Comparative Advantage and Optimal Trade Taxes*

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September 2013

Abstract

Intuitively, a country should have more room to manipulate world prices in sectors in which it has a comparative advantage. In this paper we formalize this intuition in the context of a canonical Ricardian model of international trade. We then study the quantitative importance of such considerations for the design of unilaterally optimal trade taxes in two sectors: agriculture and manufacturing.

*We thank Rodrigo Rodrigues Adao for superb research assistance. Costinot and Donaldson thank the National Science Foundation (under Grant SES-1227635) for research support. Vogel thanks the Princeton International Economics Section for their support.

1 Introduction

Two of the most central questions in international economics are "Why do nations trade?" and "How should a nation conduct its trade policy?" Since David Ricardo's pioneering work, the theory of comparative advantage is one of the most influential answers to the former question. Yet after almost two hundred years, it has had virtually no impact on answers to the latter question. In this paper we go back to Ricardo's model, a framework that has become a new workhorse model for theoretical and quantitative work in the field, and explore the relationship between comparative advantage and optimal trade taxes.

Our main result can be stated as follows. Optimal trade taxes should be uniform across imported goods and weakly monotone with respect to comparative advantage across exported goods. Examples of optimal trade taxes include (i) a zero import tariff accompanied by export taxes that are weakly increasing with comparative advantage or (ii) a uniform, positive import tariff accompanied by export subsidies that are weakly decreasing with comparative advantage. While the latter pattern accords well with the observation that countries tend to protect their least competitive sectors in practice, larger subsidies do not stem from a greater desire to expand production in less competitive sectors. Rather they reflect tighter constraints on the ability to exploit monopoly power by contracting exports. Put simply, countries have more room to manipulate world prices in their comparative-advantage sectors.

Our starting point is a canonical Ricardian model of international trade with a continuum of goods and Constant Elasticity of Substitution (CES) utility, as in Dornbusch et al. (1977), Wilson (1980), Eaton and Kortum (2002), and Alvarez and Lucas (2007). We consider a world economy with two countries, Home and Foreign. Labor productivity can vary arbitrarily across sectors in both countries. Home sets trade taxes in order to maximize domestic welfare, whereas Foreign is passive. In the interest of clarity we assume no other trade costs in our baseline model. Iceberg trade costs are incorporated later as an extension.

In order to characterize the structure of optimal trade taxes, we use the primal approach and consider first a fictitious planning problem in which the domestic government directly controls consumption and output decisions. Using Lagrange multiplier methods, we then show how to transform this infinite dimensional problem with constraints into a series of simple unconstrained, low-dimensional problems. This allows us to derive sharp predictions about the structure of the optimal allocation. Finally, we demonstrate how that allocation can be implemented through trade taxes and relate optimal trade taxes to comparative advantage.

After demonstrating the robustness of our main insights to the introduction of general preferences and trade costs, we apply our theoretical results to study the design of optimal trade policy in two sectors: agriculture and manufacturing. In our numerical examples, we find gains from trade under optimal trade taxes that are 24% larger than those obtained under laissez-faire for the agricultural case and 32% larger for the manufacturing case. Interestingly, a significant fraction of these gains arises from the use of trade taxes that are monotone in comparative advantage. Under an optimal uniform tariff, gains from trade for the agriculture and manufacturing cases would only be 16% and 9% larger, respectively, than those obtained under laissez-faire.

Our paper makes two distinct contributions to the existing literature. The first one is to study the relationship between comparative advantage and optimal trade taxes. In his survey of the literature, Dixit (1985) sets up the general problem of optimal taxes in an open economy as a fictitious planning problem and derives the associated first-order conditions. As Bond (1990) demonstrates, such conditions impose very weak restrictions on the structure of optimal trade taxes. Hence, optimal tariff arguments are typically cast in simple economies featuring only two goods or quasi-linear preferences. In such environments, characterizing optimal trade taxes reduces to solving the problem of a single-good monopolist/monopsonist and leads to the prediction that the optimal tariff should be equal to the inverse of the (own-price) elasticity of the foreign export supply curve.[1]

In a canonical Ricardian model, countries buy and sell many goods whose prices depend on the entire vector of net imports through their effects on wages. Thus the (own-price) elasticity of the foreign export supply curve no longer provides a sufficient statistic for optimal trade taxes. Nevertheless our analysis shows that for any wage level, optimal trade taxes must satisfy simple and intuitive properties. What matters for one of our main results is not the entire schedule of own-price and cross-price elasticities faced by a country acting as a monopolist, which determines the optimal level of wages in a non-trivial manner, but the cross-sectional variation in own-price elasticities across sectors holding wages fixed, which is tightly connected to a country's comparative advantage.

The paper most closely related to ours is Itoh and Kiyono (1987). They show that in a Ricardian model with Cobb-Douglas preferences, export subsidies that are concentrated on "marginal" goods may be welfare-enhancing. This resonates well with our finding that, at the optimum, export subsidies should be weakly decreasing with comparative advantage, so that "marginal" goods should indeed be subsidized more. Our analysis,

[1]This idea has a long history in the international trade literature, going back to Torrens (1844) and Mill (1844). This rich history is echoed by recent theoretical and empirical work emphasizing the role of terms-of-trade manipulation in the analysis of optimal tariffs and its implication for the WTO; see Bagwell and Staiger (1990), Bagwell and Staiger (2011) and Broda et al. (2008).

however, goes beyond the results of Itoh and Kiyono (1987) by considering a Ricardian environment with general CES utility and, more importantly, by solving for optimal trade taxes rather than providing examples of welfare-enhancing policies.[2] Beyond generality, our results also shed light on the simple economics behind optimal trade taxes in a canonical Ricardian model: taxes should be monotone in comparative advantage because countries have more room to manipulate prices in their comparative-advantage sectors.

The second contribution of our paper is technical. The problem of finding optimal trade taxes in a canonical Ricardian model is infinite-dimensional (since there is a continuum of goods), non-concave (since indirect utility functions are quasi-convex in prices), and non-smooth (since the world production possibility frontier has kinks). To make progress on this question, we follow a three-step approach. First, we use the primal approach to go from taxes to quantities. Second, we identify concave subproblems for which general Lagrangian necessity and sufficiency theorems problems apply. Third, we use the additive separability of preferences to break down the maximization of a potentially infinite dimensional Lagrangian into multiple low-dimensional maximization problems that can be solved by simple calculus. The same approach could be used to study optimal trade taxes in economies with alternative market structures, as in Bernard et al. (2003) and Melitz (2003), or multiple factors of production, as in Dornbusch et al. (1980).

Our approach is broadly related to recent work by Amador et al. (2006) and Amador and Bagwell (2013) who have used general Lagrange multiplier methods to study optimal delegation problems, including the design of optimal trade agreements, and to Costinot et al. (2013) who have used these methods together with the time-separable structure of preferences typically used in macro applications to study optimal capital controls. We briefly come back to the specific differences between these various approaches in Section 3. For now, we note that like in Costinot et al. (2013), our approach heavily relies on the observation, first made by Everett (1963), that Lagrange multiplier methods are particularly well suited for studying "cell-problems," i.e., additively separable maximization problems with constraints. Given the importance of additively separable utility in the field of international trade, we believe that these methods could prove useful beyond the question of how comparative advantage shapes optimal trade taxes. We hope that our paper will help make such methods part of the standard toolbox of trade economists.

The rest of our paper is organized as follows. Section 2 describes our baseline Ricardian model. Section 3 sets up and solves the domestic government's planning problem in this environment. Section 4 shows how to decentralize the solution of the planning prob-

[2] Opp (2009) also studies optimal trade taxes in a two-country Ricardian model with CES utility, but his analysis focuses on optimal tariffs that are uniform across goods.

lem through trade taxes and derive our main theoretical results. Section 5 establishes the robustness of our main insights to departures from CES utility and the introduction of trade costs. Section 6 applies our theoretical results to the design of optimal trade taxes in the agricultural and manufacturing sectors. Section 7 offers some concluding remarks.

2 Basic Environment

2.1 A Ricardian Economy

Consider a world economy with two countries, Home and Foreign, one factor of production, labor, and a continuum of goods indexed by i.[3] Preferences at home are represented by the Constant Elasticity of Substitution (CES) utility,

$$U \equiv \int_i u_i(c_i)di,$$

where $u_i(c_i) \equiv \beta_i \left(c_i^{1-1/\sigma} - 1\right) \Big/ (1 - 1/\sigma)$ denotes utility per good; $\sigma \geq 1$ denotes the elasticity of substitution between goods; and (β_i) are exogenous preference parameters such that $\int_i \beta_i di = 1$. Preferences abroad have a similar form with asterisks denoting foreign variables. Production is subject to constant returns to scale in all sectors. a_i and a_i^* denote the constant unit labor requirements at home and abroad, respectively. Labor is perfectly mobile across sectors and immobile across countries. L and L^* denote the inelastic labor supply at home and abroad, respectively.

2.2 Trade Equilibrium

We are interested in situations in which the domestic government imposes ad-valorem trade taxes, $t \equiv (t_i)$, that are rebated to domestic consumers in a lump-sum fashion, whereas the foreign government does not have any tax policy in place. $t_i \geq 0$ corresponds to an import tariff if good i is imported or an export subsidy if it is exported. Conversely, $t_i \leq 0$ corresponds to an import subsidy or an export tax. Here, we characterize the trade equilibrium for arbitrary taxes. Next, we will describe the domestic government's problem, of which optimal taxes are a solution.

In a trade equilibrium, consumers in both countries maximize their utility subject to

[3] All subsequent results generalize trivially to economies with a countable number of goods. Whenever the integral sign "$\int$" appears, one should simply think of a Lebesgue integral. If the set of goods is finite or countable, "$\int$" is equivalent to "$\sum$."

their budget constraints. The associated necessary and sufficient conditions are given by

$$u_i'(c_i) = \theta p_i (1+t_i), \tag{1}$$

$$\int_i p_i (1+t_i)\, d_i(\theta p_i (1+t_i))\, di = wL + T, \tag{2}$$

$$u_i^{*\prime}(c_i^*) = p_i, \tag{3}$$

$$\int_i p_i d_i^*(p_i)\, di = w^* L^*, \tag{4}$$

where w and w^* are the domestic and foreign wages; $p \equiv (p_i)$ is the schedule of world prices; $d_i(\cdot) \equiv u_i'^{-1}(\cdot)$ and $d_i^*(\cdot) \equiv u_i^{*\prime -1}(\cdot)$ are the "Frisch" demand functions for good i in the two countries; T is Home's total tax revenues; and θ is the Lagrange multiplier associated with the domestic budget constraint. Note that we have normalized prices so that the Lagrange multiplier associated with the foreign budget constraint is equal to one. We maintain this normalization throughout our analysis.

Profit maximization by firms in both countries implies

$$p_i(1+t_i) \leq w a_i, \text{ with equality if } q_i > 0, \tag{5}$$

$$p_i \leq w^* a_i^*, \text{ with equality if } q_i^* > 0, \tag{6}$$

where q_i and q_i^* denote the output of good i at home and abroad, respectively. Finally, good and foreign labor market clearing requires

$$c_i + c_i^* = q_i + q_i^*, \tag{7}$$

$$\int_i a_i^* q_i^* di = L^*. \tag{8}$$

Given a schedule of taxes $t \equiv (t_i)$, a trade equilibrium corresponds to wages, w and w^*, a schedule of world prices, $p \equiv (p_i)$, a pair of consumption schedules, $c \equiv (c_i)$ and $c^* \equiv (c_i^*)$, and a pair of output schedules, $q \equiv (q_i)$ and $q^* \equiv (q_i^*)$, such that equations (1)-(8) hold. By Walras' Law, if the previous conditions hold, then labor market clearing at home holds as well.

2.3 The Domestic Government's Problem

We assume that Home is a strategic country that sets ad-valorem trade taxes $t \equiv (t_i)$ in order to maximize domestic welfare, whereas Foreign is passive. Formally, the domestic government's problem is to choose t in order to maximize the utility of its representative consumer, U, subject to (i) utility maximization and profit maximization by domestic consumers and firms at (distorted) local prices $p_i(1+t_i)$, conditions (1), (2), and

(5); (ii) utility maximization and profit maximization by foreign consumers and firms at (undistorted) world prices p_i, conditions (3), (4), and (6); and (iii) good and labor market clearing, conditions (7) and (8). This leads to the following definition.

Definition 1. *The domestic government's problem is* $\max_t U$ *subject to (1)-(8).*

The goal of the next two sections is to characterize how unilaterally optimal trade taxes, i.e., taxes that solve the domestic government's problem, vary with Home's comparative advantage, as measured by the relative unit labor requirements a_i^*/a_i. To do so we follow the public finance literature and use the primal approach. Namely, we will first approach the optimal policy problem of the domestic government in terms of a planning problem in which domestic consumption and domestic output can be chosen directly (Section 3). We will then establish that the optimal allocation can be implemented through trade taxes and characterize the structure of these taxes (Section 4).

3 Optimal Allocation

3.1 Home's Planning Problem

Throughout this section we focus on a fictitious environment in which the domestic government directly controls domestic consumption, c, and domestic output, q. In this fully controlled economy, the domestic government's problem can be rearranged as a planning problem ignoring the three equilibrium conditions associated with utility maximization and profit maximization by domestic consumers and firms. We refer to this new maximization problem as Home's planning problem.

Definition 2. *Home's planning problem is* $\max_{c,c^*,q\geq 0,q^*\geq 0,p\geq 0} U$ *subject to (3), (4), (6), (7), (8), and the resource constraint,* $\int_i a_i q_i di \leq L$.

In order to facilitate our discussion of optimal trade taxes, we focus on the schedule of net imports $m \equiv c - q$ and the foreign wage w^* as the two key control variables of the domestic government. In the proof of the next lemma, we establish that if foreign consumers maximize their utility, foreign firms maximize their profits, and markets clear, then the equilibrium world prices and foreign output are equal to

$$p_i\left(m_i, w^*\right) \equiv \min\left\{u_i^{*\prime}\left(-m_i\right), w^* a_i^*\right\}, \tag{9}$$

$$q_i^*\left(m_i, w^*\right) \equiv \max\left\{m_i + d_i^*(w^* a_i^*), 0\right\}, \tag{10}$$

where we slightly abuse notation and adopt the convention $u_i^{*\prime}\left(-m_i\right) = \infty$ if $m_i \geq 0$.

Using the previous observation, the set of solutions to Home's planning problem can be characterized as follows.

Lemma 1. *If $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ solves Home's planning problem, then there exists $w^{0*} \geq 0$ such that $\left(m^0 = c^0 - q^0, q^0, w^{0*}\right)$ solves*

$$\max_{m,q\geq 0,w^*\geq 0} \int_i u_i(q_i + m_i)di \tag{P}$$

subject to:

$$\begin{aligned} \int_i a_i q_i di &\leq L, & (11)\\ \int_i a_i^* q_i^* \left(m_i, w^*\right) di &\leq L^*, & (12)\\ \int_i p_i(m_i, w^*) m_i di &\leq 0. & (13) \end{aligned}$$

Conversely, if $\left(m^0, q^0, w^{0*}\right)$ solves (P), then there exists a solution $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ to Home's planning problem such that $c^0 - q^0 = m^0$.

The formal proof of Lemma 1 as well as all subsequent proofs can be found in the Appendix. The first and second constraints, equations (11) and (12), correspond to resource constraints in Home and Foreign. The third constraint, equation (13), corresponds to Home's (or Foreign's) trade balance condition. It characterizes the set of feasible net imports. If Home were a small open economy, then it would take $p_i(m_i, w^*)$ as exogenously given and the solution to (P) would coincide with the free trade equilibrium. Here, in contrast, Home internalizes the fact that net import decisions affect world prices, both directly through their effects on the marginal utility of the foreign consumer and indirectly through their effects on the foreign wage.

Two technical aspects of Home's planning problem are worth mentioning at this point. First, in spite of the fact that Foreign's budget constraint and labor market clearing condition, equations (4) and (8), must bind in equilibrium, the solution to Home's planning problem can be obtained as the solution to a relaxed planning problem (P) that only features inequality constraints. This will allow us to invoke Lagrangian (necessity) theorems in Section 3.2. Second, for all values of w^* that are feasible in the sense that the set of import and output levels $m, q \geq 0$ that satisfy (11)-(13) is non-empty, Home's planning problem can be decomposed into: (i) an inner problem:

$$V(w^*) \equiv \max_{m,q\geq 0} \int_i u_i(q_i + m_i)di \tag{P_{w^*}}$$

subject to (11)-(13), that takes the foreign wage w^* as given; and (ii) an outer problem:

$$\max_{w^* \geq 0} V(w^*),$$

that maximizes the value function $V(w^*)$ associated with (P_{w^*}) over all possible values of the foreign wage. It is the particular structure of the inner problem (P_{w^*}) that will allow us to characterize the main qualitative properties of the optimal allocation.

Compared to the general problem studied in Dixit (1985), the Ricardian nature of the economy and the assumption of additively separable utility imposes additional structure on the world price schedule, as captured in equation (9). Conditional on the foreign wage w^*, the world price of each good only depends on its own net imports, as in a two-good general equilibrium model or in a partial equilibrium model. Whenever Foreign produces a good, world prices must equal foreign labor costs, regardless of how much Home sells to or buys from world markets. Whenever Foreign does not produce a good, world prices must equal the marginal utility of the foreign consumer, which then depends on the level of domestic (net) imports of that particular good.

In the next two subsections, we will take the foreign wage w^* as given and characterize the main qualitative properties of the solutions to the inner problem (P_{w^*}). Since such properties will hold for all feasible values of the foreign wage, they will hold for the optimal one, $w^{0*} \in \arg\max_{w^* \geq 0} V(w^*)$, and so by Lemma 1, they will apply to any solution to Home's planning problem. Of course, for the purposes of obtaining quantitative results we also need to solve for the optimal foreign wage, w^{0*}, which we will do in Section 6.

Two observations will facilitate our analysis of the inner problem (P_{w^*}). First, as we will formally demonstrate, (P_{w^*}) is concave, which implies that its solutions can be computed using Lagrange multiplier methods.[4] Second, both the objective function and the constraints in (P_{w^*}) are additively separable in (m_i, q_i). In the words of Everett (1963), (P_{w^*}) is a "cell-problem." Using Lagrange multiplier methods, we will therefore be able to transform an infinite dimensional problem with constraints into a series of simple unconstrained, low-dimensional problems.

[4] At a technical level, this is a key difference between our approach and the approaches used in Amador et al. (2006), Amador and Bagwell (2013), and Costinot et al. (2013). Amador et al. (2006) and Costinot et al. (2013) both analyze optimization problems that are assumed to be concave, whereas Amador and Bagwell (2013) construct Lagrange multipliers such that Lagrangians are concave. Section 5 further illustrates the usefulness of being able to identify concave subproblems by showing how our results can be extended to environments with weakly separable preferences in a straightforward manner.

3.2 Lagrangian Formulation

The Lagrangian associated with (P_{w^*}) is given by

$$\mathcal{L}\left(m, q, \lambda, \lambda^*, \mu; w^*\right) \equiv \int_i u_i\left(q_i + m_i\right) di - \lambda \int_i a_i q_i di - \lambda^* \int_i a_i^* q_i^*\left(m_i, w^*\right) di - \mu \int_i p_i(m_i, w^*) m_i di,$$

where $\lambda \geq 0$, $\lambda^* \geq 0$, and $\mu \geq 0$ are the Lagrange multipliers associated with constraints (11)-(13). As alluded to above, the crucial property of $\mathcal{L}$ is that it is additively separable in (m_i, q_i). This implies that in order to maximize $\mathcal{L}$ with respect to (m, q), one simply needs to maximize the good-specific Lagrangian,

$$\mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right) \equiv u_i\left(q_i + m_i\right) - \lambda a_i q_i - \lambda^* a_i^* q_i^*\left(m_i, w^*\right) - \mu p_i(m_i, w^*) m_i,$$

with respect to (m_i, q_i) for almost all i. In short, cell problems can be solved cell-by-cell, or in the present context, good-by-good.

Building on the previous observation, the concavity of (P_{w^*}), and Lagrangian necessity and sufficiency theorems—Theorem 1, p. 217 and Theorem 1, p. 220 in Luenberger (1969), respectively—we obtain the following characterization of the set of solutions to (P_{w^*}).

Lemma 2. *For any feasible w^*, $\left(m^0, q^0\right)$ solves (P_{w^*}) if and only if there exist Lagrange multipliers $(\lambda, \lambda^*, \mu)$ such that for almost all i, $\left(m_i^0, q_i^0\right)$ solves*

$$\max_{m_i, q_i \geq 0} \mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right) \tag{P_i}$$

and the three following conditions hold:

$$\lambda \geq 0,\ \int_i a_i q_i^0 di \leq L, \text{ with complementary slackness,} \tag{14}$$
$$\lambda^* \geq 0,\ \int_i a_i^* q_i^*\left(m_i^0, w^*\right) di \leq L^*, \text{ with complementary slackness,} \tag{15}$$
$$\mu \geq 0,\ \int_i p_i(m_i, w^*) m_i^0 di \leq 0, \text{ with complementary slackness.} \tag{16}$$

Let us take stock. We started with Home's planning problem, which is an infinite dimensional problem in consumption and output in both countries as well as world prices. We then transformed it into a new planning problem (P) that only involves the schedule of domestic net imports, m, domestic output, q, and the foreign wage, w^*, but remains infinitely dimensional (Lemma 1). Finally, in this subsection we have taken advantage of the concavity and the additive separability of the inner problem (P_{w^*}) in (m_i, q_i) to go from one high-dimensional problem with constraints to many two-dimensional, unconstrained maximization problems (P_i) using Lagrange multiplier methods.

The goal of the next subsection is to solve these two-dimensional problems in (m_i, q_i) taking both the foreign wage, w^*, and the Lagrange multipliers, $(\lambda, \lambda^*, \mu)$, as given. This is all we will need to characterize qualitatively how comparative advantage affects the solution of Home's planning problem and, as discussed in Section 4, the structure of optimal trade taxes. Once again, a full quantitative computation of optimal trade taxes will depend on the equilibrium values of $(\lambda, \lambda^*, \mu)$, found by using conditions (14)-(16), and the value of w^* that maximizes $V(w^*)$, calculations that we defer until Section 6.

3.3 Optimal Output and Net Imports

Our objective here is to find the solution $\left(m_i^0, q_i^0\right)$ of

$$\max_{m_i, q_i \geq 0} \mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right) \equiv u_i\left(q_i + m_i\right) - \lambda a_i q_i - \lambda^* a_i^* q_i^*\left(m_i, w^*\right) - \mu p_i(m_i, w^*) m_i,$$

deferring a discussion of the intuition underlying our results until we re-introduce trade taxes in Section 4. We proceed in two steps. First, we solve for the output level $q_i^0\left(m_i\right)$ that maximizes $\mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right)$, taking m_i as given. Second, we solve for the net import level m_i^0 that maximizes $\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right)$. The optimal output level is then simply given by $q_i^0 = q_i^0\left(m_i^0\right)$.

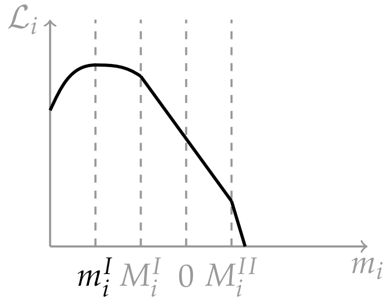
Since $\mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right)$ is strictly concave and differentiable in q_i, the optimal output level, $q_i^0\left(m_i\right)$, is given by the necessary and sufficient first-order condition,

$$u_i'\left(q_i^0\left(m_i\right) + m_i\right) \leq \lambda a_i, \text{ with equality if } q_i^0\left(m_i\right) > 0.$$

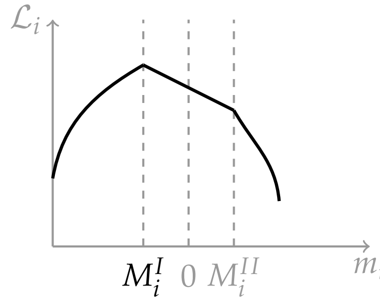
The previous condition can be rearranged in a more compact form as

$$q_i^0\left(m_i\right) = \max\left\{d_i\left(\lambda a_i\right) - m_i, 0\right\}. \tag{17}$$

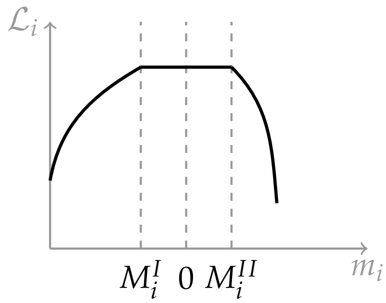
Let us now turn to our second Lagrangian problem, finding the value of m_i that maximizes $\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right)$. By using the same arguments as in the proof of Lemma 2, one can check that the previous Lagrangian is concave in m_i. However, it has two kinks. The first one occurs at $m_i = M_i^I \equiv -d_i^*(w^* a_i^*) < 0$, when Foreign starts producing good i. The second one occurs at $m_i = M_i^{II} \equiv d_i\left(\lambda a_i\right) > 0$, when Home stops producing good i. Accordingly, we cannot search for maxima of $\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right)$ by looking for stationary points. But, as we now demonstrate, this technicality is of little consequence for our approach, the end goal of which is the maximization of the Lagrangian with respect to m_i, not the location of its stationary points.



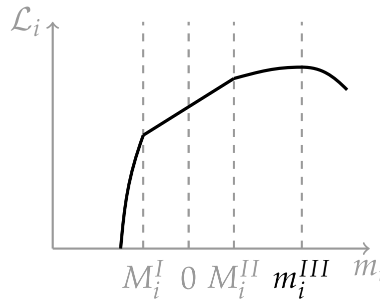
(a) $a_i/a_i^* < A^I$.



(b) $a_i/a_i^* \in [A^I, A^{II})$.



(c) $a_i/a_i^* = A^{II}$.



(d) $a_i/a_i^* > A^{II}$.

Figure 1: Optimal net imports.

To study how $\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right)$ varies with m_i, we consider separately the three regions partitioned by the two kinks: $m_i < M_i^I$, $M_i^I \leq m_i \leq M_i^{II}$, and $m_i > M_i^{II}$. First, suppose that $m_i < M_i^I$. In this region, equations (9), (10), and (17) imply

$$\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right) = u_i\left(d_i\left(\lambda a_i\right)\right) - \lambda a_i d_i\left(\lambda a_i\right) + \lambda a_i m_i - \mu m_i u_i^{*\prime}\left(-m_i\right).$$

Given CES utility (and hence $u_i^{*\prime}\left(c_i^*\right) = \beta_i^*\left(c_i^*\right)^{-1/\sigma}$), $\mathcal{L}_i$ is strictly increasing if $m_i \in \left(-\infty, m_i^I\right)$ and strictly decreasing if $m_i \in \left(m_i^I, M_i^I\right)$, with $m_i^I \equiv -\left(\frac{\sigma^*}{\sigma^*-1}\frac{\lambda a_i}{\mu \beta_i^*}\right)^{-\sigma^*}$. By definition of $M_i^I \equiv -d_i^*(w^* a_i^*) = -(w^* a_i^*/\beta_i^*)^{-\sigma^*}$, the interval $\left(m_i^I, M_i^I\right)$ is non-empty if $\frac{a_i}{a_i^*} < A^I \equiv \frac{\sigma^*-1}{\sigma^*}\frac{\mu w^*}{\lambda}$. When the previous inequality is satisfied, the concavity of $\mathcal{L}_i$ implies that Home exports m_i^I units of good i, as illustrated in Figure 1(a), whereas Foreign does not produce anything.

Second, suppose that $m_i \in \left[M_i^I, M_i^{II}\right]$. In this region, equations (9), (10), and (17) imply

$$\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right)$$
$$= u_i\left(d_i\left(\lambda a_i\right)\right) - \lambda a_i d_i\left(\lambda a_i\right) + \left(\lambda a_i - \left(\lambda^* + \mu w^*\right) a_i^*\right) m_i - \lambda^* a_i^* d_i^*\left(w^* a_i^*\right),$$

which is strictly decreasing in m_i if and only if $\frac{a_i}{a_i^*} < A^{II} \equiv \frac{\lambda^* + \mu w^*}{\lambda}$. When $\frac{a_i}{a_i^*} \in \left[A^I, A^{II}\right)$, the concavity of $\mathcal{L}_i$ implies that Home will export M_i^I units of good i, as illustrated in Figure 1(b). For these goods, Foreign is at a tipping point: it would start producing if Home's exports were to go down by any amount. In the knife-edge case, $\frac{a_i}{a_i^*} = A^{II}$, the Lagrangian is flat between M_i^I and $M_i^{II} = m_i^{III}$ so that any import level between M_i^I and M_i^{II} is optimal, as illustrated in Figure 1(c). In this situation, either Home or Foreign may produce and export good i.

Finally, suppose that $M_i^{II} \leq m_i$. In this region, equations (9), (10), and (17) imply

$$\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right) = u_i\left(m_i\right) - \left(\lambda^* + \mu w^*\right) a_i^* m_i - \lambda^* a_i^* d_i^*\left(w^* a_i^*\right),$$

which is strictly increasing if $m_i \in \left(M_i^{II}, m_i^{III}\right)$ and strictly decreasing if $m_i \in \left(m_i^{III}, \infty\right)$, with $m_i^{III} \equiv d_i\left(\left(\lambda^* + \mu w^*\right) a_i^*\right)$. By definition of $M_i^{II} \equiv d_i\left(\lambda a_i\right)$, $\left(M_i^{II}, m_i^{III}\right)$ is non-empty if $\frac{a_i}{a_i^*} > A^{II} \equiv \frac{\lambda^* + \mu w^*}{\lambda}$. When this inequality is satisfied, the concavity of $\mathcal{L}_i$ implies that Home will import m_i^{III} units of good i, as illustrated in Figure 1(d).

We summarize the above observations into the following proposition.

Proposition 1. *If $\left(m_i^0, q_i^0\right)$ solves (P_i), then optimal net imports are such that: (a) $m_i^0 = m_i^I$, if $a_i/a_i^* < A^I$; (b) $m_i^0 = M_i^I$, if $a_i/a_i^* \in \left[A^I, A^{II}\right)$; (c) $m_i^0 \in \left[M_i^I, M_i^{II}\right]$ if $a_i/a_i^* = A^{II}$; and (d) $m_i^0 = m_i^{III}$, if $a_i/a_i^* > A^{II}$, where m_i^I, M_i^I, m_i^{III}, A^I, and A^{II} are the functions of w^* and $\left(\lambda, \lambda^*, \mu\right)$ defined above.*

Proposition 1 highlights the importance of comparative advantage, i.e., the cross-sectoral variation in the relative unit labor requirement a_i/a_i^*, for the structure of optimal imports. In particular, Proposition 1 implies that Home is a net exporter of good i only if $a_i/a_i^* < A^{II}$. Using Lemmas 1 and 2 to go from (P_i) to Home's planning problem, this leads to the following corollary.

Corollary 1. *At any solution to Home's planning problem, Home produces and exports goods in which it has a comparative advantage, $a_i/a_i^* < A^{II}$, whereas Foreign produces and exports goods in which it has a comparative advantage, $a_i/a_i^* > A^{II}$.*

According to Corollary 1, there will be no pattern of comparative advantage reversals at an optimum. Like in a free trade equilibrium, there exists a cut-off such that Home exports a good only if its relative unit labor requirement is below the cut-off. However, the value of that cut-off—as well as the level of net imports—may of course be different than those in a free trade equilibrium.

4 Optimal Trade Taxes

We now demonstrate how to implement the solution of Home's planning problem using trade taxes in a decentralized market equilibrium.

4.1 Wedges

Trade taxes cause domestic and world prices to differ from one another. To prepare our analysis of optimal trade taxes, we therefore start by describing the wedges, τ_i^0, between the marginal utility of the domestic consumer, $u_i'(c_i^0) = \beta_i \left(c_i^0\right)^{-\frac{1}{\sigma}}$ and the world price, p_i^0, that must prevail at any solution to Home's planning problem:

$$\tau_i^0 \equiv \frac{u_i'\left(c_i^0\right)}{p_i^0} - 1. \tag{18}$$

By Lemma 1, we know that if $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ solves Home's planning problem, then there exists $w^{0*} \geq 0$ such that $\left(m^0 = c^0 - q^0, q^0, w^{0*}\right)$ solves (P). In turn, this implies that $\left(m^0 = c^0 - q^0, q^0\right)$ solves (P_{w^*}) for $w^* = w^{0*}$, and by Lemma 2, that there exists $\left(\lambda^0, \lambda^{0*}, \mu^0\right)$ such that for almost all i, $\left(m_i^0, q_i^0\right)$ solves (P_i). Accordingly, for almost all i, the good-specific wedge can be expressed as

$$\tau_i^0 = \frac{u_i'\left(q_i^0\left(m_i^0\right) + m_i^0\right)}{p_i\left(m_i^0, w^{0*}\right)} - 1,$$

with m_i^0 satisfying conditions (i)-(iv) in Proposition 1. Combining this observation with equations (9) and (17), we obtain

$$\tau_i^0 = \begin{cases} \frac{\sigma^*-1}{\sigma^*}\mu^0 - 1, & \text{if } \frac{a_i}{a_i^*} < A^I \equiv \frac{\sigma^*-1}{\sigma^*}\frac{\mu^0 w^{0*}}{\lambda^0}; \\ \frac{\lambda^0 a_i}{w^{0*} a_i^*} - 1, & \text{if } A^I < \frac{a_i}{a_i^*} \leq A^{II} \equiv \frac{\mu^0 w^{0*} + \lambda^{0*}}{\lambda^0}; \\ \frac{\lambda^{0*}}{w^{0*}} + \mu^0 - 1, & \text{if } \frac{a_i}{a_i^*} > A^{II}. \end{cases} \tag{19}$$

Since $A^I < A^{II}$, we see that good-specific wedges are (weakly) increasing with a_i/a_i^*. For goods that are imported, $a_i/a_i^* > A^{II}$, wedges are constant and equal to $\frac{\lambda^{0*}}{w^{0*}} + \mu^0 - 1$.

For goods that are exported, $a_i/a_i^* < A^{II}$, the magnitude of the wedge depends on the strength of Home's comparative advantage: it increases linearly with a_i/a_i^* for goods such that $a_i/a_i^* \in (A^I, A^{II})$ and attains its minimum value for goods such that $a_i/a_i^* < A^I$.

4.2 Comparative Advantage and Trade Taxes

To conclude our baseline theoretical analysis, we now demonstrate that any solution to Home's planning problem can be implemented by constructing a schedule of taxes $t^0 = \tau^0$. To do so, we only need to check that the sufficient conditions for utility maximization and profit maximization by domestic consumers and firms at (distorted) local prices $p_i^0\left(1+t_i^0\right)$ are satisfied at the solution to Home's planning problem if $t_i^0 = \tau_i^0$.

Consider first the domestic consumer's problem. By equation (18), we know that

$$u_i'\left(c_i^0\right) = p_i^0\left(1+t_i^0\right).$$

Thus for any pair of goods, i_1 and i_2, we have

$$\frac{u_{i_1}'\left(c_{i_1}^0\right)}{u_{i_2}'\left(c_{i_2}^0\right)} = \frac{1+t_{i_1}^0}{1+t_{i_2}^0}\frac{p_{i_1}^0}{p_{i_2}^0}.$$

Hence, the domestic consumer's marginal rates of substitution are equal to domestic relative prices. At a solution to Home's planning problem we also know from Lemma 1 that $\int_i m_i^0 p_i^0 di = 0$. Thus the domestic consumer's budget constraint must hold as well, which implies that the sufficient conditions for utility maximization are satisfied.

Let us now turn to the domestic firm's problem. At a solution to Home's planning problem, we have already argued in Section 3.3 that for almost all i,

$$u_i'\left(q_i^0 + m_i^0\right) \leq \lambda^0 a_i, \text{ with equality if } q_i^0 > 0.$$

Since $u_i'\left(q_i^0 + m_i^0\right) = u_i'\left(c_i^0\right) = p_i^0\left(1+t_i^0\right)$, the sufficient condition for profit maximization is satisfied, with the domestic wage in the trade equilibrium given by the Lagrange multiplier on the labor resource constraint, λ^0.

At this point, we have established that any solution to Home's planning problem can be implemented by constructing a schedule of taxes $t^0 = \tau^0$. Furthermore, by the analysis of Section 4.1, such a schedule of taxes must satisfy equation (19). Conversely, if t^0 is a solution to the domestic's government problem, then the allocation and prices in the trade equilibrium associated with taxes t^0, $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$, must solve Home's planning problem and, by utility maximization, satisfy

$$t_i^0 = \frac{u_i'\left(c_i^0\right)}{\theta p_i^0} - 1,$$

with θ the Lagrange multiplier on the domestic consumer's budget constraint. By the exact same logic as in Section 4.1, such a schedule of taxes must then be such that

$$t_i^0 \equiv \begin{cases} \frac{\sigma^*-1}{\sigma^*}\frac{\mu^0}{\theta} - 1, & \text{if } \frac{a_i}{a_i^*} < A^I \equiv \frac{\sigma^*-1}{\sigma^*}\frac{\mu^0 w^{0*}}{\lambda^0}; \\ \frac{\lambda^0 a_i}{w^{0*} a_i^* \theta} - 1, & \text{if } A^I < \frac{a_i}{a_i^*} \leq A^{II} \equiv \frac{\mu^0 w^{0*} + \lambda^{0*}}{\lambda^0}; \\ \frac{\lambda^{0*} + \mu^0 w^{0*}}{\theta w^{0*}} - 1, & \text{if } \frac{a_i}{a_i^*} > A^{II}. \end{cases} \tag{20}$$

This leads to the following characterization of optimal trade taxes.

Proposition 2. *At any solution t^0 of the domestic government's problem: (a) $t_i^0 = (1+\bar{t})\left(A^I/A^{II}\right) - 1$, if $a_i/a_i^* < A^I$; (b) $t_i^0 = (1+\bar{t})\left(\left(a_i/a_i^*\right)/A^{II}\right) - 1$, if $a_i/a_i^* \in \left[A^I, A^{II}\right]$; and (c) $t_i^0 = \bar{t}$, if $a_i/a_i^* > A^{II}$, with $\bar{t} > -1$ and $A^I < A^{II}$.*

Proposition 2 states that optimal trade taxes vary with comparative advantage as wedges do. Trade taxes are at their lowest values, $(1+\bar{t})\left(A^I/A^{II}\right) - 1$, for goods in which Home's comparative advantage is the strongest, $a_i/a_i^* < A^I$; they are linearly increasing with a_i/a_i^* for goods in which Home's comparative advantage is in some intermediate range, $a_i/a_i^* \in \left[A^I, A^{II}\right]$; and they are at their highest value, $\bar{t}$, for goods in which Home's comparative advantage is the weakest, $a_i/a_i^* > A^{II}$. Of course, the overall level of taxes is indeterminate, as captured by $\bar{t} > -1$ in the previous proposition, since only relative prices and hence relative taxes matter for domestic consumers and firms. Figure 2 illustrates two polar cases. On Panel (a), there are no import tariffs, $\bar{t} = 0$, and all exported goods are subject to an export tax that rises with comparative advantage. On Panel (b), in contrast, all imported goods are subject to a tariff $\bar{t} = \frac{A^{II}}{A^I} - 1 \geq 0$, whereas exported goods receive a subsidy that falls with comparative advantage. For expositional purposes, we focus in the rest of our discussion on the solution with zero import tariffs, $\bar{t} = 0$.

To gain intuition about the economic forces that shape optimal trade taxes, consider first the case in which foreign preferences are Cobb-Douglas, $\sigma^* = 1$, as in Dornbusch et al. (1977). In this case, $A^I = 0$ so that the first region, $a_i/a_i^* < A^I$, is empty. In the second region, $a_i/a_i^* \in \left[A^I, A^{II}\right]$, there is *limit pricing*: Home exports the goods and sets export taxes $t_i^0 < \bar{t} = 0$ such that foreign firms are exactly indifferent between producing and not producing those goods, i.e., such that the world price satisfies $p_i^0 = \lambda^0 a_i/\theta(1+t_i^0) =$

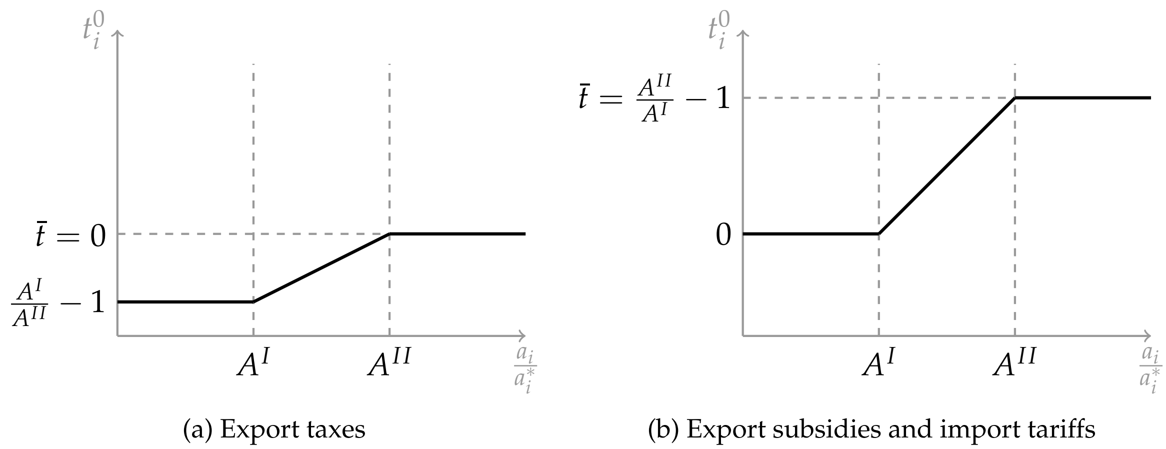


Figure 2: Optimal trade taxes.

$w^{0*}a_i^*$. The less productive are foreign firms relative to domestic firms, the more room Home has to manipulate prices, and the bigger the export tax is (in absolute value).

In the more general case, $\sigma^* \geq 1$, as in Wilson (1980), Eaton and Kortum (2002), and Alvarez and Lucas (2007), the first region, $a_i/a_i^* < A^I$, is no longer necessarily empty. The intuition, however, remains simple. In this region the domestic government has incentives to charge a *constant monopoly markup*, proportional to $\sigma^*/(\sigma^* - 1)$. Specifically, the ratio between the world price and the domestic price is equal to $1/(1 + t_i^0) = \frac{\sigma^*}{\sigma^*-1}\frac{\theta}{\mu^0}$. In the region $a_i/a_i^* \in \left[A^I, A^{II}\right]$, limit pricing is still optimal. But because A^I is increasing in σ^*, the extent of limit pricing, all else equal, decreases with the elasticity of demand in the foreign market.

4.3 Discussion

Proposition 2 is related to the results of Itoh and Kiyono (1987). As mentioned in the Introduction, they show that in the Ricardian model with Cobb-Douglas preferences considered by Dornbusch et al. (1977), export subsidies may be welfare enhancing. A key feature of the welfare-enhancing subsidies that they consider is that they are not uniform across goods and are concentrated on "marginal" goods. This is consistent with our observation that, at the optimum, export taxes should be weakly decreasing (in absolute value) with Home's relative unit labor requirements, a_i/a_i^*, so that "marginal" goods should indeed be taxed less.

Whereas our finding accords well with the observation that governments often protect a small number of less competitive industries, such targeted subsidies do not stem from a greater desire to expand production in less competitive sectors. On the contrary, they reflect tighter constraints on the ability to exploit monopoly power by contracting exports. According to Proposition 2, Home can only charge constant monopoly markups for exported goods in which its comparative advantage is the strongest. For other exported goods, the threat of entry of foreign firms lead markups to go down with Home's comparative advantage.

An interesting issue is whether the structure of optimal trade taxes crucially relies on the assumption that domestic firms are perfectly competitive. Since Home's government behaves like a domestic monopolist competing à la Bertrand against foreign firms, one may conjecture that if each good were produced by only one domestic firm, then Home would no longer have to use trade taxes to manipulate prices: domestic firms would already manipulate prices under laissez-faire. This conjecture, however, is incorrect for two reasons. The first one is that although the government behaves like a monopolist, the domestic government's problem involves non-trivial general equilibrium considerations, as reflected in the values of $(w^{0*}, \lambda^0, \lambda^{0*}, \mu^0)$. Even when Home charges a constant monopoly markup, the optimal level of the markup differs from what an individual firm would choose, i.e., $\sigma^* / (\sigma^* - 1)$. The second reason is that to manipulate prices, Home's government needs to affect the behaviors of both firms and consumers: net imports depend both on supply and demand. If each good were produced by only one domestic firm, Home's government would still need to impose good-varying consumption taxes that mimic the trade taxes described above (plus output subsidies that reflect general equilibrium considerations). Intuitively, if each good were produced by only one domestic firm, consumers would face monopoly markups in each country, whereas optimality requires a wedge between consumer prices at home and abroad, as we show above.

5 Robustness

In this section we incorporate general preferences and trade costs into the Ricardian model presented in Section 2. Our goal is twofold. First, we want to demonstrate that Lagrange multiplier methods, and in particular our strategy of identifying concave cell-problems, remain well suited to analyzing optimal trade policy in this more general environment. Second, we want to show that the central predictions derived in Section 4 do not crucially hinge on the assumption of CES utility or the absence of trade costs. To save on space, we focus on sketching alternative environments and summarizing their main implications.

5.1 Preferences

While the assumption of CES utility is standard in the Ricardian literature—from Dornbusch et al. (1977) to Eaton and Kortum (2002)—it implies strong restrictions on the demand-side of the economy: own-price elasticities and elasticities of substitution are both constant and pinned down by a single parameter, σ. In practice, price elasticities may vary with quantities consumed and patterns of substitutions may vary across goods. For instance, one would expect two varieties of cars to be closer substitutes than, say, cars and bikes.

Here we relax the assumptions of Section 2 by assuming that: (i) Home's preferences are weakly separable over a discrete number of sectors, $s \in \mathcal{S} \equiv \{1, ..., S\}$; and that: (ii) subutility within each sector, U^s, is additively separable, though not necessarily CES. Specifically, we assume that Home's preferences can be represented by the following utility function,

$$U = F\left(U^1\left(c^1\right), ..., U^S\left(c^S\right)\right),$$

where F is a strictly increasing function; $c^s \equiv (c_i)_{i \in \mathcal{I}^s}$ denotes the consumption of goods in sector s, with $\mathcal{I}^s$ the set of goods that belongs to that sector; and the subutility U^s is such that

$$U^s\left(c^s\right) = \int_{i \in \mathcal{I}^s} u_i^s(c_i) di.$$

Foreign's preferences are similar, and asterisks denote foreign variables. Section 2 corresponds to the special case in which there is only one-sector, $S = 1$, and U^s is CES, $u_i^s\left(c_i\right) \equiv \beta_i^s \left(c_i^{1-1/\sigma^s} - 1\right) \Big/ \left(1 - 1/\sigma^s\right)$.[5]

For expositional purposes, let us start by considering an intermediate scenario in which utility is not CES, but we maintain the assumption that there is only one sector, $S = 1$. It should be clear that the CES assumption is not crucial for the results derived in Sections 2.2-3.2. In contrast, CES plays a key role in determining the optimal level of net imports, $m_i^I = -\left(\frac{\sigma^*}{\sigma^* - 1} \frac{\lambda a_i}{\mu \beta_i^*}\right)^{-\sigma^*}$, in Section 3.3 and, in turn, in establishing that trade taxes are at their lowest values, $(1 + \bar{t})\left(A^I / A^{II}\right) - 1$, for goods in which Home's comparative advantage is the strongest in Section 4.2. Absent CES utility, trade taxes would still be at their highest value, $\bar{t}$, for goods in which Home's comparative advantage is the weakest and linearly increasing with a_i / a_i^* for goods in which Home's comparative advantage is in some intermediate range. But for goods in which Home's comparative advantage is the strongest, the optimal trade tax would now vary with the elasticity of demand, reflecting

[5] The analysis in this section trivially extends to the case in which only a subset of sectors have additively separable utility. For this subset of sectors, and this subset only, our predictions would remain unchanged.

the incentives to charge different monopoly markups.

Now let us turn to the general case with multiple sectors, $S \geq 1$. With weakly separable preferences abroad, one can check that foreign consumption in each sector must be a solution to

$$\max_{c^{s*}} U^{s*}\left(c^{s*}\right)$$

subject to

$$\textstyle\int_{i\in\mathcal{I}^s} p_i^s c_i^{s*} di = E^{s*}, \tag{21}$$

where E^{s*} is total expenditure on sector s goods. Accordingly, by the same argument as in Section 3.1, we can write the world price and foreign output for all $s \in \mathcal{S}$ and $i \in \mathcal{I}^s$ as

$$p_i^s\left(m_i, w^*, E^{s*}\right) \equiv \min\left\{u_i^{s*\prime}\left(-m_i\right)\theta^{s*}\left(E^{s*}\right), w^* a_i^*\right\}, \tag{22}$$

and

$$q_i^{s*}\left(m_i, w^*, E^{s*}\right) \equiv \max\left\{m_i + d_i^{s*}(w^* a_i^* / \theta^{s*}\left(E^{s*}\right)), 0\right\}, \tag{23}$$

where $\theta^{s*}\left(E^{s*}\right)$ is the Lagrange multiplier associated with (21) for a given value of E^{s*}.

As we show in our online Addendum, Home's planning problem can still be decomposed into an outer problem and multiple inner problems, one for each sector. At the outer level, the government now chooses the foreign wage, w^*, together with the sectoral labor allocations in Home and Foreign, L^s and L^{s*}, the sectoral trade deficits, T^s, subject to aggregate factor market clearing and trade balance. At the inner level, the government treats L^s, L^{s*}, T^s, and w^* as constraints and maximizes sector-level utility sector-by-sector. More precisely, , Home's planning problem can be expressed as

$$\max_{\{L^s, L^{s*}, T^s\}_{s\in\mathcal{S}}, w^*\geq 0} F\left(V^1\left(L^1, L^{1*}, T^1, w^*\right), ..., V^S\left(L^S, L^{S*}, T^S, w^*\right)\right)$$

subject to

$$\begin{aligned}\textstyle\sum_{s\in\mathcal{S}} L^s &= L,\\ \textstyle\sum_{s\in\mathcal{S}} L^{s*} &= L^*,\\ \textstyle\sum_{s\in\mathcal{S}} T^s &= 0,\end{aligned}$$

where the sector-specific value function satisfies

$$V^s\left(L^s, L^{s*}, T^s, w^*\right) \equiv \max_{m^s, q^s \geq 0} \textstyle\int_{i\in\mathcal{I}^s} u_i^s(m_i + q_i) di$$

subject to

$$
\begin{aligned}
\int_{i\in\mathcal{I}^s} a_i q_i di &= L^s, \\
\int_{i\in\mathcal{I}^s} a_i^* q_i^{s*}\left(m_i, w^*, w^* L^{s*} - T^s\right) di &= L^{s*}, \\
\int_{i\in\mathcal{I}^s} p_i^s(m_i, w^*, w^* L^{s*} - T^s) m_i di &= T^s.
\end{aligned}
$$

Given equations (22) and (23), the sector-specific problem is the same type of maximization problem as in the baseline case (program P). As in Section 3.2, we can therefore reformulate each infinite-dimensional sector-level problem into many two-dimensional, unconstrained maximization problems using Lagrange multiplier methods. Within any sector with CES utility, all of our previous results hold exactly. Within any sector in which utility is not CES, our previous results continue to hold subject to the qualification about monopoly markups discussed above.

5.2 Trade Costs

Trade taxes and subsidies are not the only forces that may cause domestic and world prices to diverge. Here we extend our model to incorporate exogenous iceberg trade costs, $\delta \geq 1$, such that if 1 unit of good i is shipped from one country to another, only a fraction $1/\delta$ arrives. In the canonical two-country Ricardian model with Cobb-Douglas preferences considered by Dornbusch et al. (1977), these costs do not affect the qualitative features of the equilibrium beyond giving rise to a range of commodities that are not traded. We now show that similar conclusions arise from the introduction of trade costs in our analysis of optimal trade policy.

We continue to define world prices, p_i, as those prevailing in Foreign and let

$$
\phi\left(m_i\right) \equiv \begin{cases} \delta, & \text{if } m_i \geq 0, \\ 1/\delta, & \text{if } m_i < 0, \end{cases} \tag{24}
$$

denote the gap between the domestic price and the world price in the absence of trade taxes.

As in our benchmark model, the domestic government's problem can be reformulated and transformed into many two-dimensional, unconstrained maximization problems using Lagrange multiplier methods. In the presence of trade costs, Home's objective is to find the solution $\left(m_i^0, q_i^0\right)$ of the good-specific Lagrangian,

$$\max_{m_i, q_i \geq 0} \mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right) \equiv u_i\left(q_i + m_i\right) - \lambda a_i q_i$$
$$- \lambda^* a_i^* q_i^*\left(m_i, w^*\right) - \mu p_i(m_i, w^*) \phi\left(m_i\right) m_i$$

where $p_i\left(m_i, w^*\right)$ is given by

$$p_i\left(m_i, w^*\right) \equiv \min\left\{u_i^{*\prime}\left(-m_i \phi\left(m_i\right)\right), w^* a_i^*\right\} \tag{25}$$

and $q_i^*\left(m_i, w^*\right)$ is given by

$$q_i^*\left(m_i, w^*\right) \equiv \max\left\{m_i \phi\left(m_i\right) + d_i^*(w^* a_i^*), 0\right\}. \tag{26}$$

Compared to the analysis of Section 3, if Home exports $-m_i > 0$ units abroad, then Foreign only consumes $-m_i/\delta$ units. Conversely, if Home imports $m_i > 0$ units from abroad, then Foreign must export $m_i \delta$ units. This explains why $\phi\left(m_i\right)$ appears in the two previous expressions.

The introduction of transportation costs leads to a new kink in the good-specific Lagrangian. In addition to the kinks associated with both $q_i^*\left(m_i, w^*\right)$ and $p_i\left(m_i, w^*\right)$ at $m_i = M_i^I/\delta \equiv -d_i^*(w^* a_i^*)/\delta$ and the kink associated with $q_i\left(m_i\right)$ at $m_i = M_i^{II} \equiv d_i\left(\lambda a_i\right)$, there is now a kink associated with both $q_i^*\left(m_i, w^*\right)$ and $p_i\left(m_i, w^*\right)$ at $m_i = 0$. As before, since we are not looking for stationary points, this technicality does not complicate our problem. When maximizing the good-specific Lagrangian, $\mathcal{L}_i\left(\cdot\right)$, we simply consider four regions in m_i space: $m_i < M_i^I/\delta$, $M_i^I/\delta \leq m_i < 0$, $0 \leq m_i < M_i^{II}$, and $m_i \geq M_i^{II}$.

As in Section 3, if Home's comparative advantage is sufficiently strong, $a_i/a_i^* \leq \frac{1}{\delta} A^I \equiv \frac{1}{\delta} \frac{\sigma^* - 1}{\sigma^*} \frac{\mu w^*}{\lambda}$, then optimal net imports are $m_i^0 = \delta^{1-\sigma^*} m_i^I \equiv -\left(\frac{\sigma^*}{\sigma^* - 1} \frac{\lambda a_i}{\mu \beta_i^*}\right)^{-\sigma^*} \delta^{1-\sigma^*}$. Similarly, if Foreign's comparative advantage is sufficiently strong, $a_i/a_i^* > \delta A^{II} \equiv \delta \frac{\lambda^* + \mu w^*}{\lambda}$, then optimal net imports are $m_i^0 = \tilde{m}_i^{II} \equiv d_i\left(\left(\lambda^* + \mu w^*\right) \delta a_i^*\right)$. Relative to the benchmark model, there is now a range of goods for which comparative advantage is intermediate, $a_i/a_i^* \in \left(\frac{1}{\delta} A^{II}, \delta A^{II}\right)$, in which no international trade takes place. For given values of the foreign wage, w^*, and the Lagrange multipliers, λ, λ^*, μ, this region expands as trade costs become larger, i.e., as δ increases.

Building on the previous observations, we obtain the following generalization of Proposition 1.

Proposition 3. *Optimal net imports are such that: (a) $m_i^0 = \delta^{1-\sigma^*} m_i^I$, if $a_i/a_i^* \leq A^I$; (b)*

$m_i^0 = \frac{1}{\delta} M_i^I$, *if* $a_i/a_i^* \in \left(A^I, \frac{1}{\delta}A^{II}\right)$; (c) $m_i^0 \in \left[\frac{1}{\delta}M_i^I, 0\right]$ *if* $a_i/a_i^* = \frac{1}{\delta}A^{II}$; (d) $m_i^0 = 0$, *if* $a_i/a_i^* \in \left(\frac{1}{\delta}A^{II}, \delta A^{II}\right)$; (e) $m_i^0 \in \left(0, \tilde{m}_i^{II}\right)$ *if* $a_i/a_i^* = \delta A^{II}$; *and* (f) $m_i^0 = \tilde{m}_i^{II}$, *if* $a_i/a_i^* > \delta A^{II}$.

Using Proposition 3, it is straightforward to show, as in Section 4.1, that good-specific wedges are (weakly) increasing with Home's comparative advantage. Similarly, as in Section 4.2, we can show that any solution to Home's planning problem can be implemented by constructing a schedule of taxes, and that the optimal trade taxes vary with comparative advantage as wedges do. In summary, our main theoretical results are also robust to the introduction of exogenous iceberg trade costs.

6 Applications

To conclude, we apply our theoretical results to two sectors: agriculture and manufacturing. Our goal is to take a first look at the quantitative importance for welfare of optimal trade taxes, both in an absolute sense and relative to simpler uniform trade taxes.

In both applications, we compute optimal trade taxes as follows. First, we use Proposition 1 to solve for optimal imports and output given arbitrary values of the Lagrange multipliers, $(\lambda, \lambda^*, \mu)$, and the foreign wage, w^*. Second, we use conditions (14)-(16) in Lemma 2 to solve for the Lagrange multipliers. Finally, we find the value of the foreign wage that maximizes the value function $V(w^*)$ associated with the inner problem. Given the optimal foreign wage, w^{0*}, and the associated Lagrange multipliers, $(\lambda^0, \lambda^{0*}, \mu^0)$, we finally compute optimal trade taxes using Proposition 2.

6.1 Agriculture

In many ways, agriculture provides the perfect environment in which to explore the quantitative importance of our results. From a theoretical perspective, market structure is as close as possible to the neoclassical ideal. From a measurement perspective, the scientific knowledge of agronomists provides a unique window into the structure of comparative advantage, as discussed in Costinot and Donaldson (2011). Finally, from a policy perspective, agricultural trade taxes are pervasive and one of the most salient and contentious global economic issues, as illustrated by the World Trade Organization's current, long-stalled Doha round.

Calibration. We start from the Ricardian economy presented in Section 2.1 and assume that each good corresponds to one of 39 crops for which we have detailed productivity data, as we discuss below. All crops enter utility symmetrically in all countries, $\beta_i =$

$\beta_i^* = 1$. Home is the United States and Foreign is an aggregate of the rest of the world (henceforth R.O.W.). The single factor of production is equipped land . We also explore how our results change in the presence of exogenous iceberg trade costs, as in Section 5.2.

The parameters necessary to apply our theoretical results are: (i) the unit factor requirement for each crop in each country, a_i and a_i^*; (ii) the elasticity of substitution, σ; (iii) the relative size of the two countries, L^*/L; and (iv) trade costs, δ, when relevant. For setting each crop's unit factor requirements, we use data from the Global Agro-Ecological Zones (GAEZ) project from the Food and Agriculture Organization (FAO); see Costinot and Donaldson (2011). Feeding data on local conditions—e.g., soil, topography, elevation and climatic conditions—into an agronomic model, scientists from the GAEZ project have computed the yield that parcels of land around the world could obtain if they were to grow each of the 39 crops we consider in 2009.[6] We set a_i and a_i^* equal to the average hectare per ton of output across all parcels of land in the United States and R.O.W., respectively; see Table 1 in our online Addendum.

The other parameters are chosen as follows. We set $\sigma = 8.1$ in line with the estimates of Costinot et al. (2012) using trade and price data for 10 major crops in 2009 from the FAOSTAT program at the FAO. We set $L = 1$ and $L^* = 13$ to match the relative acreage devoted to the 39 crops considered, as reported in the FAOSTAT data in 2009. Finally, in the extension with trade costs, we set $\delta = 1.32$ so that Home's import share in the equilibrium without trade policy matches the U.S. agriculture import share—that is, the total value of U.S. imports over the 39 crops considered divided by the total value of U.S. expenditure over those same crops—in the FAOSTAT data in 2009, 11.1%.

Results. The left and right panels of Figure 3 report optimal trade taxes on all traded crops i as a function of comparative advantage, a_i/a_i^*, in the calibrated examples without trade costs, $\delta = 1$, and with trade costs, $\delta = 1.32$, respectively.[7] The region between the two vertical lines in the right panel corresponds to goods that are not traded at the solution of Home's planning problem.

As discussed in Section 4.2, the overall level of taxes is indeterminate. Here we focus

[6]The GAEZ project constructs output per hectare predictions under different assumptions on a farmer's use of complementary inputs (e.g. irrigation, fertilizers, and machinery). We use the measure that is constructed under the assumption that irrigation and a "moderate" level of other inputs (fertilizers, machinery, etc.) are available to farmers.

[7]We compute optimal trade taxes, throughout this and the next subsection, by performing a grid search over the foreign wage w^* so as to maximize $V(w^*)$. Since Foreign cannot be worse off under trade than under autarky—whatever world prices may be, there are gains from trade—and cannot be better off than under free trade—since free trade is a Pareto optimum, Home would have to be worse off—we restrict our grid search to values of the foreign wage between those that would prevail in the autarky and free trade equilibria. Recall that we have normalized prices so that the Lagrange multiplier associated with the foreign budget constraint is equal to one. Thus w^* is the real wage abroad.

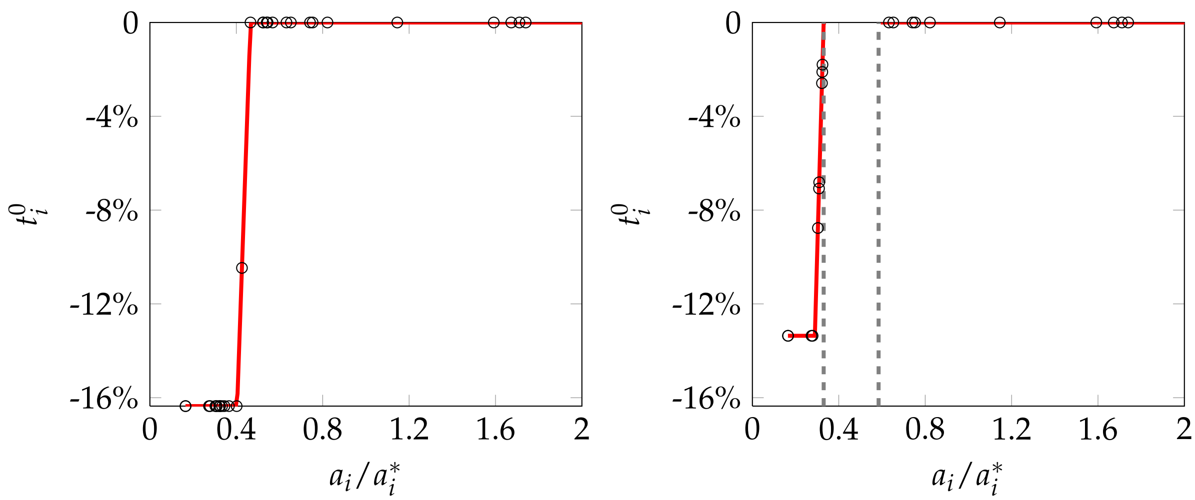


Figure 3: Optimal trade taxes for the agricultural case. The left panel assumes no trade costs, $\delta = 0$. The right panel assumes trade costs, $\delta = 1.32$.

on a normalization with zero import tariffs. Given the high elasticity of substitution across crops, $\sigma = 8.1$, the optimal monopoly markup is sufficiently low that there is little scope for limit pricing in either case: there are 2 crops out of 39 in this range in the left panel ($\delta = 1$) and and 6 out of 39 in the right panel ($\delta = 1.32$). This suggests that the additional gains associated with optimal trade taxes relative to a uniform tariff may be small in this example.

The first and second columns of Table 1 display U.S. and R.O.W. welfare gains from trade, i.e. the percentage change in total income divided by the CES price index relative to autarky, in the baseline model with no trade costs, $\delta = 1$. Three rows report the values of these gains from trade under each of three scenarios: (i) a laissez-faire regime with no U.S. trade taxes, (ii) a U.S. optimal uniform tariff, and (iii) U.S. optimal trade taxes as characterized in Proposition 2.[8] In this example, optimal trade taxes that are monotone in comparative advantage increase U.S. gains from trade in agriculture by 24% ($15.86/12.80 - 1 \simeq 0.24$) and decrease R.O.W.'s gains from trade by 59% ($1 - 0.99/2.39 \simeq 0.59$). Interestingly, in spite of the large elasticity of substitution, $\sigma = 8.1$, that plausibly applies to agricultural goods, approximately one third of the U.S. gains arise from the use of non-uniform trade taxes ($14.79/12.80 - 1 \simeq 0.16$).

The third and fourth columns of Table 1 revisit the previous three scenarios using the model with trade costs, setting $\delta = 1.32$. Not surprisingly, as the U.S. import shares goes

[8]Scenarios (i) and (ii) are computed using the equilibrium conditions (1)-(8) in Section 2.2. In scenario (i), we set $t_i = 0$ for all goods i. In scenario (ii) we set $t_i = t$ for all imported goods, we set $t_i = 0$ for other goods, and we do a grid search over t to find the optimal tariff.

	No Trade Costs ($\delta = 1$)		Trade Costs ($\delta = 1.32$)	
	U.S.	R.O.W.	U.S.	R.O.W.
Laissez-Faire	12.80%	2.39%	0.84%	0.06%
Uniform Tariff	14.79%	1.61%	0.99%	0.00%
Optimal Taxes	15.86%	0.99%	1.03%	0.00%

Table 1: Gains from trade for the agricultural case.

down from around 80% in the model without trade costs to its calibrated value of 11.1% in the model with trade costs, gains from trade also go down by an order of magnitude, from 12.8% to 0.8%. Yet, the relative importance of trade taxes that vary with comparative advantage remains fairly stable. Even with trade costs, gains from trade for the United States are 23% larger under optimal trade taxes than in the absence of any trade taxes ($1.03/0.84 - 1 \simeq 0.23$) and about one fifth of these gains arise from the use of non-uniform trade taxes ($0.99/0.84 \simeq 0.18$).

6.2 Manufacturing

There are good reasons to suspect that the quantitative results from Section 6.1 may not generalize to other tradable sectors. In practice, most traded goods are manufactured goods and elasticities of substitutions across those goods are generally much lower than across agricultural products. Everything else being equal, this should tend to increase the prevalence of limit pricing under optimal trade taxes and, therefore, to increase the gains associated with optimal trade taxes that vary with comparative advantage. We now explore the quantitative importance of such considerations.

Calibration. As in the previous subsection, we focus on the baseline Ricardian economy presented in Section 2.1 and the extension to iceberg trade costs presented in Section 5.2. Home and Foreign still correspond to the United States and R.O.W., respectively, but we now assume that each good corresponds to one of 400 manufactured goods that are produced using equipped labor.[9]

Compared to agriculture, the main calibration issue is how to set unit factor requirements. Since one cannot measure unit factor unit requirements directly for all manufactured goods in all countries, we follow the approach pioneered by Eaton and Kortum (2002) and assume that unit factor requirements are independently drawn across countries and goods from an extreme value distribution whose parameters can be calibrated to

[9]The number of goods is chosen to balance computational burden against distance between our model and models with a continuum of goods such as Eaton and Kortum (2002). We find similar results with other numbers of goods.

match a few key moments in the macro data. In a two-country setting, Dekle et al. (2007) have shown that this approach is equivalent to assuming

$$a_i = \left(\frac{i}{T}\right)^{\frac{1}{\theta}} \qquad \text{and} \qquad a_i^* = \left(\frac{1-i}{T^*}\right)^{\frac{1}{\theta}},$$

with θ the shape parameter of the extreme value distribution, that is assumed to be common across countries, and T and T^* the scale parameters, that are allowed to vary across countries. We let the good index i be equally spaced between $1/10,000$ and $1-1/10,000$ for the 400 goods in the economy.

Given the previous functional form assumptions, we choose parameters as follows. We set $\sigma = 2.6$ to match the median estimate of the elasticity of substitution across 5-digit SITC sectors in Broda and Weinstein (2006). We set $L = 1$ and $L^* = 19.5$ to match population in the U.S. relative to R.O.W., as reported by the World Bank in 2009. Since the shape parameter θ determines the elasticity of trade flows with respect trade costs, we set $\theta = 5$, which is a typical estimate in the literature; see e.g. Anderson and Van Wincoop (2004) and Head and Mayer (2013). Given the previous parameters, we then set $T = 5,194.6$ and $T^* = 1$ so that Home's share of world GDP matches the U.S. share, 26%, as reported by the World Bank in 2009. Finally, in the extension with trade costs, we now set $\delta = 1.44$ so that Home's import share in the equilibrium without trade policy matches the U.S. manufacturing import share—i.e. , total value of U.S. manufacturing imports divided by total value of U.S. expenditure in manufacturing—as reported in the OECD STructural ANalysis (STAN) database in 2009, 24.7%.

Results. Figure 4 reports optimal trade taxes as a function of comparative advantage for manufacturing. As before, the left and right panels correspond to the models without and with trade costs, respectively.

Together with the shape of the distribution of unit factor requirements, the relatively low elasticity of substitution across manufactured goods leads to more limit pricing than in the previous example. At the solution of the planner's problem, the U.S. now limit-prices 122 goods out of 400 in the absence of trade costs, $\delta = 1$, although this number falls to 33 out of 400 in the presence of trade costs, $\delta = 1.44$.

Table 2 displays welfare gains in the manufacturing sector. In the absence of trade costs, as shown in the first two columns, gains from trade for the U.S. are 32% larger under optimal trade taxes than in the absence of any trade taxes ($36.67/27.74 - 1 \simeq 0.32$), but 84% smaller for the R.O.W. ($1 - 1.03/6.64 \simeq 0.84$). This suggests large inefficiencies from terms-of-trade manipulation at the world level. In line with the observation that,

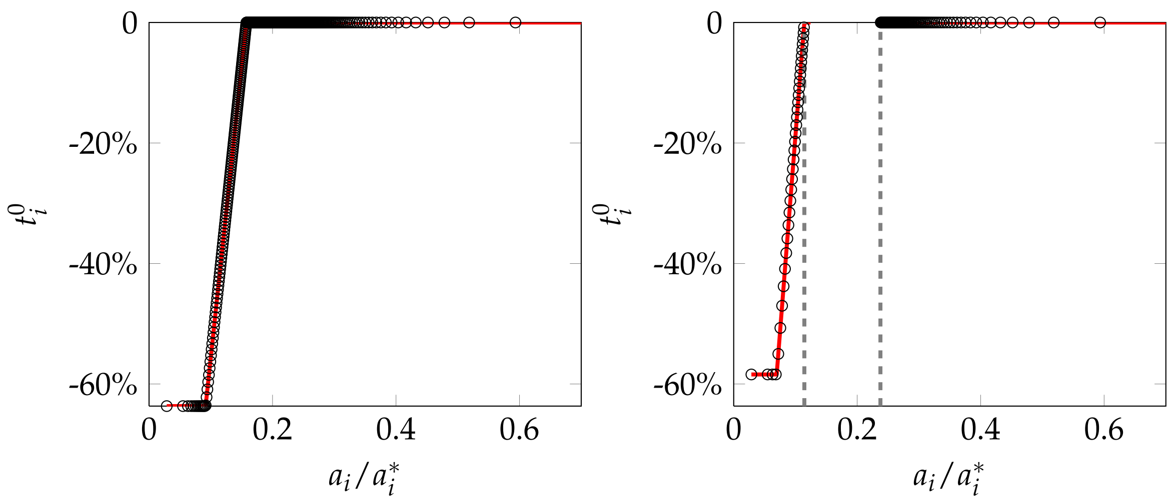


Figure 4: Optimal trade taxes for the manufacturing case. The left panel assumes no trade costs, $\delta = 0$. The right panel assumes trade costs, $\delta = 1.44$.

	No Trade Costs ($\delta = 1$)		Trade Costs ($\delta = 1.44$)	
	U.S.	R.O.W.	U.S.	R.O.W.
Laissez-Faire	27.74%	6.64%	6.18%	2.04%
Uniform Tariff	30.18%	4.85%	7.35%	1.34%
Optimal Taxes	36.67%	1.03%	9.16%	0.40%

Table 2: Gains from trade for the manufacturing case

relative to the agricultural case in Section 6.1, there is now significantly more scope for limit pricing, we see that the share of the U.S. gains arising from the use of non-uniform trade taxes is now substantial: more than two thirds ($30.18/27.74 - 1 \simeq 0.09$).

As in Section 6.1, although the gains from trade are dramatically reduced by trade costs—they go down to 6.18% and 2.04% for the U.S. and the R.O.W, respectively—the importance of non-uniform trade taxes relative to uniform tariffs remains broadly stable. In the presence of trade costs, gains from trade for the U.S., reported in the third column, are 48% larger under optimal trade taxes than in the absence of any trade taxes ($9.16/6.18 - 1 \simeq 0.48$), and around two fifths of these gains ($7.35/6.18 - 1 \simeq 0.19$) arise from the use of trade taxes that vary with comparative advantage.

In sum, the economic forces emphasized in this paper appear to be quantitatively important, at least within the scope of our simple calibrated examples. We hope that future quantitative work, in the spirit of Ossa (2011), will further explore this issue in an environment featuring a large number of countries and a rich geography of trade costs.

7 Concluding Remarks

Comparative advantage is at the core of neoclassical trade theory. In this paper we have taken a first stab at exploring how comparative advantage across nations affects the design of optimal trade taxes. In the context of a canonical Ricardian model of international trade we have shown that optimal trade taxes should be uniform across imported goods and weakly monotone with respect to comparative advantage across exported goods. Specifically, export goods featuring weaker comparative advantage should be taxed less (or subsidized more) relative to those featuring stronger comparative advantage. Our results formalize the intuition that countries should have more room to manipulate world prices in their comparative-advantage sectors.

Characterizing optimal trade taxes in a Ricardian model is technically non-trivial. As mentioned in the Introduction, the maximization problem of the country manipulating its terms-of-trade is potentially infinite-dimensional, non-concave, and non-smooth. A second contribution of our paper is to show how to use Lagrange multiplier methods to solve such problems. Our basic strategy can be sketched as follows: (i) use the primal approach to go from taxes to quantities; (ii) identify concave subproblems for which general Lagrangian necessity and sufficiency theorems problems apply; and (iii) use the additive separability of preferences to break down the Lagrangian into multiple low-dimensional maximization problems that can be solved by simple calculus. Although we have focused on optimal trade taxes in a Ricardian model, our approach is well suited to other additively separable problems. For instance, one could use these tools to compute fully optimal policy in the Melitz (2003) model, extending the results of Demidova and Rodríguez-Clare (2009) and Felbermayr et al. (2011).

Finally, we have studied the quantitative implications of our theoretical results for the design of unilaterally optimal trade taxes in agricultural and manufacturing sectors. In our applications, we have found that trade taxes that vary with comparative advantage across goods lead to substantially larger welfare gains than optimal uniform trade taxes. In spite of the similarities between welfare gains from trade across models featuring different margins of adjustment—see e.g. Atkeson and Burstein (2010) and Arkolakis et al. (2012)—this result suggests that the design of and the gains associated with optimal trade policy may crucially depend on the extent of heterogeneity at the micro level.

References

Alvarez, Fernando and Robert E. Lucas, "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade," *Journal of Monetary Economics*, 2007, *54* (6), 1726–1768.

Amador, Manuel and Kyle Bagwell, "The Theory of Optimal Delegation with an Application to Tariff Caps," *Econometrica*, 2013, *81* (4), 1541–600.

_, Ivan Werning, and George-Marios Angeletos, "Commitment versus Flexibility," *Econometrica*, 2006, *74* (2), 365–396.

Anderson, James E. and Eric Van Wincoop, "Trade Costs," *Journal of Economic Literature*, 2004, *42* (3), 691–751.

Arkolakis, Costas, Arnaud Costinot, and Andres Rodríguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, 2012, *102* (1), 94–130.

Atkeson, Andrew and Ariel Burstein, "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, 2010, *118* (3), 433–489.

Bagwell, Kyle and Robert W Staiger, "A Theory of Managed Trade," *American Economic Review*, September 1990, *80* (4), 779–95.

_ and Robert W. Staiger, "What Do Trade Negotiators Negotiate About? Empirical Evidence from the World Trade Organization," *American Economic Review*, 2011, *101* (4), 1238–73.

Bernard, Andew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum, "Plants and Productivity in International Trade," *American Economic Review*, 2003, *93* (4), 1268–1290.

Bond, Eric W, "The Optimal Tariff Structure in Higher Dimensions," *International Economic Review*, February 1990, *31* (1), 103–16.

Broda, Christian, Nuno Limao, and David E. Weinstein, "Optimal Tariffs and Market Power: The Evidence," *American Economic Review*, December 2008, *98* (5), 2032–65.

Broda, Cristian and David Weinstein, "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 2006, *121* (2), 541–585.

Costinot, Arnaud and Dave Donaldson, "How Large Are the Gains from Economic Integration? Theory and Evidence from U.S. Agriculture, 1880-2002," *mimeo MIT*, 2011.

_ , _ , and Cory Smith, "Evolving Comparative Advantage and the Impact of Climate Change in Agricultural Markets: Evidence from a 9 Million-Field Partition of the Earth," *mimeo MIT*, 2012.

_ , Guido Lorenzoni, and Ivan Werning, "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *mimeo MIT*, 2013.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum, "Unbalanced Trade," *NBER working paper 13035*, 2007.

Demidova, Svetlana and Andres Rodríguez-Clare, "Trade policy under firm-level heterogeneity in a small economy," *Journal of International Economics*, 2009, *78* (1), 100–112.

Dixit, Avinash, "Tax policy in open economies," in A. J. Auerbach and M. Feldstein, eds., *Handbook of Public Economics*, Vol. 1 of *Handbook of Public Economics*, Elsevier, 1985, chapter 6, pp. 313–374.

Dornbusch, Rudiger S., Stanley Fischer, and Paul A. Samuelson, "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *The American Economic Review*, 1977, *67* (5), 823–839.

Dornbusch, Rudiger, Stanley Fischer, and Paul Samuelson, "Heckscher-Ohlin Trade Theory with a Continuum of Goods," *Quarterly Journal of Economics*, 1980, *95* (2), 203–224.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Everett, Hugh, "Generalized Lagrange Multiplier Methods for Solving Problems of Optimal Allocation of Resources," *Operations Research*, 1963, *11* (3), 399–417.

Felbermayr, Gabriel J., Benjamin Jung, and Mario Larch, "Optimal Tariffs, Retaliation and the Welfare Loss from Tariff Wars in the Melitz Model," *CESifo Working Paper Series*, 2011.

Head, Keith and Thierry Mayer, *Gravity Equations: Toolkit, Cookbook, Workhorse*, Vol. 4 of *Handbook of International Economics*, Elsevier, 2013.

Itoh, Motoshige and Kazuharu Kiyono, "Welfare-Enhancing Export Subsidies," *Journal of Political Economy*, February 1987, *95* (1), 115–37.

Luenberger, David G., *Optimization by Vector Space Methods*, John Wiley and Sons, Inc, 1969.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

Mill, J.S, *Essays on Some Unsettled Questions of Political Economy*, London: Parker, 1844.

Opp, Marcus M., "Tariff Wars in the Ricardian Model with a Continuum of Goods," *Journal of International Economics*, 2009, *80*, 212–225.

Ossa, Ralph, "Trade Wars and Trade Talks with Data," *NBER Working Paper*, 2011.

Torrens, R., *The Budget: On Commercial Policy and Colonial Policy*, London: Smith, Elder, 1844.

Wilson, Charles A., "On the General Structure of Ricardian Models with a Continuum of Goods: Applications to Growth, Tariff Theory, and Technical Change," *Econometrica*, 1980, *48* (7), 1675–1702.

A Proofs

A.1 Lemma 1

In order to establish Lemma 1, we will make use of the following lemma that characterizes Home's planning problem as an optimization problem with both equality and inequality constraints. We will then demonstrate that the solution to this optimization problem coincides with the solution to the relaxed problem with only inequality constraints.

Lemma 0. *If $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ solves Home's planning problem, then there exists $w^{0*} \geq 0$ such that $\left(m^0 = c^0 - q^0, q^0, w^{0*}\right)$ solves*

$$\max_{m, q \geq 0, w^* \geq 0} \int_i u_i(q_i + m_i) di \tag{P'}$$

subject to:

$$\begin{aligned} \int_i a_i q_i di &\leq L, &(27)\\ \int_i a_i^* q_i^* \left(m_i, w^*\right) di &= L^*, &(28)\\ \int_i p_i(m_i, w^*) m_i di &= 0. &(29) \end{aligned}$$

Conversely, if $\left(m^0, q^0, w^{0*}\right)$ solves (P), then there exists a solution $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ of Home's planning problem such that $c^0 - q^0 = m^0$.

Proof. ($\Rightarrow$) Suppose that $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ solves Home's planning problem. By Definition 1, $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ solves

$$\max_{c, c^*, q \geq 0, q^* \geq 0, p} \int u_i\left(c_i\right) di$$

subject to (3), (4), (6), (7), (8), and the resource constraint, $\int_i a_i q_i di \leq L$. Introducing $m \equiv c - q$ to substitute for c and using (3) and (7) to substitute for c^* and q^*, $\left(m^0 = c^0 - q^0, q^0, p^0\right)$ therefore solves

$$\max_{m, q \geq 0, p \geq 0} \int u_i\left(q_i + m_i\right) di \tag{P''}$$

subject to:

$$\int_i p_i d_i^*(p_i) di = w^* L^*, \tag{30}$$

$$p_i \leq w^* a_i^*,\ d_i^*(p_i) + m_i \geq 0, \text{ with complementary slackness,} \tag{31}$$

$$\int_i a_i^* \left(d_i^*(p_i) + m_i\right) di = L^*, \tag{32}$$

$$\int_i a_i q_i di \leq L. \tag{33}$$

Note that (m, p) satisfy constraints (31) if and only if

$$p_i = \min\left\{u_i^{*\prime}\left(-m_i\right), w^* a_i^*\right\}. \tag{34}$$

Note also that if equation (34) holds, then

$$d_i^*(p_i) + m_i = \max\left\{m_i + d_i^*(w^* a_i^*), 0\right\}. \tag{35}$$

Using equations (9), (10), (34), and (35) to substitute for p_i and $d_i^*(w^* p_i) + m_i$, we obtain that if $\left(m^0 = c^0 - q^0, q^0, p^0\right)$ solves (P''), then there must exist $w^{0*} \geq 0$ such that $\left(m^0 = c^0 - q^0, q^0, w^{0*}\right)$ solves

$$\max_{m,q\geq 0,w^*\geq 0} \int u_i\left(q_i + m_i\right) di$$

subject to:

$$\int_i p_i\left(m_i, w^*\right) d_i^*(p_i) di = w^* L^*, \tag{36}$$

$$\int_i a_i^* q_i^*(m_i, w^*) di = L^*, \tag{37}$$

$$\int_i a_i q_i di \leq L. \tag{38}$$

By construction, $q_i^*(m_i, w^*) = 0$ if $p_i\left(m_i, w^*\right) \neq w^* a_i^*$. Thus constraint (37) is equivalent to $\int_i p_i\left(m_i, w^*\right) q_i^*(m_i, w^*) di = w^* L^*$. Together with constraint (36), the previous constraint implies constraint (29). Hence constraints (36)-(38) are equivalent to constraints (27)-(29). This implies that if $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ solves Home's planning problem, then there exists $w^{0*} \geq 0$ such that $\left(m^0 = c^0 - q^0, q^0, w^{0*}\right)$ solves (P').

($\Leftarrow$) Suppose that $\left(m^0, q^0, w^{0*}\right)$ solves (P'). To construct a solution $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ of Home's planning problem such that $c^0 - q^0 = m^0$, one can proceed as follows. Given m^0 and w^{0*}, construct the schedule of equilibrium prices p^0 using equations (9), the schedule of foreign consumption c^{0*} using equation (3), and the schedule of foreign output using equation (10). Since $\left(m^0, w^{0*}\right)$ satisfy constraints (28) and (29), the foreign budget constraint (4) and the foreign labor market clearing condition (8) hold. Since p_i^0 and q_i^{0*} have been constructed using (9) and (3), we have $p_i^0 \leq w^* a_i^*$, with equality if $q_i^{0*} > 0$ so that condition (6) is satisfied. We also have $q_i^{0*} = m_i^0 + d_i^*(p_i^0)$ so that the good market clearing condition (7) is satisfied as well. To conclude, set $q^0 = \arg\max_{q\geq 0}\left\{\int_i u_i\left(q_i + m_i^0\right) di \middle| \int_i a_i q_i di \leq L\right\}$ and $c^0 = q^0 + m^0$. $\left(c^0, c^{0*}, q^0, q^{0*}, p^0\right)$ maximizes U and satisfies all the constraints of Home's planning problem. So it is a solution to Home's planning problem such that $c^0 - q^0 = m^0$. □

We are now ready to establish Lemma 1.

Proof. [Proof of Lemma 1] Let us show that $\left(m^0, q^0, w^{0*}\right)$ solves (P') if and only if $\left(m^0, q^0, w^{0*}\right)$ solves the relaxed problem (P).

($\Rightarrow$) We proceed by contradiction. Suppose that $\left(m^0, q^0, w^{0*}\right)$ solves (P'), but does not solve (P). If $\left(m^0, q^0, w^{0*}\right)$ is not a solution to (P), then there must exist a solution $\left(m^1, q^1, w^{1*}\right)$ of (P) such that at least one of the two constraints (12) and (13) is slack. There are three possible cases. First, constraints (12) and (13) may be simultaneously slack. In this case, starting from m^1, one could strictly increase imports for a positive measure of goods by a small amount, while still satisfying (11)-(13). This would strictly increase utility and contradict the fact that $\left(m^1, q^1, w^{1*}\right)$ solves (P). Second, constraint (12) may be slack, whereas constraint (13) is binding. In this case, starting from m^1 and w^{*1}, one could strictly increase imports for a positive measure of goods and decrease the foreign wage by a small amount such that (13) still binds. Since (11) is independent of m and w^* and (12) is slack to start with, (11)-(13) would still be satisfied. Since domestic utility is independent of w^*, this would again increase utility and contradict the fact that $\left(m^1, q^1, w^{1*}\right)$ solves (P). Third, constraint (13) may be slack, whereas constraint (12) is binding. In this case, starting from m^1 and w^{*1}, one could strictly increase imports for a positive measure of goods and increase the foreign wage by a small amount such that (12) still binds. For the exact same reasons as in the previous case, this would again contradict the fact that $\left(m^1, q^1, w^{1*}\right)$ solves (P).

($\Leftarrow$) Suppose that $\left(m^0, q^0, w^{0*}\right)$ solves (P). From the first part of our proof we know that at any solution to (P), (12) and (13) must be binding. Thus $\left(m^0, q^0, w^{0*}\right)$ solves (P').

At this point, we have shown that the solutions to (P) and (P') coincide. Combining this observation with Lemma 0, we obtain Lemma 1. □

A.2 Lemma 2

Proof. [Proof of Lemma 2] ($\Rightarrow$) Suppose that $\left(m^0, q^0\right)$ solves (P_{w^*}). Let us first demonstrate that (P_{w^*}) is a concave maximization problem. Consider $f_i(m_i) \equiv p_i(m_i, w^{0*})m_i$. By equation (9), we know that

$$f_i(m_i) = \begin{cases} m_i w^* a_i^*, & \text{if } m_i > -d_i^*(w^* a_i^*), \\ m_i u_i^{*\prime}(-m_i), & \text{if } m_i \leq -d_i^*(w^* a_i^*). \end{cases}.$$

For $m_i > -d_i^*(w^* a_i^*)$, we have $f_i'(m_i) = w^* a_i^*$. For $m_i < -d_i^*(w^* a_i^*)$, $\sigma^* \geq 1$ implies $f_i'(m_i) = \left(1 - \frac{1}{\sigma^*}\right) \beta_i^* \left(-m_i\right)^{-\frac{1}{\sigma^*}} > 0$ and $f_i''(m_i) = \frac{1}{\sigma^*}\left(1 - \frac{1}{\sigma^*}\right) \beta_i^* \left(-m_i\right)^{-\frac{1}{\sigma^*}-1} > 0$. Since

$$\lim_{m_i \to^+ -d_i^*(w^* a_i^*)} f'(m_i) = w^* a_i^* > \lim_{m_i \to^- -d_i^*(w^* a_i^*)} \left(1 - \frac{1}{\sigma^*}\right) w^* a_i^*,$$

f_i is strictly convex and increasing for all i.

Now consider $g_i(m_i) \equiv a_i^* q_i^*\left(m_i, w^{0*}\right)$. By equation (10), we know that

$$g_i(m_i) = \begin{cases} m_i a_i^* + a_i^* d_i^*\left(w^* a_i^*\right), & \text{if } m_i > -d_i^*(w^* a_i^*), \\ 0, & \text{if } m_i \leq -d_i^*(w^* a_i^*). \end{cases}$$

For $m_i > -d_i^*(w^* a_i^*)$, we have $g_i'(m_i) = a_i^*$. For $m_i < -d_i^*(w^* a_i^*)$, $g'(m_i) = 0$. Thus g_i is strictly convex and increasing for all i.

Since u_i is strictly concave in (m_i, q_i), $a_i q_i$ is linear in q_i, and f_i and g_i are convex in m_i, the objective function is a concave functional, whereas the constraints are of the form $G(m, q) \leq 0$, with G a convex functional. Accordingly, Theorem 1, p. 217 in Luenberger (1969) implies the existence of $(\lambda, \lambda^*, \mu) \geq 0$ such that $\left(m^0, q^0\right)$ solves

$$\max_{m,q\geq 0} \mathcal{L}\left(m, q, \lambda, \lambda^*, \mu; w^*\right) \equiv \int_i u_i\left(q_i + m_i\right) di - \lambda \int_i a_i q_i di$$
$$- \lambda^* \int_i a_i^* q_i^*\left(m_i, w^*\right) di - \mu \int_i p_i(m_i, w^*) m_i di.$$

and the three following conditions hold:

$$\begin{aligned} \lambda\left(L - \int_i a_i q_i^0 di\right) &= 0, \\ \lambda^*\left(L^* - \int_i a_i^* q_i^*\left(m_i^0, w^*\right) di\right) &= 0, \\ \mu\left(\int_i p_i(m_i^0, w^*) m_i^0 di\right) &= 0. \end{aligned}$$

Since $\left(m^0, q^0\right)$ satisfy constraints (11)-(13), conditions (14)-(16) must hold. To conclude, note that if $\left(m^0, q^0\right)$ solves $\max_{m,q\geq 0} \mathcal{L}\left(m, q, \lambda, \lambda^*, \mu; w^*\right)$, then for almost all i, $\left(m_i^0, q_i^0\right)$ must solve

$$\max_{m_i,q_i\geq 0} \mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right) \equiv u_i\left(q_i + m_i\right) - \lambda a_i q_i$$
$$- \lambda^* a_i^* q_i^*\left(m_i, w^*\right) - \mu p_i(m_i, w^*) m_i.$$

($\Leftarrow$) Now suppose that $\left(m_i^0, q_i^0\right)$ solves (P_i) for almost all i with λ, λ^*, μ such that conditions (14)-(16) hold. This implies

$$\left(m^0, q^0\right) \in \arg\max_{m,q\geq 0} \mathcal{L}\left(m, q, \lambda, \lambda^*, \mu; w^*\right).$$

Suppose first that all Lagrange multipliers are strictly positive: $\lambda > 0$, $\lambda^* > 0$, $\mu > 0$, then conditions (14)-(16) imply

$$\begin{aligned} \int_i a_i q_i^0 di &= L, \\ \int_i a_i^* q_i^*\left(m_i^0, w^*\right) di &= L^*, \\ \int_i p_i(m_i^0, w^*) m_i^0 di &= 0. \end{aligned}$$

Thus Theorem 1, p. 220 in Luenberger (1969) immediately implies that $\left(m^0, q^0\right)$ is a solution to (P_{w^*}). Now suppose that at least one Lagrange multiplier is equal to zero. For expositional purposes suppose that $\lambda = 0$, whereas $\lambda^* > 0$ and $\mu > 0$. In this case, we have

$$\left(m^0, q^0\right) \in \arg\max_{m,q\geq 0} \mathcal{L}\left(m, q, 0, \lambda^*, \mu; w^*\right)$$

and

$$\begin{array}{rcl} \int_i a_i^* q_i^* \left(m_i^0, w^*\right) di & = & L^*, \\ \int_i p_i(m_i^0, w^*) m_i^0 di & = & 0. \end{array}$$

Thus Theorem 1, p. 220 in Luenberger (1969) now implies that $\left(m^0, q^0\right)$ is a solution to

$$\max_{m,q \geq 0} \int_i u_i \left(q_i + m_i\right) di$$

subject to:

$$\begin{array}{rcl} \int_i a_i^* q_i^* \left(m_i, w^*\right) di & \leq & L^*, \\ \int_i m_i p_i(m_i, w^*) di & \leq & 0. \end{array}$$

Since $\int_i a_i q_i^0 di \leq L$ by condition (14), $\left(m^0, q^0\right)$ is therefore also a solution to (P_{w^*}). The other cases can be dealt with in a similar manner. $\square$