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# Theoretical Issues in International Borrowing

Jeffrey Sachs

INTERNATIONAL FINANCE SECTION
DEPARTMENT OF ECONOMICS
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# 1 INTRODUCTION

The current crisis in international lending points up a lesson relearned several times in the past hundred and fifty years: the international loan markets function very differently from the textbook model of competitive lending. In the simple model, borrowers have ready access to loans at a given interest rate; they enter the loan market to finance all investment projects with positive present value at the prevailing interest rate; and they use loans to equate the marginal utility of consumption at various points in time. Actual lending behavior is far from this rosy view. Borrowers are extensively rationed in the international markets: they may be unable to obtain credit at any price, much less the posted market price. Highly profitable investment projects may be blocked for want of foreign capital or, worse, may be abandoned midstream after creditors withdraw capital in a sudden loss of confidence. Various institutions, such as the International Monetary Fund and the Bank for International Settlements, have an acknowledged role in maintaining stability in the loan markets, even though such institutions are superfluous in the simple model.

The large gap between theory and practice has led to a search for new theoretical concepts to explain actual loan behavior. A number of recent models have taken seriously the possibility of debt repudiation (or "sovereign risk") on loans to developing-country governments, and have shown that such a risk radically alters the behavior of borrowers and lenders. (McDonough, 1982, surveys many of these new models.) The presence of sovereign risk can help to explain credit rationing, debt rescheduling, conditionality, and even the maturity structure of international obligations (see Eaton and Gersovitz, 1981; Sachs and Cohen, 1982). Other models (e.g., Stiglitz and Weiss, 1981) have shown that credit rationing may arise for other reasons as well, such as lenders' inability to evaluate the risk categories of potential borrowers. Still other models (e.g., Kharas, 1981) have explored the interaction of the domestic financial systems in

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developing countries with international borrowing in order to derive more useful policy guidelines for international borrowing decisions. These new models help not only to restore the relevance of a central class of economic models but also to shed light on actual loan-market behavior. The theoretical advances can also help us to define the proper role of the IMF in the present debt crisis and of the banks and borrowing countries in a very imperfect market.

This Study discusses some of the elements needed for a richer and more realistic model of international lending to developing-country governments. It draws on recent work and also offers some new results aimed at highlighting what is right and wrong with the standard model of international lending.

The textbook case is both insightful and a natural starting point for discussion. I then proceed to three areas of that model to show how more realistic assumptions can fundamentally alter our views of borrower and lender behavior. The first revision comes in modeling the borrowing country itself. The textbook case treats the borrower as a "representative agent" maximizing utility subject to a budget constraint based on national wealth. Since most international borrowing is by governments or government enterprises, the textbook model implicitly assumes that governments have unlimited taxing power over national wealth. I follow Kharas (1981) in introducing a limit to the government's taxing authority and show that optimal borrowing strategies may be quite different from standard prescriptions. In particular, governments should no longer borrow in order to finance all investment projects with a positive present value at world interest rates. Such a strategy almost surely leads to slow growth and a credit squeeze.

The second extension to the textbook model pertains to the assumption about loan repayments. The standard model assumes that loans are repaid as a long as resources are available to repay them. Implicitly, it assumes that the costs to a country of repudiating its debts always outweigh the benefits. There is no doubt that the costs of debt repudiation are high, in both economic and diplomatic terms. Nevertheless, governments have at times preferred unilateral debt repudiation (or at least a unilateral debt moratorium) to the arduous and politically unpopular task of servicing a heavy debt burden, even when the debt servicing was technically feasible. The textbook model is therefore extended by giving the borrower the option of debt repudiation. The costs and benefits of debt repudiation are made explicit, and all market participants are assumed to understand them.

A number of important implications are then derived. Most obviously, the repudiation risk leads to an upward-sloping supply of funds to the borrowing country, and to credit rationing once a high level of indebtedness is reached. Less intuitively, inefficiencies arise in the dynamic behavior of the borrowing country, since the default risk distorts several of its incentives. From a given set of initial conditions, there typically exists an optimal sequence of policies that maximizes the borrower's utility as viewed from the initial period. However, there are often incentives that make it unlikely that the borrower will actually pursue that optimal sequence over time. Relative to the path that would maximize *ex ante* expected utility, a debtor may be induced (1) to overborrow period to period, (2) to overinvest in risky projects at the expense of safe projects, and (3) to overconsume and underinvest.

The problems arise from the inability of the borrower to pre-commit itself to certain behavior once a loan is arranged. For example, in order to attract large loans at low rates, a borrower may try to convince lenders that it will act prudently and avoid overly risky investments. After the loan is made, however, the borrower can often reduce the expected burden of the debt by going ahead with the risky projects (or by overborrowing, or overconsuming, etc.). Since creditors anticipate these actions, they charge a risk premium for them in the original loan contract. Not surprisingly, a borrower is best off when it can convincingly forswear these actions and thereby reduce its initial cost of borrowing. In domestic capital markets, bond covenants are used for that purpose. In the international markets, convincing ways to forswear such costly behavior have yet to be found.

The third extension to the basic model pertains to loan-supply behavior. The competitive-loan model typically ignores the institutional structure of lending. Credit is assumed to be supplied elastically at a given interest rate, and little attention is paid to whether that credit comes from a bank loan or a bond flotation, a single lender or a syndicate, etc. Actually, the nature of intermediation can be of profound importance. Inherent risk considerations coupled with prudential lending rules for banks have made the syndication the preferred form of lending. A standard loan to a borrowing country may be extended by several hundred participating banks. The problem with the syndication is that member banks do not necessarily act in their collective best interest when key contingencies arise, as would be the case if the loan were made by a single bank operating in a competitive environment.

Many actions that a syndicate should take have "public good" features. For example, an efficient loan package may require banks to monitor a country's economic performance after a loan is made, but the banks may have no way to share the cost of monitoring. Even if the need for monitoring is clear, each bank may try to be a free rider on the monitoring expenditures of other banks, leading to insufficient supervision. More important, an efficient loan package sometimes requires banks to refinance the debts of a heavily indebted country at below-market rates, in order to keep it from imminent default. Collectively, the need for rescheduling may be clear, but, once again, each individual bank may try to withdraw its own credits in order to leave the debt burden to the other banks.

The most dramatic breakdown of loan supply comes in a panic, where a fundamentally sound economy is forced into default by a shortage of credit. This type of market failure can result from the rational behavior of a large number of small lenders. Each bank rationally bases its loan-supply decision on the actions of the other banks. Suppose, for example, that a country needs a large loan to tide it over a short-run fall in income. If all banks but one stop lending, only a very large loan from the remaining bank can save the country from default. But such a large loan may be precluded by risk or regulatory considerations. Thus, if no other banks lend to the country, the given bank may also choose to stop lending; if the other banks were to continue their loans, the given bank could safely contribute a share of the total credit line. In such a situation, two equilibria may be present. In the favorable case, all banks continue to lend; in a panic, all banks stop lending because the others have stopped lending, and the "healthy" country finds itself forced into default.

There are several ways to overcome the collective-action problems of the creditors, though none is costless or foolproof. Syndicated loans typically include loan managers who provide public-good services, such as monitoring or legal representation, in return for management fees. Also, in the course of reschedulings informal "fair share" rules have emerged under which various burdens are divided on the basis of the banks' existing shares in the total loans to a rescheduling country. Finally, institutions like the IMF (and, to a much lesser degree, the BIS and leading central banks) assume some of the public-good aspects of international lending, such as monitoring and enforcement costs.

I illustrate these considerations in a series of models of international lending to a developing-country sovereign borrower. The models are kept simple to illustrate the main points as clearly as possible. I make no attempt at generality and little attempt at putting all the various models together. Each remains a single facet of an evolving general model. Chapter 2 introduces the standard borrowing model. Chapters 3, 4, and 5 consider the three major extensions of that model: imperfections in the borrowing economy, risks of debt repudiation, and collective-action problems of the creditors.

## 2 THE BASIC MODEL OF INTERNATIONAL BORROWING

Consider the standard borrowing problem facing a social planner of a small open economy (for examples, see Bardhan, 1967; Blanchard, 1983; Sachs, 1981). The economy produces a pure traded good in amount $Q_t$ in period $t$, according to a production function $Q_t$ equal to $F(K_t, L_t)$. The labor supply is exogenous (or perfectly elastic with a fixed wage $w$). The capital stock evolves according to the equation $K_{t+1} = K_t(1-d) + I_t$, where $I$ is gross investment and $d$ is the rate of depreciation. In the closed economy, total spending (the sum of consumption and investment) is equal to output. In the open economy, spending can be augmented by foreign borrowing. It is typical to assume that the economy can engage in one-period international loans at given world interest rates. Let $D_{t+1}$ be the stock of loans undertaken at time $t$ for repayment at time $t + 1$, and let $r$ be the real rate of interest on the loan (I will simplify substantially and assume that $r$ is fixed through time). With national output given by $Q_t$, national income (denoted by $Y_t$) is given by $Q_t$ net of interest payments on debt coming due: $Y_t = Q_t - rD_t$. Total consumption in period $t$ is national income minus investment plus net new borrowing: $C_t = Y_t - I_t + (D_{t+1} - D_t)$.

The model is closed by specifying the terms of borrowing. How much can the borrower borrow? The standard assumption here is that the borrower can attract any loan that can feasibly be paid back. The incentive to make the necessary loan repayments is simply assumed. In a finite-horizon version of the model, say $T$ periods, it is also assumed that there is no last-period debt, so that $D_{T+1}$ does not exceed zero. The borrower faces the set of constraints:

$$\begin{aligned} D_2 &= (1+r)D_1 + C_1 + I_1 - Q_1 , \qquad (1)\\ D_3 &= (1+r)D_2 + C_2 + I_2 - Q_2 , \\ &\vdots \\ D_{T+1} &= (1+r)D_T + C_T + I_T - Q_T , \\ D_{T+1} &< 0 . \end{aligned}$$

By substituting the $D_2$ equation into the $D_3$ equation, the $D_3$ equation into the $D_4$ equation, and so forth, we can rewrite (1) as

$$\sum_{i=1}^{T}(1+r)^{-(i-1)}C_i \leqslant \sum_{i=1}^{T}(1+r)^{-(i-1)}(Q_i - I_i) - (1+r)D_1 . \tag{2}$$

This equation implicitly defines an upper bound to borrowing. Note that

$$\sum_{i=1}^{T}(1+r)^{-(i-1)}C_i$$

must be nonnegative. Therefore, we can bring $(1+r)D_1$ to the left-hand side of the inequality in (2) to obtain

$$(1+r)D_1 \leqslant \max_{I_i} \sum_{i=1}^{T}(1+r)^{-(i-1)}(Q_i - I_i) . \tag{3}$$

In any period $t$, we could perform the same manipulations for the remaining $T - t$ periods, to find

$$(1+r)D_t \leqslant \max_{I_i} \sum_{i=t}^{T}(1+r)^{-(i-t)}(Q_i - I_i) . \tag{4}$$

Equation (4) states a very important point:

> In order for debt repayment to be feasible, that is, for $D_{T+1}$ not to exceed zero, indebtedness at any time must be less than national productive wealth, where the latter is defined as the maximum discounted value of GDP net of investment in the remaining periods.

If the constraint in (4) is ever strictly binding, it implies that consumption is zero along the *entire* remaining growth path. For that reason, it is unlikely that an optimal borrowing program involves borrowing $D_t$ up to its maximum feasible level. It is a simple matter to transform (4) to the appropriate expression for the infinite-horizon case. Feasibility conditions for repayment in that case are found simply by replacing $T$ with $\infty$ in (4):

$$(1+r)D_t \leqslant \max_{I_i} \sum_{i=t}^{\infty}(1+r)^{-(i-t)}(Q_i - I_i) \quad (\textit{infinite-horizon case}) . \tag{5}$$

Now, let us state the full borrowing problem:

*The Borrowing Problem in Finite Horizon* (6)

max $U(C_1, C_2, \ldots, C_T)$,

subject to

$$Q_t = F(K_t, L_t) \,,$$
$$K_{t+1} = K_t(1-d) + I_t \,,$$
$$C_t = (Q_t - rD_t) - I_t + (D_{t+1} - D_t) \,,$$
$$D_t \leq \max \sum_{i=t}^{T} (1+r)^{(t-i-1)}(Q_i - I_i) \,.$$

$K_1, D_1$ are given; $L_t$ is given for all $t$.

The infinite-horizon problem is found simply by substituting $\infty$ for $T$ in (6). This problem has been heavily explored, in various guises and degrees of sophistication, in the economics and planning literature. When stated as in (6), the problem results in the following necessary conditions of optimization:

*Optimal Borrowing in the Finite Horizon* (7)

The solution to (6) is a set of sequences $\{C_1, C_2, \ldots, C_T\}\{I_1, \ldots, I_{T-1}\}$, and $\{D_1, \ldots, D_T\}$ that satisfy the conditions in (6) together with

(a) $U_i = \partial U / \partial C_i = \lambda(1+r)^{-(i-1)}$ ,

(b) $\partial F / \partial K_i = r + d$ for $i = 2, \ldots, T-1$ ,
$\partial F / \partial K_T = 1 + r$ ,

(c) $\sum_{i=1}^{T} (1+r)^{-(i-1)} C_i = \sum_{i=1}^{T} (1+r)^{-(i-1)}(Q_i - I_i) - (1+r)D_1$ .

$\lambda$ is the marginal utility of wealth, equal to the increase in $U$ resulting from a unit reduction in initial indebtedness $(1+r)D_1$. There are three main conditions for optimal borrowing. First (7)(a) states that the international loan market should be used to equate the marginal utility of consumption in each period, $U_i$, with a discounted marginal utility of wealth $\lambda(1+r)^{-i}$. Second, (7)(b) states that investments should be undertaken in each period (except the last) in order to equate the marginal product of capital, $\partial F/\partial K_i$, with the cost of capital, $r + d$. Finally, (7)(c) holds that the discounted value of total consumption equals the discounted value of total productive wealth net of initial indebtedness. We see from (2) that this condition is equivalent to assuming that $D_{T+1}$ equals zero.

We have so far interpreted the optimal borrowing problem as that of a social planner choosing $\{D_1,\ldots,D_T\}$ to maximize a social-welfare function. According to standard principles of welfare economics, there is a second interpretation, that (7)(a) – (c) characterize the *market* equilibrium of the economy under conditions of perfect competition. Assuming that $U(C_1,\ldots,C_T)$ is the utility function of the representative household, and that production and investment decisions are undertaken by competitive firms operating with perfect foresight, the decentralized equilibrium will replicate the social planner's policies. (For a formal demonstration of this equivalence in the infinite-horizon case, see Abel and Blanchard, 1983.) In general, the equivalence of centralized and decentralized equilibria will *not* hold once the model is extended to allow for various credit-market imperfections.

The conditions (7)(a) – (c) are also properties of richer models, for example, those that allow for productivity shocks or terms-of-trade fluctuations. When carefully interpreted, they describe many of the standard guidelines in the development literature for foreign borrowing. For example, (7)(a) is really a prescription to smooth consumption over time relative to income by borrowing when output is low relative to trend and paying back loans, on net, when output is high (see Sachs, 1981 and 1982b for this interpretation). The country should borrow in order to finance consumption during a temporary drop in income but not during a permanent drop in the trend of income. On close analysis, (7)(a) makes explicit the IMF dictum to finance temporary shocks but "adjust" to permanent shocks. Equation (7)(b) states the standard cost-benefit condition for investment projects in a small open economy. Regardless of the consumption stream, the planner should invest so as to equate the marginal product of capital, evaluated at world market prices, with the cost of capital, also at world market prices. A nearly equivalent condition is that all projects should be undertaken with positive present value at the world market prices and interest rates.

A useful simplification for solving the borrowing model is to write utility as additively separable:

$$U(C_1,C_2,\ldots,C_T) = \sum_{i=1}^{T} u(C_i)(1+\delta)^{-i}. \tag{8}$$

That is, total utility (upper-case $U$) is the sum of subutilities (lower-case $u$) based on consumption in each period. These subutilities are discounted according to the subjective rate of time preference, $\delta$. With this formulation, (7)(a) becomes $u_i = \lambda(1+\delta)^i/(1+r)^i$. From this point forward, I will assume additive separability.

A second transformation is also helpful. Let $V(K_t,D_t)$ be the maximum value of utility that a borrower can achieve starting with $K_t,D_t$. It is found by plugging the solution from (6) into the utility function. Then a $T$-period or infinite-horizon problem can be reduced to a two-period or three-period problem. Thus, for example, the problem stated in (6) can be written as

$$\max\ u(c_1) + u(c_2)/(1+\delta) + V(K_3,D_3)/(1+\delta)^2\ , \qquad (6')$$

subject to

$$Q_t = F(K_t,L_t) \quad (\text{where } t = 1,2)\ ,$$
$$K_{t+1} = K_t(1-d) + I_t\ ,$$
$$C_t = (Q_t - rD_t) - I_t + (D_{t+1} - D_t)\ .$$

$K_1,D_1$ are given; $L_t$ is given for all $t$.

The first-order conditions are now

$$u_1(C_1) = -(1+r)V_D(K_3,D_3)/(1+\delta)^2, \qquad (7')$$

(where $u_i$ signifies $\partial u/\partial C_i$)

$$u_2(C_2) = -V_D(K_3,D_3)/(1+\delta),$$
$$F_K(K_2) = r + d\ ,$$
$$V_K(K_3,D_3) = -V_D(K_3,D_3)\ .$$

$V_D(K_3,D_3)$ is the marginal disutility of an increment of debt in period 3. Since each unit of debt requires $1 + r$ in repayment and thus reduces wealth by $1 + r$, $-(1+r)V_D(K_3,D_3)$ measures the marginal utility of wealth in period 3. Discounted to the present, the marginal utility of wealth in the first period is therefore $-(1+r)V_D(K_3,D_3)/(1+\delta)^2$. This is what I called $\lambda$ in the original solution given in (7). With this interpretation we see that $U_1(C_1) = \lambda$, as before. Also $u_2(C_2)/(1+\delta)$, which is $U_2$ in (7), is equal to $(1+r)^{-1}\lambda$, as before.

## 3 INTERNATIONAL BORROWING WITH DOMESTIC FINANCIAL CONSTRAINTS

I now expand the model to allow for an explicit role for public-sector financial variables. For simplicity I illustrate the results in the three-period version of the model just introduced.

Kharas (1981), Katz (1982), and others have pointed out that the pure borrowing model should differentiate between the private and public sectors and take seriously the empirical fact that most international lending to developing countries is to the public sector, or to the private sector with public-sector guarantees. In these circumstances, debt-servicing capacity depends not only on national wealth but on the public sector's ability to tax that wealth. Moreover, domestic capital markets in the borrowing country tend to be highly segmented and imperfect, so that the public sector must use its borrowing powers to bring about an efficient level of aggregate investment in the economy.

Thus, let us suppose that the private sector in the developing country saves a fixed fraction of post-tax income, which is available for private investment, while the government uses its taxing and borrowing authority to supplement private investment and/or private consumption (see Arrow and Kurz, 1970, Chap. VI, for a similar setup in a closed economy). Private investors have no direct access to the international loan market. The government taxes domestic output at rate $\tau_t$, which may change over time. This rate must be less than one and may be less than zero if the government is making net income transfers to the private sector. Also, we allow for the possibility of a ceiling to the tax rate, $\bar{\tau}$, so that $\tau_t \leqslant \bar{\tau} \leqslant 1$. The ceiling may reflect political or economic constraints (e.g., tax evasion at high tax rates) on the government's ability to raise revenues.

With domestic output given by $Q_t$, tax revenues are $\tau_t Q_t$, and private-sector savings are $s(1-\tau_t)Q_t$. Private consumption is given by $C_t$ equal to $(1-s)(1-\tau_t)Q_t$. In any period, the government borrows $D_{t+1}$ and repays $(1+r)D_t$. Total investment in the economy equals private savings plus tax revenue plus net foreign-resource inflow:

$$I_t = s(1 - \tau_t)Q_t + \tau_t Q_t + [D_{t+1} - (1 + r)D_t]. \tag{9}$$

As written, it appears that all foreign borrowing is used for investment rather than consumption, but this is true only as an accounting matter. Suppose, for example, that the government wants to raise private consumption while holding investment levels fixed. It merely raises $D_{t+1}$ while reducing $\tau_t$ sufficiently to keep $I_t$ constant; in that case, the borrowing finances consumption 100 per cent on the margin.

As in the standard borrowing problem, the government faces a ceiling on external indebtedness. But now the ability to repay debt depends on the government's taxing authority as well as on national wealth. The new constraint is that $D_t$ must be less than or equal to the maximum level of tax revenues net of government investment. Government investment is $I_t$ minus private investment, $s(1-\tau_t)Q_t$. Thus, by summing over (9), we derive

$$D_t(1+r) \leq \max_{\tau, I} \sum_{i=1}^{t} (1+r)^{-(i-t)}[\tau_t Q_t \; - \; I_t \; + \; s(1-\tau_t)Q_t] \; . \qquad (4')$$

Note that if $\bar{\tau}$ equals one (i.e., there is no constraint on the maximum tax rate), so that $\tau_t = \bar{\tau} = 1$ in (4′), the borrowing constraint reduces to (4). For $\bar{\tau}$ significantly less than one, the borrowing constraint in (4′) is likely to be far more restrictive than the constraint based on national wealth alone. Having noted this point, we will now study the general model assuming that (4′) does *not* bind along the optimal path for the economy.

Once again, we calculate the optimal financial policy of the government, assuming that it tries to maximize an intertemporal utility function of the form $u(C_1) + u(C_2)/(1+\delta) + V(K_3, D_3)/(1+\delta)^2$. In this case, however, the government's policy instruments are taxes and foreign borrowing rather than investment and foreign borrowing, as in the previous model. Taxes are now manipulated to achieve the desired level of national savings, whereas in the previous model that savings rate is chosen directly. (Remember, though, that the equilibrium in the basic model could also be interpreted as the market equilibrium under conditions of perfect competition. The correspondence of market and optimal equilibria will no longer apply here.)

*The Basic Public-Finance Problem with International Borrowing* (10)

$$\max_{D_2, D_3, \tau_1, \tau_2} \quad u(C_1) + u(C_2)/(1+\delta) + V(K_3, D_3)/(1+\delta)^2 \, ,$$

subject to

$$
\begin{aligned}
Q_t &= F(K_t, L_t) , \\
K_{t+1} &= K_t(1-d) + I_t , \\
C_t &= (1-s)(1-\tau_t)Q_t , \\
I_t &= s(1-\tau_t)Q_t + \tau_t Q_t + D_{t+1} - (1+r)D_t , \\
\tau_t &\leqslant \bar{\tau} \leqslant 1 .
\end{aligned}
$$

$D_1, K_1$ are given.

As long as tax rates are completely flexible (i.e., $\bar{\tau} = 1$), the solution to this problem is identical to the social planner's solution of Chapter 2, since the dynamic budget constraint facing the government is no different whether it chooses $C_t$ and $I_t$, as before, or $\tau_t$ and $I_t$, as here.

To show this formally, simply substitute for $C_1$, $C_2$, $K_3$, and $D_3$ in the utility function in (10),[1] to find

$$
\begin{aligned}
U = {} & u[(1-s)(1-\tau_1)F(K_1)] + u[(1-s) \\
& \cdot (1-\tau_2)F(K_1(1-d) + I_1)]/(1+\delta) + V(K_1(1-d)^2 \\
& + I_1(1-d) + I_2, (1+r)^2 D_1 + (1+r)[I_1 - \tau_1 F(K_1) - s(1-\tau_1)F(K_1)] \\
& + I_2 - \tau_2 F[K_1(1-d) + I_1] - s(1-\tau_2)F[K_1(1-d) + I_1)]/(1+\delta)^2 .
\end{aligned}
$$

Now, the first-order conditions are given by

$$
\begin{aligned}
&\text{(a) } \partial U/\partial \tau_1 = 0 \Rightarrow u_1 = -(1+r)V_D/(1+\delta)^2 , \qquad (11) \\
&\text{(b) } \partial U/\partial \tau_2 = 0 \Rightarrow u_2 = -V_D/(1+\delta) , \\
&\text{(c) } \partial U/\partial I_1 = 0 \Rightarrow (1-s)(1-\tau_2)F_K(K_2) \cdot u_2 + (1-d)V_K/(1+\delta) \\
&\qquad + (1+r)V_D/(1+\delta) - [\tau_2 + s(1-\tau_2)]F_K(K_2) \\
&\qquad \cdot V_D/(1+\delta) = 0 , \\
&\text{(d) } \partial U/\partial I_2 = 0 \Rightarrow V_K = -V_D .
\end{aligned}
$$

By substituting (11)(a), (b), and (d) into (11)(c), we find that $F_K(K_2)$ equals $r + d$, as before; the conditions (11)(a), (b), and (d) are exactly as before. Thus, the demonstration is complete.

[1] The following substitutions are made:

$$
\begin{aligned}
C_1 &= (1-s)(1-\tau_1)F(K_1) , \\
C_2 &= (1-s)(1-\tau_2)F(K_2) = (1-s)(1-\tau_2)F[K_1(1-d) + I_1] , \\
K_3 &= K_1(1-d)^2 + I_1(1-d) + I_2 , \\
D_3 &= (1+r)^2 D_1 + (1+r)[I_1 - \tau_1 F(K_1) - s(1-\tau_1)F(K_1)] \\
&\quad + [I_2 - \tau_2 F(K_2) - s(1-\tau_2)F(K_2)] .
\end{aligned}
$$