

PRINCETON STUDIES IN INTERNATIONAL FINANCE NO. 35

The Disturbances Approach to the Demand for International Reserves

F. Steb Hipple

INTERNATIONAL FINANCE SECTION
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I. THE DISTURBANCES APPROACH IN THE LITERATURE ON THE DEMAND FOR INTERNATIONAL RESERVES

Governments accumulate and hold stocks of international reserves primarily for the purpose of financing external disequilibria that otherwise would disrupt the domestic economy. These external disequilibria usually arise from two sources: First, fluctuations will occur more or less regularly in the balance of payments owing to seasonal or temporal differences in the business cycle in different nations, and, second, persistent deficit or surplus positions will develop in certain countries as a result of diverging rates of price inflation, income growth, and economic development in different nations.

Broadly speaking, two approaches to measuring the size of external disequilibria have appeared in the literature on the demand for international reserves. They may be called the *transactions approach*, where the size of disequilibria is measured in terms of the mean level of some class of external transactions (usually import expenditures), and the *disturbances approach*, where the size of disequilibria is measured in terms of the variations in the level of some class of external transactions. No one measure has won general acceptance under the disturbances approach similar to the acceptance given import expenditures under the transactions approach.

This paper is a study of the demand for international reserves utilizing the disturbances approach to measuring the size of external disequilibria.¹ Chapter I is a review of the theoretical and empirical work of several authors that has led to the present study.² In Chapter II a general model of reserve demand is developed based upon the major points discussed in Chapter I. And in Chapter III the study is concluded with several regressions run upon a sample of sixty-one nations for the period 1960-65. The results of these regressions are very encouraging.

¹This paper had its origin in a dissertation done at Southern Methodist University (1972) entitled "The Demand for International Reserves." I need to thank Professors Carter Murphy, Stephen Guisinger, Bart Trescott, and Tom Johnson for their guidance and assistance. The author of course bears full responsibility for all errors of commission and omission.

²The reader is referred to Grubel (1971) and Williamson (1973) for surveys of the literature on the demand for international reserves. Williamson's comments on pp. 688-697 are especially pertinent to this study.

For the purposes of this study, "international reserves" are defined as the "official reserves" reported by the International Monetary Fund. Such reserves include a government's holdings of gold and foreign exchange plus three lines of credit with the Fund — the super gold tranche position, the gold tranche position, and special drawing rights.

Major studies of the demand for reserves using the transactions approach have been those by the International Monetary Fund (1953, 1958), Triffin (1959), Brown (1964), Machlup (1966), Thorn (1967), Courchene and Youssef (1967), Hawkins and Rangarajan (1970), Clark (1970b), and Flanders (1971). The early studies by the Fund, Triffin, Brown, and Machlup stated reserves as a ratio to some transactions variable, usually imports, in an attempt to see if the ratios displayed sufficient clustering to be interpreted as a "demand" for reserves. The consensus, as expressed by Machlup, was that no such clustering could be observed. The later studies all employed regression analysis. Thorn, and Courchene and Youssef, regressed reserves against imports and found a statistically significant relationship. Hawkins and Rangarajan regressed IMF quotas against the sum of export and import transactions (and other variables) with little success. Clark took the ratio of reserves to the sum of export and import transactions and with little success regressed this ratio against several explanatory variables. Flanders regressed reserve/import ratios against several explanatory variables but also found few relationships of significance.

Major studies of the demand for reserves using the disturbances approach have been those by Nagabhushanam and Sastry (1962), Heller (1964, 1966), Brown (1964), Kenen and Yudin (1965), Machlup (1966), Hodjera (1969), Clark (1970a, 1970b), Kelly (1970), Flanders (1971), and Archibald and Richmond (1971). The studies by Brown, Machlup, Clark, and Flanders used both approaches. Brown and Machlup computed the ratio of reserves to some measure of disturbances but found no clustering in these ratios. Nagabhushanam and Sastry, Heller (1964), Kenen and Yudin, Hodjera, and Archibald and Richmond were primarily interested in the stochastic relationship of the reserve stock to some measure of disturbances and were very successful in refining this approach. Heller (1966), Clark, and Kelly, while also considering the stochastic relationships, were interested in linking reserve demand to a broad range of explanatory variables. All three of the last-named studies yielded mixed results. Flanders included a measure of disturbances among the explanatory variables in her study on reserve/import ratios.

In the remainder of this chapter, the studies by Nagabhushanam and Sastry, Kenen and Yudin, Heller (1966), Kelly, Clark, Archibald and Richmond, and some others will be discussed in more detail. A common system of notation has been developed and will be introduced as the discussion proceeds.

*Nagabhushanam and Sastry, "A Stochastic Model
for Foreign-Exchange Reserves"*

Two Indian economists, K. Nagabhushanam and M. Perayya Sastry (1962), in an innovative but overlooked study, first used the disturbances approach to estimate a demand function for reserves. The demand for reserves, they pointed out, is analogous to the demand for water in a reservoir. There is a branch of inventory analysis on water storage called "dam theory," which they applied to reserve demand. If the inflow of water to a reservoir is stable in a stochastic sense, then it may be described by a probability frequency distribution. The operator of the dam determines the stock of water in the reservoir by releasing water from the reservoir. It is assumed that the operator is rational, releasing water according to some "release rule" that reflects his demand function for the stock of water in the reservoir. Given data on inflows, outflows, and stock levels, the release rule can be estimated.

Nagabhushanam and Sastry assumed that India's receipts of foreign exchange were exogenously determined and stochastically stable over time, and that the Indian government completely controlled foreign-exchange expenditures. Thus the source of disturbances to be covered by reserves was the variation in total external receipts. The release rule was assumed to be a variable function of the inherited reserve stock and current foreign-exchange earnings. Based upon these assumptions, they estimated the demand function for international reserves of the Indian government using quarterly data for the years 1940-60. The importance of this study, aside from being the first of its type, is that it anticipates by several years the methodology used by Kelly (1970) in his very important study of reserve demand.

Kenen and Yudin, "The Demand for International Reserves"

Peter B. Kenen and Elinor B. Yudin (1965) conducted the first multinational study of reserve demand under the disturbances approach. They

began by treating changes in reserves R as a stochastic process of the Markov type:

$$\begin{aligned} (R_t - R_{t-1}) &= p(R_{t-1} - R_{t-2}) + Q_t \\ 0 < p < 1 \quad Q_t &= N[E(Q), V(Q)]. \end{aligned} \quad (1.1)$$

The change in reserves in period t is $(R_t - R_{t-1})$. This change is composed of two elements: the first is a "carry-forward" element $p(R_{t-1} - R_{t-2})$ from the change in reserves in period $(t - 1)$; and the second is a random disturbance Q_t (for disequilibrium) generated by a normal distribution with a nonzero mean $E(Q)$ and an independent variance $V(Q)$. Kenen and Yudin point out that the balance of payments is a sum of many separate transactions. While each transaction may not be drawn from a normal population, the central-limit theorem suggests that the sum of the transactions will have a normal distribution. Since changes in reserves are an approximation of the balance of payments, they assumed that Q_t was normally distributed.

It is important to note that under (1.1) the variance of reserves $V(R)$ is never stable at the limit.³ This apparently disturbed Kenen and Yudin, and they went on to note (p. 247) that governments could stabilize the reserve stock by changing the value of $E(Q)$. In that case, $E(Q)$ is no longer a truly stochastic element in reserve changes. This difficulty in the formulation of the stochastic process was remedied by Clark (1970a) when he stated reserve changes as a lagged-adjustment process. It should also be noted that Kenen and Yudin treated reserve changes as a flow-stochastic equation (R_{t-1} is to the left of the equality sign), while later writers use a stock-stochastic equation (R_{t-1} is to the right of the equality sign). While this leads to obvious differences in empirical work (the dependent variable is changes in reserves as opposed to reserve levels), the estimated values of coefficients should not be affected by this difference.

Equation (1.1) may be restated as

$$\begin{aligned} (R_t^* - R_{t-1}) &= E(Q) + p(R_{t-1} - R_{t-2}) + [Q_t - E(Q)] \\ 0 < p < 1 \quad [Q_t - E(Q)] &= N[0, V(Q)]. \end{aligned} \quad (1.2)$$

Note that the mean is zero for the $[Q_t - E(Q)]$ distribution. Equation (1.2) can then be written as

$$(R_t - R_{t-1}) = a_0 + a_1 (R_{t-1} - R_{t-2}) + e_t. \quad (1.3)$$

Using (1.3) as a regression equation, Kenen and Yudin estimated the

³This point is discussed further by Archibald and Richmond (1971, pp. 246-247).

values of the a_0 and a_1 coefficients and the parameters of the error distribution for fourteen trading nations, using monthly reserve data for the period 1958–62. From a_0 came estimates of $E(Q)$; from a_1 came estimates of p ; from $V(e)$ came estimates of $V(Q)$; and from $S(e)$, the “standard error” of (1.3), came estimates of $S(Q)$.

Kenen and Yudin then assumed that a nation’s reserve-demand function was of the form

$$R = b_0 - b_1E(Q) + b_2p + b_3S(Q). \quad (1.4)$$

The size of disturbances is represented by the standard deviation of disturbances $S(Q)$. Using the values of $E(Q)$, p , and $S(Q)$ for each nation calculated by (1.3), they performed cross-sectional regressions for the beginning and end of their sample period 1958–62. Since b_1 , the coefficient for $E(Q)$, came out with the wrong (positive) sign, it was deleted and a second set of regressions performed. The only variable to show any significance in the second set of regressions, and the only one to show significance (at the 5 per cent level) in all four regressions, was $S(Q)$. Kenen and Yudin then added explanatory variables to represent holding cost H (per capita income) and “liquid liabilities,” but only $S(Q)$ continued to show any explanatory significance.

The contributions of this paper to the literature are extremely important. First, a technique was developed for estimating the variance of disturbances $V(Q)$. Note that $V(Q)$ is the same parameter under the assumption of a zero mean for disturbances in (1.2) and under the assumption of a nonzero mean for disturbances in (1.1). By equating the nonzero mean for disturbances with the trend component in reserve changes, Clark (1970b) and Archibald and Richmond (1971) built their own analyses around the Kenen and Yudin technique. Second, the significance of the $V(Q)$ parameter was amply demonstrated by Kenen and Yudin in their regressions.

Heller, “Optimal International Reserves”

H. Robert Heller (1966) performed the first formal cost-benefit study of reserve demand. He wanted to estimate optimal reserves for sixty member nations of the IMF, those for which he could assemble data. In scope, his study was the broadest since the 1958 IMF study of reserve adequacy. Heller defined optimal reserves $E(R)^*$ as that average stock of reserves which will maximize national income over time. To optimize reserves, the marginal benefit of each reserve unit is compared with the marginal cost.

Reserve units should be accumulated until the marginal benefit equals the marginal cost.

The marginal cost of holding reserves H is the rate of return r foregone by not transforming reserves into real physical capital. The higher this cost of holding reserves, the smaller the optimal reserve stock $E(R)^*$. To measure holding costs, Heller assumed heroically that r was a uniform 5 per cent for all nations. The marginal benefit of holding reserves is the avoided economic cost of adjusting to a deficit not covered by reserves. (The cost of adjusting to a surplus was not considered in the literature until later.) The marginal benefit of holding reserves takes the form (cA) , where c is the probability of having to use the marginal reserve unit,⁴

$$c = P[R < 0 | E(R), V(R)] \quad c > 0, \quad (1.5)$$

and A is the change in income Y necessary to change temporarily the balance of payments by one unit, thereby altering the reserve stock by one unit. To measure adjustment cost A for a particular country, Heller assumed a purely Keynesian posture. A deficit not covered by reserves would be handled by reducing income to reduce imports sufficiently to eliminate the deficit. Under this assumption, A is the reciprocal of the marginal propensity to import, or $(1/MPM)$. In lieu of good estimates of MPM , Heller used the average propensity to import in 1963. To derive the probability c of using the marginal reserve unit, Heller used a continuous approximation to a binary treatment of the reserve variations experienced by each nation over the period 1949–63. Thus his measure of disturbances describes the fluctuations in the reserve stock. The last unit of reserves in the optimal stock would equate marginal cost and marginal benefit ($r = c/MPM$).

Heller went on to estimate optimal reserves for each of his sixty nations for the end of 1963 and to test regional and world reserve adequacy by aggregating his results. He found that world reserves were adequate by his criterion but badly distributed. The advanced nations possessed excessive reserves, while the developing nations suffered shortages. Heller's results have been criticized by Clower and Lipsey (1968, p. 591) as overstating the size of optimal reserves. They point out that, under Heller's formulation, the probability of using the last reserve units in the current year is quite low, and that these units should not be accumulated and

⁴To attach a certain probability of usage to each reserve unit as Heller has done, one must suppose that reserve units are accumulated and used on a first-in, first-out (FIFO) basis. As reserve units are accumulated, the value of $E(R)$ will increase and the probability c of using the last acquired unit of reserves will fall.

held. For our purposes, the main contribution of Heller's study is his methodology, which provides an element missing in the models of Kelly (1970) and Clark (1970a).

Kelly, "The Demand for International Reserves"

Michael G. Kelly (1970) began his study on the demand for international reserves by setting out a utility-maximization model. The model is based upon the explanatory variables of disturbances Q , holding cost H , and adjustment cost A . Kelly assumed that disturbances are neutralized by a combination of reserve variability and income variability. The division of this task is shown by a policy parameter k (for Kelly), where a zero value places the whole burden upon reserves while unity places the whole burden upon income.⁵ Paralleling Nagabhushanam and Sastry (1962), Kelly envisioned a current-account world where the level of exports is exogenously determined and can be expressed as a frequency function, and the government determines reserve holdings by its control over imports. The source of disturbances Q is export variability, as shown by $S(X)$ or $V(X)$; the cost of adjustment A is the reciprocal of the marginal propensity to import ($1/MPM$). The variances of reserves and income are

$$V(R) = (1 - k)^2 V(X) \quad 0 < k < 1 \quad (1.6)$$

$$V(Y) = (k/MPM)^2 V(X). \quad (1.7)$$

To link mean reserves $E(R)$ with reserve variance $V(R)$, Kelly assumed that governments wish to maintain some low probability of reserve exhaustion c^* , contained in the distribution (1.5). He chooses the algebraic form

$$c^* = fV(R)/E(R)^2 \quad f > 0. \quad (1.8)$$

Substituting (1.6) into (1.8) yields

$$E(R)^2 = (f/c^*)(1 - k)^2 V(X). \quad (1.9)$$

Solving (1.7) for k^2 , substituting into (1.9), and simplifying,

$$E(R) = (f/c^*)^{1/2} [S(X) - (MPM)S(Y)]. \quad (1.10)$$

The cost of holding reserves H is the reduction in mean income $E(Y)$ incurred by holding part of the nation's assets as reserves instead of productive capital:

$$rE(R) = E(Y)_m - E(Y), \quad (1.11)$$

⁵Kelly's treatment of the policy parameter has been simplified here.

where r is the foregone rate of return and $E(Y)_m$ is the maximum possible income.

Kelly employed a quadratic utility function showing increasing marginal disutility to both income reductions and income variations:

$$U = -a[E(Y)_m - E(Y)]^2 - b[(Y) - E(Y)]^2. \quad (1.12)$$

Substituting (1.11) into (1.12) yields

$$U = -ar^2E(R)^2 - bV(Y). \quad (1.13)$$

The general solution to the model is obtained by using (1.7) to replace $V(Y)$ in (1.13), then maximizing (1.13) subject to (1.10) with respect to $E(R)$ and $S(Y)$. The optimal level of reserves $E(R)^*$ is

$$E(R)^* = S(X)/[(c^*/f)^{1/2} + (f/c^*)^{1/2} (MPM)^{2r^2} (a/b)]. \quad (1.14)$$

Optimal reserves vary directly with the measure of disturbances $S(X)$, directly with the cost of adjustment ($1/MPM$), and inversely with holding cost r . Note that the solution is achieved only by specifying exogenously the desired probability of reserve exhaustion c^* .

Kelly turned next to a statistical test of his model. He assembled annual data for forty-six nations for the years 1953–65. He used as his measure of disturbances for each nation in each year the standard deviation of its exports $S(X)$ calculated from the preceding five-year period. He tried two separate indirect measures of holding costs based on capital scarcity: per capita national income and external dividend and interest payments and receipts. His rationale for using per capita income was that high per capita income should indicate plentiful capital (low marginal productivity). Income from external investment should also measure capital scarcity, because a nation rich in capital should be an exporter of capital. To measure adjustment cost, he used the average propensity to import APM . Finally, for reserves he used "official reserves" as defined by the IMF. The reserve-stock observation, for each country in each year, was a mean value calculated from the preceding period of one to six years. The length of the averaging period varied inversely with the nation's average propensity to import, to take into account lags in adjustment.

Kelly attempted regressions in both linear and logarithmic form. The logarithmic form yielded the best fit, and these are the results he reported. His first regressions used the entire forty-six-country sample over the thirteen annual cross-sections, and all coefficients were significant at the 5 per cent level, but the cost of adjustment (average propensity to import)

carried the wrong (positive) sign.⁶ The sample nations were then split into (1) advanced countries and less developed countries and (2) relatively open countries and relatively closed countries. When the regressions were repeated on these subsamples, the cost of adjustment now acquired the correct (negative) sign; however, two measures of holding cost (per capita income and external dividend and interest payments) now carried incorrect signs. Only a few coefficients were nonsignificant. Regression runs on the annual cross-sections of the data yielded similar results. Kelly's study presents the most successful empirical results so far in the literature on the demand for reserves. He can be criticized primarily for omitting wealth as an explanatory variable, and for defining his source of disturbances as fluctuations in current-account expenditures. Even so, like Kenen and Yudin (1965), Kelly's measure of disturbances is the most successful variable in his regressions.

Clark, "Optimum International Reserves and the Speed of Adjustment" and "Demand for International Reserves: A Cross-Country Analysis"

Peter B. Clark (1970a) began his analysis of reserve demand by setting out a utility-maximization model based upon the explanatory variables of disturbances Q , holding cost H , adjustment cost A , and wealth W . He structured his model around the Kenen and Yudin (1965) procedure, decomposing the change in reserves for any period into two components: (1) a change in reserves due to a current random disturbance and (2) a change in reserves stemming from disturbances in previous periods. Clark identified the first component with the capital account and the second component with the current account. In respect to the current account, he assumed that the level of exports is exogenously determined and fixed in amount and that the government controls imports by varying national income. Thus the cost of adjustment A is again the reciprocal of the marginal propensity to import ($1/MPM$). The difference between export revenues and import expenditures shows the change in reserves for each period attributable to action by the government to compensate for disturbances in the capital account in previous periods. Clark represents this by a Koyck distributed-lag response equation, where the reserve stock R_t at the end of period t is equal to the inherited reserve stock R_{t-1} plus (1)

⁶Kelly did not use ($1/APM$) in his regressions, but just APM ; thus the proper sign is negative. However, it will be assumed in Chapter III that the propensity to import is not a measure of the cost of adjustment but a measure of "external vulnerability." On this assumption, the coefficient carries the correct (positive) sign.

the current random disturbance Q_t , and (2) the proportion L of the difference between inherited reserves R_{t-1} and optimal reserves $E(R)^*$:⁷

$$\begin{aligned} R_t &= L[E(R)^* - R_{t-1}] + R_{t-1} + Q_t \\ 0 < L < 1 \quad Q_t &= Q[0, V(Q)]. \end{aligned} \quad (1.15)$$

Clark assumed that disturbances were generated by a distribution with a zero mean and constant variance $V(Q)$. The relationship of $V(Q)$ to the variance of reserves $V(R)$ and the variance of income $V(Y)$ is

$$V(R) = V(Q)/L(2 - L) \quad (1.16)$$

$$V(Y) = LV(Q)/(2 - L)(MPM)^2. \quad (1.17)$$

The cost of adjustment A enters equation (1.17) in the form $(1/MPM)^2$. When the speed of adjustment L is unity, the variance of income is maximized and income receives most of the burden in handling disturbances. When L is close to zero, the variance of income is very small, and reserves receive most of the burden in handling disturbances.⁸ To provide a linkage between mean reserves $E(R)$ and reserve variance $V(R)$, Clark next defined c , the probability of reserve exhaustion, from (1.5). Clark does not follow Kelly in assuming c to be an exogenous parameter c^* . To give an algebraic expression to (1.5), Clark used the Chebychev inequality as an equality:

$$c = V(R)/2E(R)^2. \quad (1.18)$$

Substituting (1.16) into (1.18) yields

$$c = V(Q)/L(2 - L) 2E(R)^2. \quad (1.19)$$

Clark defined the cost of holding reserves as

$$E(Y) = E(Y)_m - rE(R), \quad (1.20)$$

where $E(Y)_m$ is maximum income, which depends upon the amount of wealth W , and r is the foregone rate of return on reserves.⁹

Clark next introduced a linear utility function of the form

$$U = a_0 + a_1E(Y) - a_2S(Y) - a_3c, \quad (1.21)$$

⁷Clark, like Kelly (1970) and Archibald and Richmond (1971), expresses reserve changes as a stock-stochastic equation rather than follow Kenen and Yudin (1965) and use a flow-stochastic equation. In equation (1.15), transfer R_{t-1} to the left of the equality sign and compare with (1.1).

⁸Compare Clark's policy parameter L in (1.16) and (1.17) with Kelly's policy parameter k in (1.6) and (1.7).

⁹Compare (1.18) with (1.8), (1.19) with (1.9), and (1.20) with (1.11).

which shows utility for increased income $E(Y)$ and disutility for income variations $S(Y)$ and higher levels of probability c for reserve exhaustion.¹⁰ By entering c into his utility function, Clark did not have to assume, like Kelly, that c is an exogenous parameter. Substituting (1.17), (1.19), and (1.20) into (1.21), and maximizing with respect to $E(R)$ and L , yields the general solution of the model. Unfortunately, the solution is too complex to be reduced to a single equation for optimal mean reserves $E(R)^*$ like Kelly's (1.14). Instead, by total differentiation Clark established that the proper relationships exist between optimal reserves and the independent variables. He also went on to establish the causal relationships between the speed of adjustment L and the independent variables:

$$\begin{aligned} E(R)^* &= R(Q, H, A, W) \\ R_Q > 0; R_H < 0; R_A > 0; R_W > 0. \end{aligned} \quad (1.22)$$

$$\begin{aligned} L &= (Q, H, A, W) \\ L_Q < 0; L_H > 0; L_A < 0; L_W < 0. \end{aligned} \quad (1.23)$$

Clark (1970b) then turned to an empirical test of his lagged-adjustment model. Since his solution was in terms of two variables, $E(R)$ and L , he needed to explain observed values for both. He used a sample of thirty-eight nations for the period 1958–67. Clark first defined his reserve variable as “official reserves” plus the IMF credit tranches. Using monthly data on “reserves,” he calculated the linear trend in reserves for each sample nation. Defining $E(R)^*$ in (1.15) as the trend value, he then estimated the value of L for each nation. In one nation, the estimate of L was negative, and in seventeen it was not significantly different from zero at the 5 per cent significance level (indicating no adjustment whatsoever to disturbances). This procedure also provided estimates of the measure of disturbances $S(Q)$ for each nation.¹¹ The cost of adjustment was represented by the marginal propensity to import MPM , which was also estimated by a set of regressions. Wealth was represented by per capita income in 1963, and holding cost was deleted from the regressions. Finally, optimal reserves $E(R)^*$ was the mean reserve level for the period 1958–67.

Thus, Clark ran two sets of regressions, one for $E(R)^*$ and one for L . In the regression for optimal reserves, the cost of adjustment MPM took the

¹⁰Compare (1.21) with Kelly's (1.12). Note that Clark uses a linear utility function, while Kelly uses a quadratic utility function. The quadratic form is to be preferred, since it can show increasing or decreasing marginal utility.

¹¹The procedure used by Clark is detailed below in the section on “Disturbances (Q)” in Chapter III.