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The Theory of Forward Exchange

Egon Sohmen

INTERNATIONAL FINANCE SECTION
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THE THEORY OF FORWARD EXCHANGE

INTRODUCTION

After years of comparative neglect, increasing attention has recently again been paid to the theory of forward exchange markets. In view of their importance for public policy, this interest appears to be well deserved. In this paper, a concise survey of the subject will be presented emphasizing a few unduly neglected aspects.[1]

The traditional treatment of the theory of forward exchange has been in terms of one spot and one single forward market. The extension to any number of forward markets does not merely introduce additional variables into the model, but gives rise to new phenomena that are of considerable interest for policy matters. The widespread view that forward covering by exporters and importers amounts to a type of "insurance" that involves a risk premium, hence additional cost, will be shown to be incorrect. Another aspect to be discussed in greater detail is the response of forward exchange markets to monetary policy and the consequent effects on aggregate demand.

Forward-exchange operations will be classified according to the *function* of a particular transaction rather than the *person* who

[1] One of the earliest inquiries into the economic aspects of forward markets was Keynes' *Tract on Monetary Reform* (London: Macmillan, 1923), Chap. III, Part IV. Two monographs by Paul Einzig are devoted to forward exchange markets: *The Theory of Forward Exchange* (London: Macmillan, 1937) and his postwar book, *A Dynamic Theory of Forward Exchange* (London: Macmillan, 1962). Another important contribution is J. Spraos, "The Theory of Forward Exchange and Recent Practice," *Manchester School of Economics and Social Studies*, Vol. 21 (1953), pp. 87-117. The most detailed treatment of the simultaneous determination of equilibrium in the spot and one forward exchange market is presented in S. C. Tsiang's article, "The Theory of Forward Exchange and Effects of Government Intervention on the Forward Exchange Market," *International Monetary Fund Staff Papers*, Vol. 7 (1959), pp. 75-106. Essentially the same basic model was described, though in less detail and with the main emphasis on other policy aspects, in my doctoral dissertation (Massachusetts Institute of Technology, 1958). The chapter on forward markets is reproduced with minor changes in my *Flexible Exchange Rates* (Chicago: University of Chicago Press, 1961), Chap. IV. Peter B. Kenen has recently probed more deeply into the microeconomic underpinnings of forward-exchange theory in his paper, "Trade, Speculation, and the Forward-Exchange Rate," in Robert E. Baldwin *et al.*, *Trade, Growth, and the Balance of Payments*, Essays in Honor of Gottfried Haberler (Chicago: Rand McNally & Co., 1965), pp. 143-169. A useful survey in Swedish is B. Hansen's "Kursbildningen pa valutamark naderna," *Ekonomisk Tidskrift*, Vol. 63 (1961), pp. 173-200. Other literature references on specific aspects of the theory of forward exchange will be given in the appropriate contexts below.

undertakes it. For the purpose of theoretical analysis, it is most convenient to separate market transactions into "pure" hedging, "pure" arbitrage, and "pure" speculation, although real-world people will often perform two or more of these functions in the same transaction. Exporters and importers, for example, may occasionally or habitually carry exchange risks, in which case they also perform the functions of speculators. The approach chosen here follows the traditional practice of most writers on the theory of forward exchange.[2]

[2] Kenen is an important exception (in Baldwin *et al.*, *op.cit.*, p. 145). It is purely a matter of notational convenience whether the "functional" or the "personal" approach is chosen. The same assumptions concerning the behavior of market participants can obviously be incorporated in either one of the possible varieties in which a model of forward exchange markets can be set up, and they will then necessarily have to give the same answers. See also fn. 9.

1. COMMERCIAL TRADE AND FORWARD EXCHANGE MARKETS

What exactly happens when an exporter or importer purchases or sells forward exchange in the course of his regular business? Many people are inclined to believe that the forward sale of the expected foreign-exchange proceeds by an exporter is made possible only because the exchange risk is transferred to somebody else. Other participants in the forward market, so this reasoning continues, will be willing to assume this risk only against payment of an appropriate risk premium. For those who argue in this way, it appears obvious that forward covering by a commercial trader inevitably increases the cost of his transactions. In the view of many laymen, the spot rate of exchange for a currency is the strategic variable on which forward transactions are, in the last analysis, really based and to which the "risk premium" involved in a delayed payment in a foreign currency is merely added if the trader hedges by forward coverage.

To overcome this erroneous conception of forward exchange markets, let us first consider a model of a forward market that is fed *exclusively* by the transactions of exporters and importers—in other words, a market from which speculation as well as interest arbitrage are entirely absent. Let us assume that all those exporters and importers whose regular commercial contracts call for payment in, say, three months, never want to carry an exchange risk. To achieve this objective, they must always sell their expected foreign-exchange proceeds (or buy the foreign-exchange equivalent of their expected payments) on the three-month forward exchange market.[3]

It is important to realize, first of all, that these "pure" traders (and all commercial traders in the real world as well who contract for deferred payment but do not want to assume an exchange risk) are entirely unaffected by the level and the anticipated future movements

[3] Kenen is quite correct in pointing out the artificiality of the conventional view of what constitutes an exchange risk (*ibid*, fn. 5). To hold all of one's monetary assets in the currency of the country in which one happens to reside most of the time may involve a higher exchange risk, properly defined, than to distribute them among many currencies. Whoever holds anything at all incurs the risk of seeing its exchange ratio to other objects of economic value decline as well as rise over time. To meet the conventional arguments on their own grounds, I shall nevertheless throughout this paper follow the usual convention that an exchange risk is present whenever a net position in a *foreign* currency is held.

of the spot rate of exchange. Assuming that all other variables on which their behavior depends, such as domestic and foreign prices, interest rates, etc., are given, the export and import contracts they are ready to negotiate will depend exclusively on the level of the three-month *forward* rate. The properties of the demand and supply functions of forward exchange will then be determined by the supply and demand elasticities of the export and import commodities being traded by the group of exporters and importers who deal on this market.

Under our introductory assumption that commercial traders are the only parties negotiating contracts on the forward exchange markets (which also implies the absence of official interventions in forward markets), the equilibrium level of the forward rate will be determined by the condition that commercial supply and demand of forward exchange shall be equal at that rate. Commercial traders would, under these assumptions, settle all forward-exchange deals between themselves. No exporter or importer in our model carries an exchange risk, in spite of the fact that there is no other market participant who would take over the exchange risks "unloaded" by commercial traders.

As will be shown later, the verdict that forward coverage does not involve an intrinsic risk premium remains correct *even if* there is speculation in the forward market (see Section 4.7 below).

What has been said here about the behavior of commercial traders who have contracted for payment in three months is likely to hold for all forward exchange markets—that is, for markets calling for payment in six, nine, or twelve months, and longer periods. Only for contracts requiring immediate payment (which should normally be a small fraction of all international trade) is the spot rate of exchange the strategic variable determining the level of trading activity of non-speculating exporters and importers. For all other transactions, "pure" traders who shun exchange risks will always have to use the forward exchange market of the maturity date closest to the moment at which their payments (or receipts) fall due.[4] In the absence of both specula-

[4] Certain credit operations can serve as a substitute for forward transactions by commercial traders. An exporter who expects to receive payment in a foreign currency in three months is able to dodge the exchange risk in the following way: he may borrow the same amount of foreign currency now, sell it spot, and repay his debt in three months with the receipts from his exports. This alternative to a forward sale is a rather complicated process and is usually only available to large internationally known corporations. Even for them, it will usually be more ex-

tion and arbitrage, the forward-exchange rates for every maturity, as well as the spot rate, would be determined independently of each other by the transactions of commercial exporters and importers.[5]

Much of what follows (in particular, the analysis of monetary policy in Chapter V) will crucially depend on what has been concluded above about the behavior of commercial traders who are unwilling to assume net positions in foreign currencies. It is in this respect that my analysis differs most from that of other recent writers on the subject. In Tsiang's model, it is assumed that all exporters and importers hedge on the single forward market.[6] This would only portray the real world reasonably correctly if all trading contracts called for payment at the maturity date of the unique type of forward-exchange contract that is available. Kenen assumes in the main part of his article that all trading contracts call for cash payment.[7] Later, he briefly explores the changes that would be called for in his model if payment were always stipulated in 90 days.[8]

The essential similarity (and consequent unrealism) of all these models is that they assume that all traders use the market with the same maturity date at all times. The decisive feature of commercial forward activity is likely to be that hedging is taking place *on different markets for different trading contracts*, and that deliveries of the commodities bought and sold in these contracts will presumably affect domestic business activity at different times.[9] The assumption that the

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pensive and cumbersome than forward coverage. We shall therefore limit its discussion to this footnote.

[5] The careful reader will raise one apparently serious objection to this statement. Would not divergences between the spot and the various forward exchange rates lead to postponement or acceleration of imports or exports so as to import at the lowest possible prices and to export at the highest possible prices in terms of domestic currency? Commercial demand and supply on a particular forward market would therefore, so one might be tempted to argue, not be independent of the level of other forward rates and the spot rate of exchange, as claimed in the text. A detailed discussion of this aspect would lead us into areas that will only be covered later; we shall return to this question in Section 4.5 below.

I am indebted to Professor Yasukichi Yasuba of the University of Osaka for having called my attention to this point.

[6] Tsiang, *op.cit.*, p. 94. [7] In Baldwin *et al.*, *op.cit.*, p. 145. [8] *Ibid.*, p. 163.

[9] Kenen is apparently unaware of the essential difference between our basic assumptions concerning traders' behavior when he contrasts his own conclusions as to the behavior of traders on forward exchange markets with mine (*ibid.*, pp. 156-158). He also appears to have interpreted the assumption that trade responds to changes of forward rates as having been intended to encompass, in addition,

commodity demand and supply functions underlying the demand and supply functions of commercial hedgers for forward exchange have the usual shape (in other words, that the demand elasticities of the respective groups of commodities are negative, supply elasticities positive) does not, of course, imply that the demand and supply elasticities of *forward exchange* derived from them must necessarily exhibit the same signs. This will be taken to be the normal case in most of the subsequent discussion (see, however, Section 4.3 below), but by no means as a necessary consequence of the assumption that commercial trade responds (however little) to changes of forward-exchange rates.

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an assumption that the elasticities of commercial excess demand for *forward exchange* always exhibit the "normal" signs. This was not intended.

2. INTEREST ARBITRAGE

2.1. *Competitive Arbitrage under Full Currency Convertibility*

With free currency convertibility, covered interest arbitrage provides a link between the spot and the forward markets of different maturities. If the transfer of funds from one country to another did not involve any (opportunity) cost or risk and if the interest rates for loans of all possible maturities were the same at home and abroad, it is easily seen that equilibrium would require that the spot rate and all forward rates be identical.[10] If the forward rate for any maturity were higher than the spot rate, it would obviously pay to use one's own or borrowed funds to purchase the currency in question on the spot market and sell it simultaneously at the higher forward rate, keeping these funds in a foreign bank (where it is assumed they earn the same interest as in domestic banks) until the day when the forward contract becomes due. Competitive arbitrage will make the profit margin disappear.

If interest rates differ, the analysis becomes slightly more complicated. We shall use the following notation:

r_o ..... spot rate of exchange at time 0.

r_t ..... forward rate of exchange for delivery at time t (t is expressed in terms of fractions of a year).

i_d and i_f ..... domestic and foreign rates of interest for instantaneous compounding of interest. Though compounding is always discontinuous in practice, the notation is considerably simplified by the assumption of continuous compounding. The modifications required to change to the discontinuous case are elementary.

δ ..... forward discount on domestic currency, expressed as *per cent per annum* of the spot rate of exchange,

$$\text{that is,} \quad \delta = \frac{r_t - r_o}{r_o \cdot t} \quad .$$

With a domestic interest rate i_d, one unit of domestic currency will grow to $e^{i_d t}$ at time t (where e is the basis of the natural logarithms)

[10] As usual, we are here concerned with "pure" interest rates, net of any risk of default.

when invested at home. If capital movements between countries are unrestricted, funds can alternatively be transferred abroad. Given spot and forward exchange rates r_o and r_t, a foreign interest rate i_f, and assuming initially that transfer is costless, covered interest arbitrage will yield $\frac{r_t}{r_o} e^{i_f t}$ units of domestic currency at time t for every unit invested abroad. Competitive arbitrage will tend to make the returns from both types of investment equal in equilibrium. This leads to the equation

$$\frac{r_t}{r_o} = 1 + t\delta = e^{t(i_d - i_f)} \qquad (1)$$

or, to a first-order approximation,

$$\delta = i_d - i_f . \qquad (2)$$

Equation (2) expresses the familiar fact that, under the assumed idealized conditions, the premium or discount (*per annum*) tends to equal the interest differential, the currency of the country with the lower interest rate being the one that exhibits a forward premium. It is to be noted that the fulfillment of (2) requires not only the absence of transfer charges and risks, but also perfect competition among interest arbitrageurs.

2.2. *Modifications*

In the real world, various risks and transfer charges will prevent exact equality in (2). The willingness of arbitrageurs to move their liquid funds from one country to another can be expected to be a (rising) function of the covered interest differential, $i_d - i_f - \delta$ (where the direction of movement of arbitrage funds depends on the sign of that difference). The less-than-perfect elasticity of supply of arbitrage funds that can be observed in the real world finds its principal explanation in the rising opportunity cost of arbitrage compared to the alternative uses of the same funds, given the increasing risk of investing more and more funds in the same asset. Abstract reflection as well as empirical observation suggests, however, that the opportunity cost of arbitrage should normally not rise very steeply. Appreciable divergences between forward premium and interest differential have usually occurred only when exchange controls were in effect, or when their imposition appeared imminent. Recent contributions to the theory of forward exchange have perhaps overstated

the degree to which deviations of forward premia from interest differentials can occur under normal conditions.[11]

Apart from outright controls, obstacles to interest arbitrage often take the less obvious form of moral discouragement by central banks of all forward operations not directly linked to commercial trade. In most countries, central banks find it relatively easy to make such discouragement effective. Its avowed purpose is usually the prevention of speculation. There is little doubt that many central bankers are sincerely convinced that any forward operation not directly linked to commercial trade must be of a speculative nature, and that all speculation ought to be discouraged as much as possible. The effect of moral dissuasion is precisely the same as that of direct controls: the link between spot and forward markets is, if not completely broken, at least weakened, and forward premia may diverge from interest differentials by appreciable margins.

It is a mistake to believe that speculation alone is sufficient to produce an excessive divergence between interest differentials and forward premia. Whenever speculative short sales of a currency make it depreciate on the forward markets by more than the discount indicated by the difference in interest rates of the two countries, covered interest arbitrage becomes profitable and reduces or eliminates the excessive discount. One might perhaps wonder why interest arbitrageurs would ever want to undertake forward purchases of a currency that is generally expected to depreciate (as they would have to in order to reduce its forward discount). Arbitrageurs who undertake forward purchases of a currency simultaneously sell it on the spot market, however. Failure to arbitrage would imply that they continue to hold this currency anyway, and this must, under the conditions we have assumed, always be a less profitable alternative, regardless of whether or not speculators' anticipations of depreciation turn out to be correct. Holders of the currency may well decide to

[11] As a rule, professional interest arbitrage is performed predominantly by banks and large international corporations. They are in the best position to observe the movements of exchange rates and interest rates in the different countries from day to day and have command over sufficiently large funds to make arbitrage transactions pay even when the profit margins are very small. According to Keynes, profitable arbitrage required a minimum difference of ½ per cent per annum between forward premium and interest differential during the 1920's. (*Tract on Monetary Reform*, [London: Macmillan, 1923], p. 128.) Today, funds move when the difference between forward premium and interest differential is only a fraction of this figure. See P. Einzig, "Some Recent Changes in Forward Exchange Practices," *Economic Journal*, Vol. 70 (1960), pp. 86-88.

sell it spot without forward coverage, but this would again tend to reduce the forward discount.

There is usually more than one interest rate in each country. The interest differential relevant to a particular forward market may therefore not be unambiguously determinable. The most realistic assumption presumably is that the interest rate relevant for a forward market of a given maturity is the interest return on the asset with lowest yield among all assets with the same maturity date. This will obviously be the asset with minimal risk.

2.3. *Triangular Arbitrage*

The introduction of more currencies does not change the substance of our argument. Triangular spot arbitrage in conjunction with bilateral interest arbitrage between each pair of countries will tend to ensure that the forward cross rates between all currencies are exactly in line with one another. If the other two types of arbitrage work effectively, there is thus no need for triangular forward arbitrage, and no scope for any profits from it.[12]

Let us illustrate by the relations between three financial centers: New York, London, and Frankfurt. We shall denote the interest rates ruling in the three centers by i_N, i_L and i_F and exchange rates by $(r)_{\substack{L \\ N}}$ etc., where the upper subscript denotes the center whose currency is quoted (in our standard fashion) in the center indicated by the lower subscript. Thus, $(r_o)_{\substack{L \\ N}}$ is the spot price of one pound sterling in terms of U.S. dollars.

From (1) above, we can state the equilibrium conditions for bilateral interest arbitrage between every pair of financial centers (assuming again, so as not to complicate this part of the analysis

[12] See esp. J. Spraos, "The Theory of Forward Exchange and Recent Practice," *Manchester School of Economics and Social Studies*, Vol. 21 (1953), pp. 89-90; and H. G. Grubel, "A Multicountry Model of Forward Exchange: Theory, Policy, and Empirical Evidence 1955-1961," *Yale Economic Essays*, Vol. 3 (1963), pp. 117-118. Grubel appears to regard the transmission to other currencies of effects brought about by official interventions of the monetary authorities in the forward market for one foreign currency as a possibly dangerous consequence of triangular arbitrage. But pairs of currencies obviously cannot (and should not) be compartmentalized in a system of multilateral convertibility. Since triangular *forward* arbitrage is rendered unnecessary by the bread-and-butter business of arbitrageurs, triangular *spot* arbitrage and bilateral interest arbitrage (a fact of which Grubel is well aware), it is difficult to understand, moreover, why he attaches such importance to the potential consequences of triangular forward arbitrage. See also Grubel's paper, "A Neglected Aspect of Forward Exchange Theory and Policy," *The Journal of Finance*, Vol. 18 (1963), pp. 537-548.

unnecessarily, that arbitrage funds are in perfectly elastic supply, that risks are absent, and that transfer costs are zero):

$$\left(\frac{r_t}{r_o}\right)_{\substack{L\\N}} = e^{t(i_N - i_L)} \quad ,$$

$$\left(\frac{r_t}{r_o}\right)_{\substack{F\\N}} = e^{t(i_N - i_F)} \quad , \qquad (3)$$

$$\left(\frac{r_t}{r_o}\right)_{\substack{L\\F}} = e^{t(i_F - i_L)} \quad .$$

It is seen without difficulty that conditions (3) imply

$$\left(\frac{r_t}{r_o}\right)_{\substack{L\\N}} = \left(\frac{r_t}{r_o}\right)_{\substack{F\\N}} \cdot \left(\frac{r_t}{r_o}\right)_{\substack{L\\F}} . \qquad (4)$$

Triangular spot arbitrage, on the other hand, ensures that

$$(r_o)_{\substack{L\\N}} = (r_o)_{\substack{F\\N}} \cdot (r_o)_{\substack{L\\F}} \quad , \qquad (5)$$

which, together with the preceding equation, implies

$$(r_t)_{\substack{L\\N}} = (r_t)_{\substack{F\\N}} \cdot (r_t)_{\substack{L\\F}} \quad , \qquad (6)$$

as was to be shown. It is easily seen that, alternatively, triangular spot and forward arbitrage together with interest arbitrage between two pairs of centers would, in equilibrium, establish the correct forward premium between the remaining pair of currencies even without any direct capital movements between the two financial centers.