying the utility curves in this manner, the inverse relationship between $E(R)$ and $E(Y)$ can be shown in Figure 1(b), which represents the cost of holding reserves. See Kelly (1970, p. 659) for a different geometric treatment omitting the holding-cost graph.

value of L as exogenously given at $L = L'$, one can find a *desired* mean level of reserves $E(R)'$ that will maximize social welfare. This first stage is shown in Figure 1 below. Second, permitting L to vary in its range of zero to unity, one can choose from the desired mean levels of reserves that level which yields the highest social welfare of all. This will be the *optimal* mean level of reserves $E(R)^*$. This procedure yields the optimal rate of adjustment L^* as part of the general solution of the model. This second stage of the analysis is shown in Figure 2 below.

Finding Desired Mean Reserves

By fixing the lagged-adjustment parameter at $L = L'$, the values of $V(R)$ and $V(Y)_1$ are set at $V(R)'$ and $V(Y)_1'$ by equations (2.6) and (2.7). It is important to note that the income variance $V(Y)_1'$ is necessary to stabilize reserve variance at $V(R)'$. This is shown in Figures 1(d) and 1(e). The levels of $V(Y)_1$ and $V(R)$ are independent stochastically of the level of mean income $E(Y)$ and mean reserves $E(R)$. The structural linkage between them is provided by the probability of reserve exhaustion. When a nation holds a mean reserve stock $E(R)$ and faces a reserve variance $V(R)$, the probability c that the stock will be exhausted is[5]

$$c = P[R < 0 \mid E(R), V(R)] \qquad c > 0 . \tag{2.10}$$

The relationship of c to $E(R)$ and $V(R)$ may be shown as[6]

$$c = fV(R)'/E(R)^2 \qquad f > 0 . \tag{2.11}$$

Given the variance of reserves $V(R)'$ set by $L = L'$, the probability of reserve exhaustion c will decline as $E(R)$ becomes larger. Following Heller (1966, pp. 297–304), the government should amass reserves on a cost-benefit basis until social welfare is maximized.

The cost of holding reserves is measured in terms of reduced income. The maximum mean level of income $E(Y)_m$ is

$$E(Y)_m = rW \qquad r > 0 , \tag{2.12}$$

where r is the income yielded by a unit of wealth W. By holding some of a society's wealth as reserves, not as income-producing assets, mean income will be reduced by[7]

$$E(Y)_m - E(Y) = rE(R) . \tag{2.13}$$

[5]Equation (2.10) is the same as (1.5) and is also found in Kelly (1970, p. 658) and Clark (1970a, p. 363).

[6]Compare (2.11) with Kelly's (1.8) and Clark's (1.18).

[7]Equation (2.13) is the same as Kelly's (1.11) and Clark's (1.20).

FIGURE 1

THE GENERAL MODEL: FINDING DESIRED MEAN RESERVES
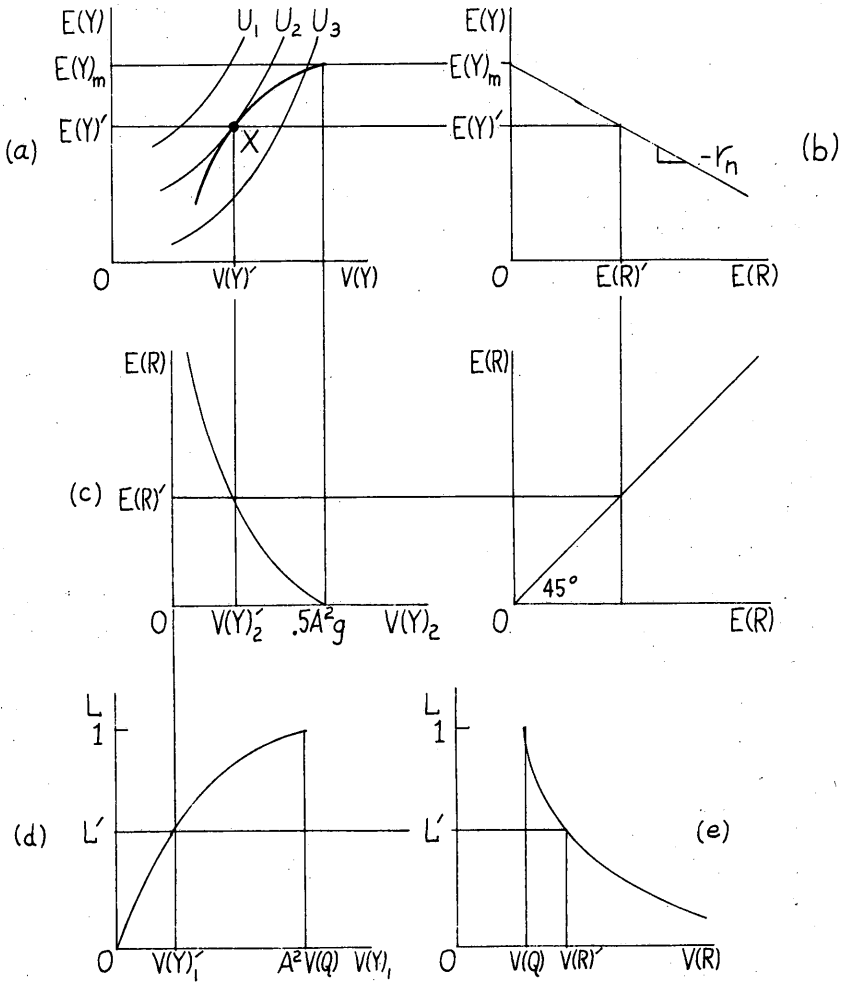
However, the reserve stock is not wholly "sterile," since part of the reserve stock — the foreign-exchange component — is usually invested at some rate of return. This will reduce r by some amount, making the actual cost of holding reserves

$$E(Y)_m - E(Y) = r_n E(R) \,, \tag{2.14}$$

where r_n is the "net" cost of holding each reserve unit. This is shown in Figure 1(b), where the transformation curve between income $E(Y)$ and reserves $E(R)$ has an intercept of $E(Y)_m$ and a slope of $-r_n$. The transformation curve for $E(Y)$ and $E(R)$ will move upward as (1) holding cost r_n is lowered or (2) the wealth W of the society increases.[8]

The benefit of holding reserves is to minimize the reductions in income necessary to eliminate deficits that would exhaust the reserve stock. These variations in income to cover deficits will create a second variance in income $V(Y)_2$. The level of this second variance will be a direct function of the probability c of reserve exhaustion:[9]

$$V(Y)_2 = A^2 gc \qquad g > 0 . \tag{2.15}$$

Total income variance $V(Y)$ is the sum of $V(Y)_1$ and $V(Y)_2$:

$$V(Y) = V(Y)_1 + V(Y)_2 . \tag{2.16}$$

Under the provisional assumption that $L = L'$, the income variance $V(Y)_1$ — which stabilized reserve variance — is set at $V(Y)_1 = V(Y)_1'$, and may be ignored in this part of the analysis.

Substituting (2.11) for c in (2.15) yields

$$V(Y)_2 = A^2 gfV(R)'/E(R)^2 , \tag{2.17}$$

where $V(Y)_2$ is an *inverse* function of the mean level of reserves. This is shown in Figure 1(c). Substituting (2.17) for $V(Y)_2$ in (2.16), and noting that $V(Y)_1 = V(Y)_1'$, yields

$$V(Y) = V(Y)_1' + A^2 gfV(R)'/E(R)^2 , \tag{2.18}$$

where $V(Y)$ is now an inverse function of the mean level of reserves. Equation (2.18) represents the benefit of holding reserves. A larger reserve stock reduces the variability of income.

Comparing equations (2.13) and (2.18), it is clear that, for each level of reserves $E(R)$, there is a corresponding unique combination of mean income $E(Y)$ and income variance $V(Y)$. The locus of these combinations is

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[8]This is not, however, where wealth W enters the demand function for reserves. It enters instead through the parameter a of the utility function (2.9). See the discussion below concerning equations (2.20) to (2.22) and also (2.26) and (2.27).

[9]The second variance in income $V(Y)_2$ is caused by the distribution of Y becoming negatively skewed to cover deficits that would exhaust the reserve stock. Also, the level of $E(Y)$ would be lowered and there would be induced effects upon $E(R)$ and $V(R)$. However, as $E(R)$ becomes larger, the distributions of Y and R would become symmetrical, and the induced effects would diminish. To simplify matters, this complex process is shown only by increases in $V(Y)$ over $V(Y)_1$. At the limit, $V(Y)$ would approximate $V(Y)_1$, as shown in Figure 2(c) below.

derived in Figure 1(a), where the combination at point X yields the highest level of welfare. From X can be found the desired level of reserves $E(R)'$ in Figure 1(b).

In algebraic terms, after several substitutions into the utility function (2.9), the following equation is being maximized:

$$U = - ar_n^2 E(R)^2 - bA^2 L'V(Q)/(2 - L')$$
$$- bA^2 gfV(Q)/L'(2 - L')E(R)^2 . \tag{2.19}$$

Maximizing (2.19) with regard to $E(R)$ yields desired mean reserves $E(R)'$ when L is fixed at L':

$$E(R)' = [bA^2 gfV(Q)/ar_n^2 L'(2 - L')]^{1/4} . \tag{2.20}$$

Comparing equation (2.20) with equation (2.1), it can be seen that the dependent variable $E(R)'$ has the proper relationship to the exogenous variables $V(Q)$, r_n, and A, but wealth W is apparently missing. This is because the effect of wealth upon reserve demand enters through the parameter a of the utility function (2.9) for reductions in mean income. In wealthier nations, the disutility of a unit loss in income will be lower than in a less wealthy nation. Note that the concept of wealth used here is aggregate wealth, not per capita relative wealth. The relationship between the disutility parameter a and aggregate wealth W is of the form

$$a = h/W \qquad h > 0 . \tag{2.21}$$

Substituting (2.21) into (2.20) yields

$$E(R)' = [bA^2 gfV(Q)W/hr_n^2 L'(2 - L')]^{1/4} . \tag{2.22}$$

Now all exogenous variables are represented in their proper relationship to $E(R)'$.

The model presented so far and shown in Figure 1 is Heller's (1966) analysis within the Clark (1970a) and Kelly (1970) framework. Heller's model is actually a special case of the general model in which a stable variance for reserves $V(R)'$ has been established by the income variance $V(Y)'_1$.

Finding Optimal Mean Reserves

The second stage of the analysis removes the constraint on the value of the lagged-adjustment parameter L, permitting it to vary across its range from zero to unity. As L varies, the point X in Figure 1(a) will trace out a curve showing the desired values for $E(Y)$ and $V(Y)$ for each level of L. There is a unique set of these desired values for each L value, which is

FIGURE 2

THE GENERAL MODEL: FINDING OPTIMAL MEAN RESERVES
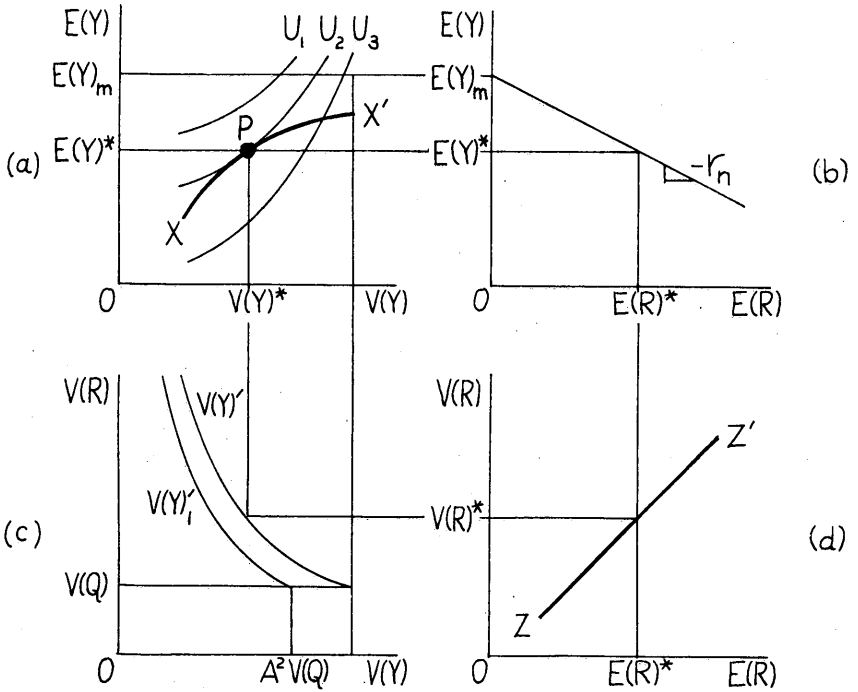
shown as the XX' curve in Figure 2(a). The ZZ' curve in Figure 2(d) shows the companion values of $E(R)'$ and $V(R)'$ for each level of L. Comparing the XX' curve with the social indifference map in Figure 2(a) shows which of the *desired* solutions is also the *optimal* solution yielding the highest possible welfare.

The transformation curve $V(Y)'$ in Figure 2(c) relates $V(R)$ on the ZZ' curve with $V(Y)$ on the XX' curve. It is derived as follows: For each level of L, there is a unique pair of values for $V(R)$ and $V(Y)_1$. The locus of these values is the $V(Y)_1'$ curve in Figure 2(c). The $V(Y)_1'$ curve will shift with the variance of disturbances $V(Q)$ and the cost of adjustment A. Note that $V(Y)_1 = A^2V(Q)$ when $V(R) = V(Q)$, and that $V(Y)_1$ approaches zero as $V(R)$ becomes larger. This relationship is of the form

$$V(Y)_1 = [V(Q)/V(R)] \, A^2V(Q) = A^2V(Q)^2/V(R) . \qquad (2.23)$$

In addition, at each level of $V(R)$ there is a unique desired level of $V(Y)_2'$:

This can be calculated by substituting $E(R)'$ from (2.22) into (2.17). However, this form of income variance will be very small and is shown as the interval between $V(Y)'_1$ and total income variance $V(Y)'$. The $V(Y)'$ curve in Figure 2(c) replaces Figures 1(c), 1(d), and 1(e), and permits the direct comparison of the XX' and ZZ' curves through the $V(R)$ and $V(Y)$ linkage.

The tangency solution at point P in Figure 2(a) is the general solution for the model. From point P can be read the optimal values for mean reserves $E(R)^*$, mean income $E(Y)^*$, reserve variance $V(R)^*$, and income variance $V(Y)^*$. Recalling that the optimal solution is that desired solution which yields the greatest social welfare, the optimal value L^* of the lagged-adjustment parameter can be found by substituting $V(R)^*$ into equation (2.6).

It is also possible to achieve an explicit algebraic solution for the model. Substituting (2.17) and (2.23) into (2.16), $V(Y)$ can be written as a function of both $V(R)$ and $E(R)$:

$$V(Y) = A^2V(Q)^2/V(R) + A^2gfV(R)/E(R)^2 . \tag{2.24}$$

Substituting (2.14) and (2.24) into the utility function (2.9), social utility U can be written as a function of mean reserves $E(R)$ and reserve variance $V(R)$:

$$U = - ar_n^2E(R)^2 - bA^2V(Q)^2/V(R) - bA^2gfV(R)/E(R)^2 . \tag{2.25}$$

Maximizing (2.25) with respect to $E(R)$ and $V(R)$, then solving for $E(R)$, one can obtain an explicit algebraic solution for optimal mean reserves:[10]

$$E(R)^* = [V(Q)^2b^2A^4gf/a^2r_n^4]^{1/6} . \tag{2.26}$$

Using equation (2.21) to introduce wealth W yields

$$E(R)^* = [V(Q)^2b^2A^4gfW^2/h^2r_n^4]^{1/6} , \tag{2.27}$$

where all the major explanatory variables — disturbances $V(Q)$, wealth W, adjustment cost A, and holding cost r_n — are represented in the proper relationship to $E(R)^*$.

The Relationship of the General Model to Earlier Models in the Literature

The general model of the demand for international reserves developed above is a synthesis and expansion of the models of Heller (1966, pp.

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[10]The solution to the general model is a conditional maximum, as shown by the second-order conditions. The first minor is $[-2ar_n^2 - 6bA^2gfV(R)E(R)^{-4}]$, which is clearly a negative quantity. The second minor is $\{4abr_n^2A^2V(Q)^2V(R)^{-3} + 4b^2A^4gfE(R)^{-4}[3V(Q)^2V(R)^{-2} - gfE(R)^{-2}]\}$, of which the sign is ambiguous. The sign will be positive if at the very least $[3V(Q)^2V(R)^{-2} > gfE(R)^{-2}]$, and this inequality is likely to hold.

297–304), Kelly (1970, pp. 657–660), and Clark (1970a). Both Kelly and Clark derived their reserve demand functions from general-equilibrium models by maximizing a utility function. Kelly achieved an explicit algebraic solution, while Clark had to settle for an implicit solution of the demand equation for reserves. However, Kelly's explicit solution was achieved by introducing an unnecessary constraint into the model. He assumed that the probability of reserve exhaustion c was exogenously specified and used this as a constraint upon utility maximization. This procedure was similar to the first stage above, where the lagged-adjustment parameter was exogenously set at $L = L'$.

Clark went further and achieved a general and unconstrained solution for his model. He did this by entering the probability of reserve exhaustion c into his utility function as a third parameter, in addition to income reductions and income variations. He maximized his utility function with respect to mean reserves $E(R)$ and the lagged-adjustment parameter L. His solution was so complex, however, that he had to settle for an implicit statement of the reserve demand function, and he was led to conclude that he was dealing with a simultaneous solution in the two variables $E(R)$ and L, both of which had to be explained by his regressions (1970b).

The model of reserve demand developed above shows that an explicit general solution is possible in terms of $E(R)$ alone. This is made possible by two modifications of the analysis. First, following Heller's approach (1966, pp. 297–304), the benefit of holding reserves is expressed as reduced income variability $V(Y)_2$. Clark's income variability is the $V(Y)_1$ term in equation (2.7) necessary to stabilize $V(R)$. He never realized that overall income variability is of the form shown in equation (2.16). By introducing into the analysis the term $V(Y)_2$ from equation (2.15), income variance $V(Y)$ can be represented as an inverse function of the mean level of reserves $E(R)$. This eliminates the need to follow Kelly in assuming that the probability of reserve exhaustion c is an exogenous constant, or to follow Clark and make c a third parameter of the social-utility function. The second modification in the analysis was to eliminate the L parameter, and the consequently complex algebraic formulations, by introducing equation (2.23) into the general solution and taking advantage of the fact that, for every value of L, there is a unique pair of values for $V(R)$ and $V(Y)_1$. These two modifications permitted the explicit general solution for the demand for reserves in the form shown by (2.27).

III. THE ESTIMATION OF THE DEMAND
FOR INTERNATIONAL RESERVES

Quantification of the Variables

In Chapter II, the model of reserve demand was developed in terms of four explanatory variables: (1) a measure Q of disturbances; (2) a measure H of the cost of holding reserves; (3) a measure A of the cost of adjustment to disturbances; and (4) a measure W of the nation's aggregate wealth. The empirical work will add four new explanatory variables to the list: (5) a measure D of the level of economic development; (6) a measure V of the degree of external vulnerability; (7) a measure F of the degree of exchange-rate flexibility; and (8) a measure T of expectations regarding the future need for reserves.

Thus, at the outset, the demand for reserves, measured by the mean stock of reserves $E(R)$, will be written as

$$E(R) = R(Q, H, A, W, D, V, F, T)$$
$$R_Q > 0 \; ; R_H < 0 \; ; R_A > 0 \; ; R_W > 0 \; ;$$
$$R_D > 0 \; ; R_V > 0 \; ; R_F < 0 \; ; R_T > 0 \; . \tag{3.1}$$

The four explanatory variables used in the formal model of reserve demand seem to be the most important, based upon a priori analysis and previous empirical work. The new explanatory variables added here are to make the first set of regressions a "state of the art" attempt to explain empirically the demand for reserves. The level of economic development D is widely discussed in the literature (see Flanders, 1971), while external vulnerability V (the openness of the economy) is suggested by certain writers and by empirical results. Exchange-rate flexibility F determines the basic need for reserves, and expectations T is an effort to introduce dynamic considerations into an otherwise static analysis.

The first section of this chapter will discuss the quantification of these nine variables. This is quite easy in some cases, but somewhat remote proxy variables must be employed in others. The second section of the chapter contains the regression equations testing the explanatory variables for statistical significance. The chapter ends with a discussion of the contributions of the study as a whole.

The data for the variables are taken from International Monetary Fund and World Bank sources. The variables in the regressions are either

monetary magnitudes or pure numbers. The monetary magnitudes are in terms of U. S. dollars. Since the regressions will be cross-sectional across nations, data averaging is required so that the value of each variable for each nation can be considered a "typical" value. The six-year period 1960–65 was selected for averaging purposes in order to use fully certain World Bank data. Also, this was a period of general currency convertibility and international financial stability. A sample of sixty-one nations is available for this period.

Reserves (R). This variable is the dependent variable and is not difficult to quantify. This study will follow the usual practice in the literature and use the IMF definition of "official reserves." Official reserves include a government's holdings of gold and convertible foreign exchange together with its super gold tranche, gold tranche, and SDR credit lines with the IMF. The agencies holding reserves for a government include the central bank, ministries of finance, and sometimes certain other public financial institutions. The only significant exception to this practice in the literature is Clark (1970b, pp. 583–584), who adds the IMF credit tranches to official reserves.

In the demand function, reserves have been defined as an average level $E(R)$. Since reserve levels are changing continuously, the reserve stock at any point in time cannot be considered to be "typical" of the level of reserves demanded and held by a nation. Averaging is called for, and one must then decide how often to observe the variable within the averaging period. Official reserve data are published on the bases of end-of-month, end-of-quarter, and end-of-year. To determine the appropriate frequency, a sensitivity test was conducted on data for three countries, using end-of-month, end-of-quarter, and end-of-year observations over the six-year period. There was no real difference between the end-of-month and end-of-quarter mean reserve levels, but end-of-year mean reserve levels were markedly different. Thus the mean of end-of-quarter observations is used as the mean reserve level for each nation in the sample. The reserve variable, of course, is a monetary magnitude.

Disturbances (Q). The measure of disturbances is derived by a technique first worked out by Kenen and Yudin (1965) and perfected by Clark (1970b, pp. 579–583). Following Clark, equation (2.5) can be restated as

$$R_t = LE(R)^* + (1 - L)R_{t-1} + Q_t \,. \tag{3.2}$$

Now, let $E(R)^*$ be the trend value of reserves at t:

$$E(R)^*_t = R_0 + qt \,. \tag{3.3}$$

Substituting into (3.2) yields

$$R_t = LR_0 + Lqt + (1 - L)R_{t-1} + Q_t . \qquad (3.4)$$

This modification is necessary to set the expected disturbance equal to zero in accordance with (2.2).[1] Equation (3.4) is of the form

$$R_t = a_0 + a_1t + a_2R_{t-1} + e_t . \qquad (3.5)$$

Used as a regression equation, equation (3.5) yields estimates of the a_i and the parameters of the error distribution for each nation. To parallel the calculation of mean reserves, end-of-quarter data over the six-year averaging period were used in the regression for each nation. The parameter a_2 in equation (3.5) is an estimate of $(1 - L)$ in equation (3.4), and the distribution of e_t is an estimate of the distribution of Q_t. To be precise, $V(e)$ is an estimate of $V(Q)$, and $S(e)$ is an estimate of $S(Q)$. The standard deviation $S(e)$ of the e distribution is used in preference to the variance $V(e)$ as the measure of Q so that Q will be expressed on a scale comparable to the mean reserve levels $E(R)$. The Q variable is a monetary magnitude and is directly related to the dependent variable $E(R)$.

Holding Cost (H). Holding cost is the opportunity cost of holding part of the nation's wealth as reserves rather than as physical capital. Holding cost should be expressed in marginal terms and will equal the rate of return lost by not transferring a reserve unit into real capital.[2] The appropriate rate of return is that on newly invested capital — the marginal efficiency of investment *MEI*. The overall relationship between reserve holdings and holding cost is inverse; a higher cost will reduce the mean level of reserves demanded by a nation.

[1]Equation (3.4) should be compared with Kenen and Yudin's (1.2). Clark handles reserve changes $(R_t - R_{t-1})$ with a stock-stochastic equation by placing R_{t-1} to the right of the equality sign.

[2]It has been noted by certain writers (Heller, 1966, p. 299; Flanders, 1971, pp. 27–28; also the section on "Finding Desired Mean Reserves" in Chapter II above) that the cost of holding reserves is a "net" cost. The foreign-exchange component of reserves is usually held as short-term paper in some financial center, which will offset in part the foregone income from not investing reserves into physical capital. Heller assumed that any such premium earned by foreign exchange would be cancelled by the loss in liquidity; thus, the proper measure for holding cost would be the unadjusted *MEI* used here. However, to test the possibility that a "net" measure of holding cost would be superior, two "net" measures of H have been tried. They are (1) $H = (1 - f^*)(MEI)$, and (2) $H = (MEI) - r'f^*$, where f^* is the proportion of reserves held as foreign exchange over 1960–65 and r' is the average of ninety-day bill rates in London and New York over the period. Neither measure was any more successful than *MEI* alone.

No study in the literature has ever attempted directly to estimate *MEI* values for the cost of holding reserves H. Instead, various proxy variables for capital scarcity have been employed. Yet a statistical proxy for *MEI* is available from World Bank data. It is the reciprocal of the "gross marginal capital-output ratio." For the period 1960 through 1965, this ratio is the difference in gross national product between 1960 and 1965 divided by aggregate gross domestic investment for the period 1959 through 1964. The one-year lag is to allow for start-up time on newly implanted capital. This series attributes all increases in output to the capital input and therefore overstates the marginal efficiency of investment. However, the series would still be expected to capture *relative* differences in capital productivity at the margin. This estimate of *MEI*, used in the regressions to represent H, is a pure number and is inversely related to the dependent variable.

Adjustment Cost (*A*). Adjustment cost, like holding cost, is a marginal concept. It is the variation in income Y necessary to neutralize one dollar of disturbance. The relationship between adjustment cost and the mean level of reserves is direct. High adjustment cost means that a substantial swing in national income is necessary to neutralize a disturbance, and a nation would want to hold larger reserves as protection against such disequilibria.

It is usual to classify adjustment policies into expenditure-changing policies and expenditure-switching policies, and each of them has its costs. The studies by Heller (1966, pp. 297–299), Clark (1970a, pp. 357–360), and Kelly (1970, pp. 657–658) have taken a purely Keynesian approach to measuring adjustment cost. The cost is the change in income required to alter expenditure on imports sufficiently to eliminate the disturbance, without allowance for internally induced changes in exports or foreign repercussions on the balance of payments. The analytical measure of adjustment cost becomes the reciprocal of the marginal propensity to import ($1/MPM$), while in empirical work, the average propensity to import *APM* is often used in place of the marginal propensity. The smaller the propensity, the larger the changes in income needed to effect adjustment, and the larger optimal holdings of reserves should be. However, empirical results using expenditure-changing measures of the cost of adjustment have been disappointing. Moreover, governments seem reluctant to manipulate income to achieve balance-of-payments effects when easier and less costly methods of relief are available via expenditure switching affecting primarily the terms of trade. Therefore, this study will

measure adjustment cost in terms of expenditure-switching policies.[3] Unfortunately, no direct measure of the cost of adjustment by expenditure switching is available, and so a proxy variable must be employed.

Hawkins and Rangarajan (1970, pp. 884–886) suggest that the expenditure-switching cost of adjustment will vary inversely to the average proportion of GNP originating in manufacturing. This measure embodies the presumption that an industrialized nation will possess greater internal factor mobility and hence have a lower cost of adjustment. Thus the cost of adjustment A under expenditure-switching policies will be measured by the average proportion of output *not* orginating in manufacturing over the sample period. The cost of adjustment is a pure number and is directly related to the level of reserves.

Wealth (W). A recent development in the literature has been to consider the demand for reserves as a function of a nation's total wealth. Wealth can be viewed as a portfolio of assets, including reserves, and, as wealth increases, the demand for each asset may be expected to increase. Thus the relationship between total wealth and reserve holdings is direct. Since wealth is the abstract measure of the real productive capacity of a society, some measure of output is a good proxy for wealth. Following Heller (1970), this will be mean GNP over the period. The wealth variable is a monetary magnitude and is directly related to the dependent variable $E(R)$.

Development (D). It is the usual practice in the literature to assume that the reserve-holding behavior of the advanced countries is essentially different from that of the less developed countries. The more advanced nations are supposed to hold proportionately larger reserves as a result of their level of development. This is apart from the level of aggregate wealth W or the level of industrialization used to represent the cost of adjustment A. It may be deemed a "relative wealth" effect upon reserve holdings. The usual approach to this question is to section the sample nations into a group of advanced nations and a group of less developed nations to see what differences emerge in the estimated coefficients of the subgroupings. This procedure will also be followed here; however, it would be interesting to see if the effect of economic development upon reserve holdings can be captured by a specific variable representing

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[3]An effort to include the cost of adjustment via expenditure-changing methods in the regressions was unsuccessful, and the results are not reported. The marginal propensity to import *MPM* was calculated as the ratio of the change in imports of goods and services between 1960 and 1965 over the change in GNP for the same period.

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economic development. The level of economic development can be shown by the mean income per capita over the averaging period 1960–65. Per capita income is a monetary magnitude and should be directly related to the level of mean reserves $E(R)$.

External Vulnerability (V). The vulnerability of an economy to external disruption will depend upon the openness of the economy. The more open the economy is, the larger the reserve stock should be to act as a buffer against external forces (Altman, 1956, pp. 136–140). Following the practice in the literature, this study will use the average propensity to import *APM* for each nation over 1960–65 to represent the degree of openness of its economy. External vulnerability V is a pure number and is directly related to mean reserves $E(R)$.

Some earlier studies have used *APM* as a measure of the cost of adjustment. Heller (1966) and Kelly (1970) have used *APM* to represent the cost of adjustment under expenditure-changing policies. Since *APM* is related inversely to cost, the relation of *APM* and reserves should also be inverse. Kelly had ambiguous empirical results, with *APM* often taking the incorrect (positive) sign. In this study, as already discussed, the cost of adjustment will be represented by a proxy variable for cost under expenditure-switching policies. However, Hawkins and Rangarajan (1970, pp. 885–886) have suggested that *APM* could also be a proxy for *expenditure-switching* cost of adjustment. They argue that a more open economy would have a higher cost of adjustment under expenditure-switching policies, since the policies would affect a wider segment of the economy. In this case, the proper sign for *APM* would be positive, and *APM* would represent the cost of adjustment. This idea must be rejected; a more open economy would have a lower cost of adjustment, since adjustment policies would be effective in a broader segment of the economy. This is the argument behind the assumption that nations with a higher marginal propensity to import have a lower cost of adjustment. If *APM* is to be used as a measure of the cost of adjustment, the proper sign is negative. The correct role for *APM* seems to be as a measure of vulnerability, where the proper sign is positive.

Exchange-Rate Flexibility (F). If a nation is willing to change its exchange rate frequently, it will have a reduced need for reserves. The nation's exchange-rate policy can be shown by an index of exchange-rate flexibility. Unfortunately, the index is rather crude and is based upon a subjective evaluation of actual exchange-rate policy. A zero value represents a firm dedication to fixed exchange rates; a value of two means a willingness to use parity change or even flexible exchange rates to achieve

adjustment. A value of one indicates an exchange-rate policy somewhere between these extremes. The period for the index was lengthened from 1960–65 to 1955–70, since it was felt that the shorter period was not an adequate sample to expose a nation's attitude toward maintaining fixed exchange rates. The index is a pure number and, as defined, will be inversely related to mean reserves.

Expectations (*T*). A government's expectations concerning future reserve needs will affect current reserve holdings, since the accumulation or decumulation of reserves can be accomplished only with some lag. The growth rate of GNP over the sample period will be used as a proxy for expectations, the assumption being that a higher growth rate will be associated with a higher future need for reserves. The use of the growth rate of income for expectations is suggested by Heller's (1970) success in explaining reserve holdings by income levels alone. This variable is a pure number and is directly related to mean reserves.

In summary, nine variables will be used in the regressions. The *monetary variables* are reserves $E(R)$ ("official reserves" as defined by the IMF), disturbances Q [the standard error of equation (3.5)], aggregate wealth W (GNP), and the level of economic development D (per capita income). The *pure-number variables* are holding cost H (an interest rate based upon the marginal efficiency of investment), adjustment cost by expenditure switching A (a proportion showing the percentage of GNP not originating in manufacturing), external vulnerability V (the average propensity to import, showing the openness of the economy), exchange-rate flexibility F (a trinary index reflecting exchange-rate policy), and expectations T (the percentage growth rate in GNP). The monetary magnitudes are in millions of U.S. dollars.

The Regressions

Five tables show the most significant results of the various regressions performed on the data. Each regression has been run on seven different groups of countries:

ALL: 61 nations (total sample)
ALL': 59 nations (total sample omitting financial centers)
AC: 25 advanced countries
AC': 23 advanced countries (advanced countries omitting financial centers)
LDC: 36 less developed countries
LA: 17 Latin-American less developed countries
AA: 19 Asian and African less developed countries.

Since the behavior of financial centers (the United States and United Kingdom) is usually presumed to be different from that of other countries, the ALL (total sample) and AC (advanced countries) groups of nations are shown with the financial centers omitted in the ALL' and AC' groups. The LDC group of less developed countries is broken down into a Latin-American group, LA, and an Asian-African group, AA. The Latin-American nations are "old" less developed countries, long independent and removed from the colonial experience. The Asian-African nations are "new" less developed countries, only recently independent.

The t statistic is given under each coefficient in the tables. Based upon a two-tailed test, the coefficients are appraised at the 1, 2, 5, and 10 per cent levels of significance, with any lower level labeled "non" for "nonsignificant." There is a question as to whether the proper test of significance should be two-tailed or one-tailed. The convention in the literature has unfortunately been to use two-tailed tests. When theory tells us that a coefficient should carry a certain sign, it is this possibility that we wish to test, not the more general hypothesis that the coefficient is significantly different from zero (see Clark, 1970b, p. 589). Our concern is first whether the estimated coefficient takes the expected sign, and second whether the coefficient is significantly different from zero. Thus the one-tailed test should be used.[4] However, to conform to convention the levels of significance reported in the tables are based upon a two-tailed test. The levels of significance are given at the 1, 2, 5, and 10 per cent levels, which refine to the 0.5, 1, 2.5, and 5 per cent levels of significance under a one-tailed test *when* the coefficient takes the expected sign. An asterisk (*) is used to identify a coefficient with an incorrect sign. For each regression as a whole, R^2, F, degrees of freedom DF, and sum of squared errors SSE are given. Each regression was run in both linear and logarithmic forms, with the linear form usually showing the best fit to the data.[5]

The first regressions were "state of the art" runs, in which mean reserves were regressed against all the explanatory variables suggested by a priori analysis and previous studies. The results are shown in Table 1. The most successful explanatory variables are wealth W and disturbances Q, which generally carry the correct signs and show high levels of signifi-

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[4]The two-tailed test should be used when the null hypothesis is that the coefficient is equal to zero (an equality). The one-tailed test should be used when the null hypothesis is that the coefficient is, say, less than zero (an inequality).

[5]The results in linear and logarithmic forms were largely the same. In the logarithmic form, the performance of disturbances Q was unchanged, while that of wealth W deteriorated (see Table 4). Only exchange-rate flexibility F, among all the variables, was significant solely in the logarithmic form (see Table 4).

TABLE 1

STATE-OF-THE-ART REGRESSIONS

Sample	Q	H	A	W	D	V	F	T	Summary Statistics
ALL	11.70	4.29*	−12.07*	.0261	−.1016*	19.78	−26.88	−5.29*	$R^2 = .974$
t	7.92	.55	−1.24	20.56	−.59	2.59	−.32	−.12	$F = 241.2\ (1\%)$
%	1	non	non	1	non	2	non	non	$DF = 52$
									$SSE = .101\ (10^8)$
ALL'	13.69	7.45*	−11.52*	.0293	−.1830*	25.76	−11.89	−47.93*	$R^2 = .925$
t	7.12	1.14	−1.46	4.94	−1.28	3.98	−.18	−1.29	$F = 77.4\ (1\%)$
%	1	non	non	1	non	1	non	non	$DF = 50$
									$SSE = .632\ (10^7)$
AC	11.24	44.13*	−29.49*	.0256	.1796	26.87	46.96*	60.01	$R^2 = .978$
t	4.19	1.27	−1.37	12.14	.47	1.52	.20	.53	$F = 88.2\ (1\%)$
%	1	non	non	1	non	non	non	non	$DF = 16$
									$SSE = .720\ (10^7)$
AC'	11.28	36.00*	−26.87*	.0372	−.0587*	29.38	40.36*	−85.88*	$R^2 = .941$
t	3.34	1.34	−1.64	3.52	−.20	2.18	.23	−.83	$F = 27.9\ (1\%)$
%	1	non	non	1	non	5	non	non	$DF = 14$
									$SSE = .364\ (10^7)$
LDC	4.69	−1.27	10.48	.0077	.2973	−6.89*	−21.91	24.94	$R^2 = .739$
t	2.22	−.54	2.41	2.01	2.62	−1.94	−.82	1.53	$F = 9.5\ (1\%)$
%	5	non	5	10	2	10	non	non	$DF = 27$
									$SSE = .275\ (10^6)$
LA	−6.54*	−1.74	29.96	.0494	.7661	−7.38*	16.97*	1.43	$R^2 = .934$
t	−1.24	-.50	4.55	2.98	3.64	−1.19	.69	.05	$F = 14.3\ (1\%)$
%	non	non	1	2	2	non	non	non	$DF = 8$
									$SSE = .386\ (10^5)$
AA	−.10*	−1.83	1.29	.0103	.2788	−8.64*	−72.00	28.70	$R^2 = .744$
t	.03	−.51	.20	2.30	1.28	−1.94	−1.46	1.17	$F = 3.6\ (5\%)$
%	non	non	non	5	non	10	non	non	$DF = 10$
									$SSE = .109\ (10^6)$

cance. External vulnerability V acquires the wrong sign in several runs but is statistically significant with the correct sign in the ALL, ALL', and AC' runs. Adjustment cost A shows similar results, being significant with the correct sign in just the LDC and LA runs. The remaining variables in general are nonsignificant and acquire the wrong signs in the first four runs but have correct signs in the last three runs for the less developed countries. This indicates possible differences in the reserve-holding behavior of the advanced countries as opposed to the less developed countries.

There are many explanatory variables in the regressions in Table 1, and the significance of a number of them is hard to determine. In an attempt

TABLE 2

REGRESSIONS ON WEALTH AND DISTURBANCES

Sample	W	Q	Summary Statistics
ALL	.0250	11.93	$R^2 = .969$
t	21.71	10.37	$F = 895.4$ (1%)
%	1	1	$DF = 58$
			$SSE = .120$ (10^8)
ALL'	.0204	14.85	$R^2 = .898$
t	3.49	8.36	$F = 245.1$ (1%)
%	1	1	$DF = 56$
			$SSE = .868$ (10^7)
AC	.0251	11.78	$R^2 = .969$
t	14.30	5.82	$F = 344.3$ (1%)
%	1	1	$DF = 22$
			$SSE = .101$ (10^8)
AC'	.0272	13.48	$R^2 = .895$
t	2.84	4.54	$F = 84.8$ (1%)
%	2	1	$DF = 20$
			$SSE = .649$ (10^7)
LDC	.0054	7.00	$R^2 = .599$
t	1.45	3.81	$F = 24.7$ (1%)
%	non	1	$DF = 33$
			$SSE = .422$ (10^6)
LA	−.0143*	12.83	$R^2 = .714$
t	−1.07	3.05	$F = 17.5$ (1%)
%	non	1	$DF = 14$
			$SSE = .168$ (10^6)
AA	.0073	5.68	$R^2 = .535$
t	1.86	1.82	$F = 9.2$ (1%)
%	10	10	$DF = 16$
			$SSE = .198$ (10^6)

to explain reserve holdings more conclusively, the regressions were restricted to fewer variables. The results of this approach were successful and are shown in the remaining tables. In particular, the regression results for the less developed nations in Table 1 do not hold up in the remaining tables, while the results for the advanced nations are largely unchanged. Since only W and Q showed consistent explanatory power, in Table 2 reserves were regressed upon these two variables alone. The explanatory power of these two variables, taken by themselves, is very impressive. Only for the less developed countries does explanatory power decline, and even here the overall regressions are still significant at the 1 per cent level. A mean square error test was conducted on the results in Tables 1

TABLE 3

REGRESSIONS ON WEALTH, DISTURBANCES,
AND EXTERNAL VULNERABILITY

Sample	W	Q	V	Summary Statistics
ALL	.0256	12.11	18.38	$R^2 = .973$
t	23.26	11.19	2.97	$F = 680.2 \ (1\%)$
%	1	1	1	$DF = 57$
				$SSE = .104 \ (10^8)$
ALL'	.0279	13.43	20.46	$R^2 = .918$
t	4.97	8.17	3.76	$F = 206.5 \ (1\%)$
%	1	1	1	$DF = 55$
				$SSE = .690 \ (10^7)$
AC	.0256	12.55	21.56	$R^2 = .972$
t	14.80	6.21	1.57	$F = 245.7 \ (1\%)$
%	1	1	non	$DF = 21$
				$SSE = .900 \ (10^7)$
AC'	.0349	12.91	28.60	$R^2 = .922$
t	3.88	4.90	2.57	$F = 74.6 \ (1\%)$
%	1	1	2	$DF = 19$
				$SSE = .482 \ (10^7)$
LDC	.0059	7.06	1.01	$R^2 = .601$
t	1.45	3.77	.33	$F = 16.0 \ (1\%)$
%	non	1	non	$DF = 32$
				$SSE = .420 \ (10^6)$
LA	−.0142*	12.82	.17	$R^2 = .715$
t	−.94	2.93	.02	$F = 10.8 \ (1\%)$
%	non	1	non	$DF = 13$
				$SSE = .168 \ (10^6)$
AA	.0065	5.49	−1.73*	$R^2 = .542$
t	1.45	1.70	−.46	$F = 5.9 \ (1\%)$
%	non	non	non	$DF = 15$
				$SSE = .195 \ (10^6)$

and 2 to see how explanatory power was affected by the deletion of six variables. In Table 2, the coefficients of the six were restricted to zero. The F statistics are: ALL, 1.68; ALL′, 3.00; AC, 1.07; AC′, 1.85; LDC, 2.40; LA, 4.40; and AA, 1.36. Excepting ALL′ and LA, the restrictions are nonsignificant at the 5 per cent level, suggesting that the deleted variables as a group have no explanatory power.

The next step was to see if any single deleted variable would be significant in conjunction with W and Q. Wealth W and disturbances Q were therefore regressed together with every other explanatory variable. Different variables were found to be marginally significant for the advanced

TABLE 4

REGRESSIONS ON WEALTH, DISTURBANCES,
AND EXCHANGE-RATE FLEXIBILITY

Sample	Log W	Log Q	Log F	Summary Statistics
ALL	.2967	.8749	−.7637	R^2 = .904
t	2.64	6.02	−2.83	F = 178.9 (1%)
%	2	1	1	DF = 57
				SSE = 16.09
ALL′	.2682	.9101	−.7475	R^2 = .890
t	2.19	6.01	−2.74	F = 148.2 (1%)
%	5	1	1	DF = 55
				SSE = 15.69
AC	.2754	.8182	−1.3814	R^2 = .885
t	1.62	3.86	−2.41	F = 54.1 (1%)
%	non	1	5	DF = 21
				SSE = 6.04
AC′	.1734	.9216	−1.3343	R^2 = .865
t	.81	3.89	−2.27	F = 40.5 (1%)
%	non	1	5	DF = 19
				SSE = 5.54
LDC	.2350	.9347	−.4604	R^2 = .832
t	1.52	4.59	−1.45	F = 52.9 (1%)
%	non	1	non	DF = 32
				SSE = 8.80
LA	.1889	1.0244	−.2162	R^2 = .894
t	.58	2.68	−.46	F = 36.7 (1%)
%	non	2	non	DF = 13
				SSE = 3.69
AA	.3149	.4544	−.5970	R^2 = .690
t	2.12	1.70	−1.14	F = 11.1 (1%)
%	5	non	non	DF = 15
				SSE = 2.86

nations (Tables 3 and 4) and the less developed nations (Table 5). Referring to Table 3, external vulnerability V (the average propensity to import) took the correct sign in all runs except AA but showed significant explanatory power only in the AC' run (23 advanced countries omitting financial centers). Referring to Table 4, exchange-rate flexibility F, in the logarithmic form, took the correct sign in all runs but was statistically significant only for the advanced nations (AC and AC'). For the less developed countries, Table 5 shows that the expenditure-switching cost of adjustment A (output not originating in manufacturing) acquires the correct sign and significance (5 per cent under a one-tailed test in the

TABLE 5

REGRESSIONS ON WEALTH, DISTURBANCES,
AND COST OF ADJUSTMENT

Sample	W	Q	A	Summary Statistics
ALL	.0253	11.03	−8.55*	R^2 = .969
t	21.24	7.81	−1.09	F = 599.2 (1%)
%	1	1	non	DF = 57
				SSE = .118 (10^8)
ALL'	.0205	14.26	−5.68*	R^2 = .899
t	3.50	7.42	−.83	F = 162.7 (1%)
%	1	1	non	DF = 55
				SSE = .857 (10^7)
AC	.0257	10.28	−28.68*	R^2 = .972
t	14.62	4.66	−1.50	F = 243.3 (1%)
%	1	1	non	DF = 21
				SSE = .908 (10^7)
AC'	.0275	12.15	−26.07*	R^2 = .908
t	2.98	4.09	−1.63	F = 62.2 (1%)
%	1	1	non	DF = 19
				SSE = .569 (10^7)
LDC	.0051	8.26	6.34	R^2 = .641
t	1.45	4.39	1.93	F = 19.1 (1%)
%	non	1	10	DF = 32
				SSE = .378 (10^6)
LA	.0012	11.05	13.32	R^2 = .790
t	.09	2.89	2.17	F = 16.3 (1%)
%	non	2	5	DF = 13
				SSE = .124 (10^6)
AA	.0074	5.58	−.47*	R^2 = .536
t	1.79	1.65	−.10	F = 5.8 (1%)
%	10	non	non	DF = 15
				SSE = .198 (10^6)

LDC run) for the less developed countries as a whole and the Latin-American nations in particular. The Asian-African nations, in contrast, seem to hold reserves primarily as a function of wealth W (see Tables 2 and 4 also). This suggests that expenditure switching is the favored adjustment tool for the Latin-American group of older less developed countries. All other variables were nonsignificant when regressed separately with wealth and disturbances.

To summarize, it seems that the demand for international reserves can be described in terms of just a few variables. The demand for reserves in general (ALL and ALL$'$) and by the advanced nations (AC and AC$'$) is very well explained by wealth W, disturbances Q, the degree of external vulnerability V, and exchange-rate flexibility F. The demand for reserves by the less developed nations (LDC, LA, and AA) is partially explained by wealth W, disturbances Q, and the expenditure-switching cost of adjustment A.

A final question is whether the demand for reserves by advanced nations is somehow different from the demand for reserves by less developed nations. The data examined here suggest that this is so. Certain variables are significant for one group but not for the other, and take opposite signs for the two groups (refer to Table 1).

Conclusions

As a way of explaining the levels of international reserves actually held by various nations, the disturbances approach to measuring external disequilibria has been much more successful than the transactions approach. This study shows that the models of reserve demand developed by Heller (1966), Kelly (1970), and Clark (1970a) can be synthesized and reconciled into a general model of reserve demand. The regression results of this study are also an improvement upon earlier studies. The variables are well behaved and either acquire significance with the proper sign or remain nonsignificant at the 5 per cent level. The results are robust and remain largely unchanged through different regressions. Thus our knowledge about the determinants of reserve holdings among nations has been expanded by the results of this and previous studies. This study also suggests that the reserve-holding behavior of developed nations is different from that of less developed nations. Variations in reserve holdings among advanced countries are almost fully explained by wealth, the size of disturbances, external vulnerability, and exchange-rate flexibility. Variations in reserve holdings among less developed countries are partially

explained by the size of disturbances, wealth, and the cost of adjustment under expenditure-switching policies.

I still hold to the view that governments determine their level of reserves by some sort of cost-benefit analysis. What the findings of this study suggest is that certain elements entering into this analysis are the same from nation to nation and thus have no explanatory power in regard to relative levels of reserve holdings. These same elements, however, would have significance in explaining the world level of international reserves or the level for large blocs of nations. A study of this possibility would be an important next step suggested by the findings of this study.

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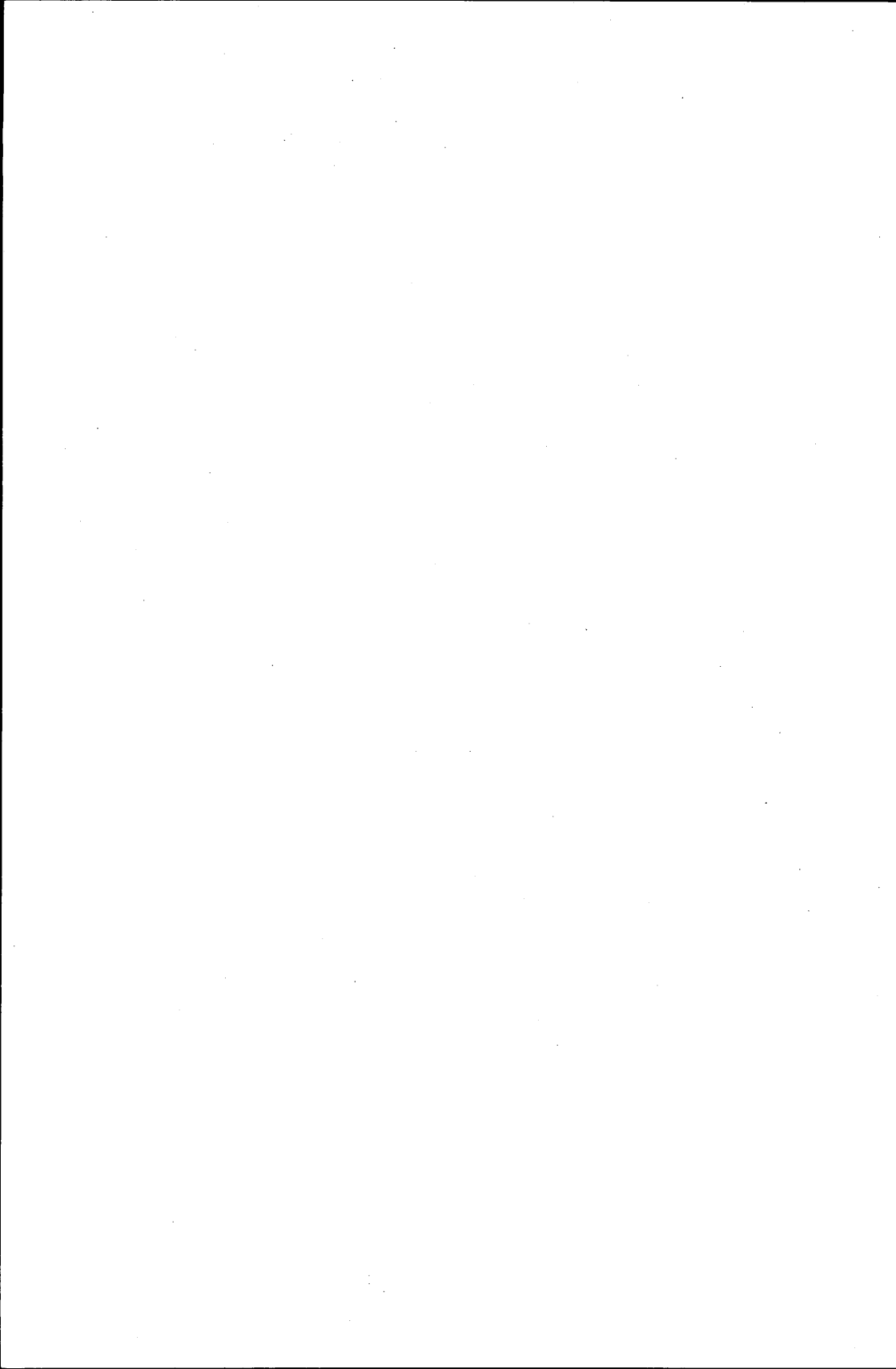
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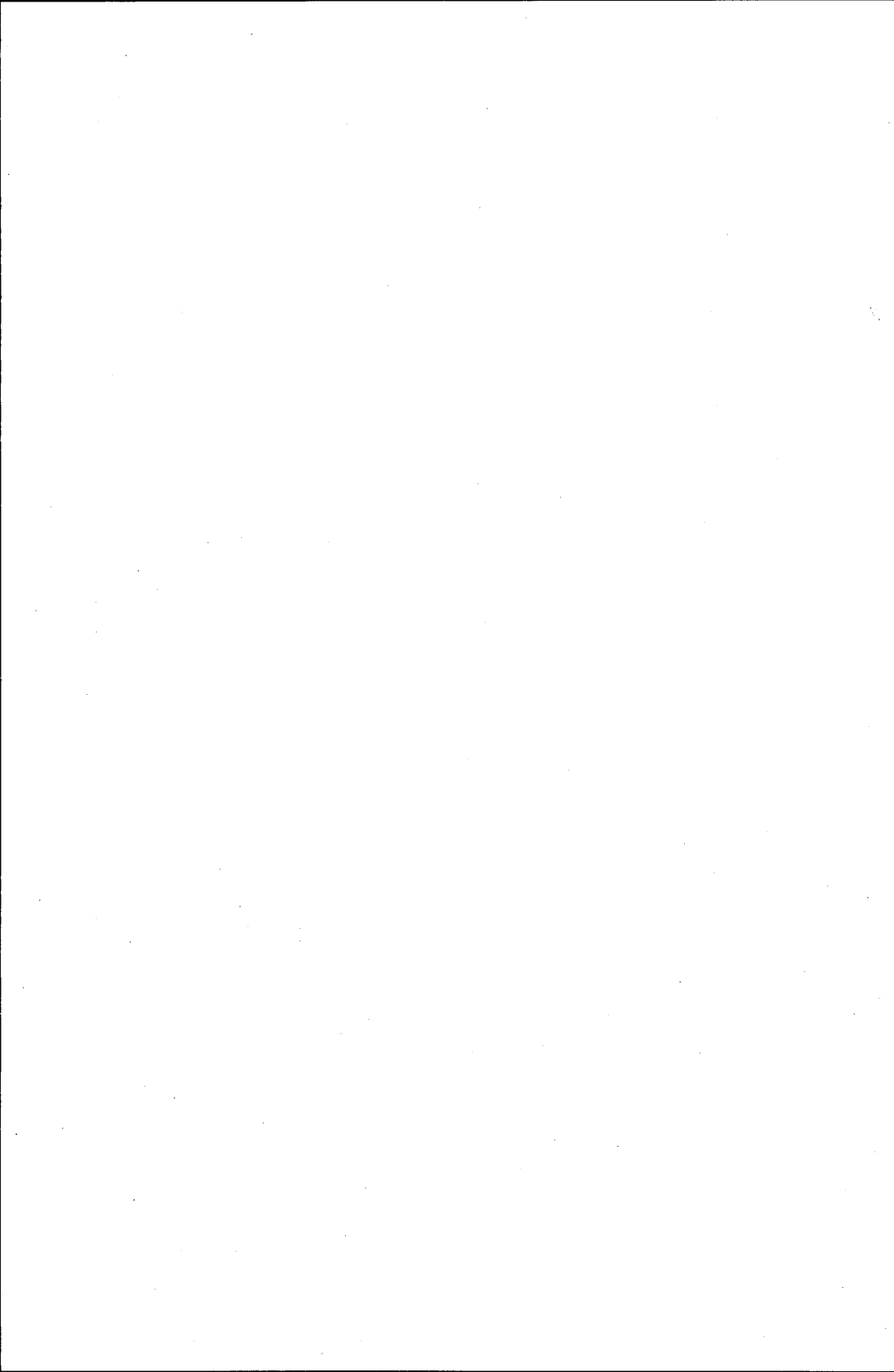29. Robert W. Oliver, *Early Plans for a World Bank*. (Sept. 1971)

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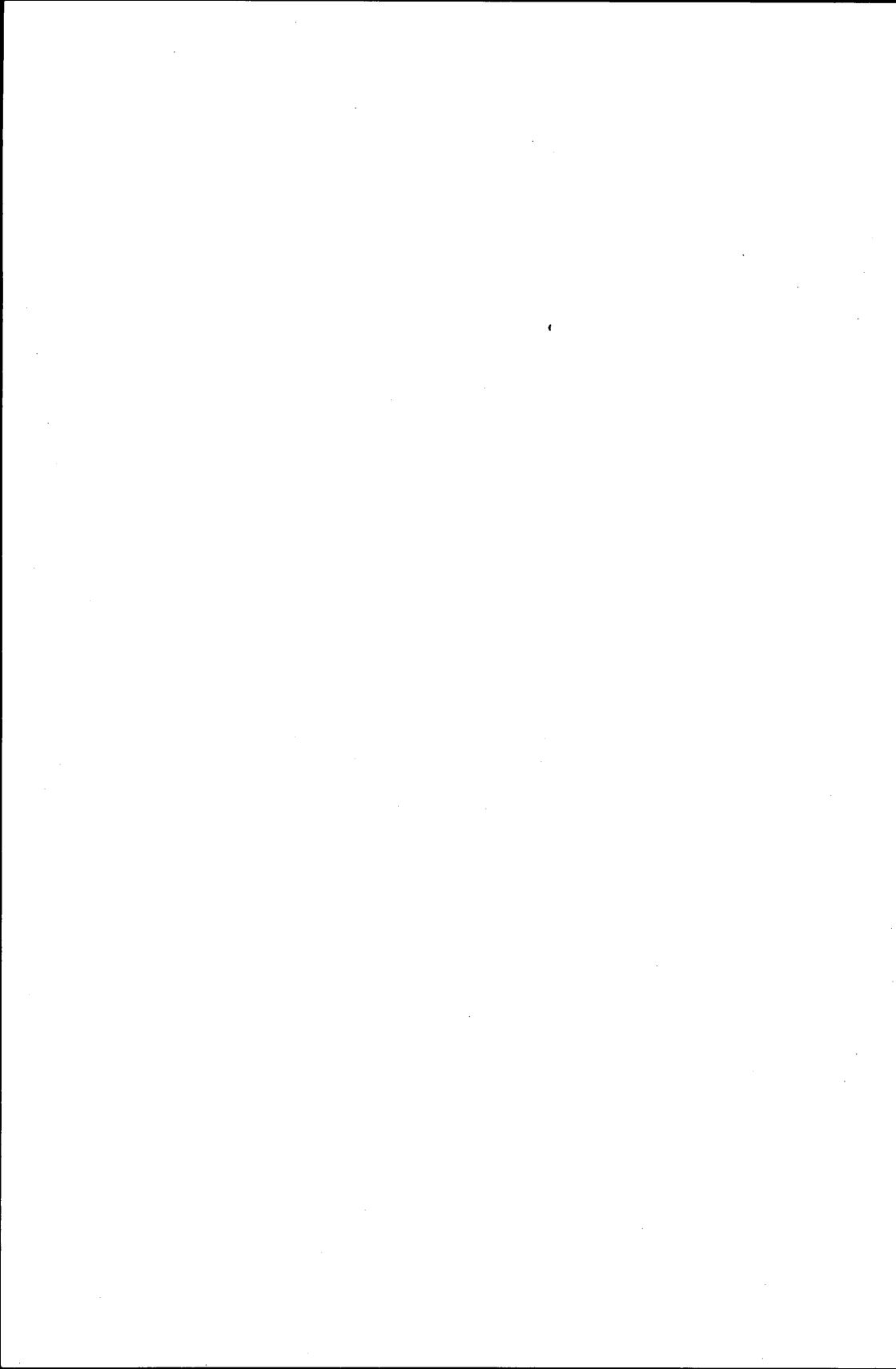
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