Estimating the Productivity Gains from Importing*

[Preliminary - Comments welcome]

Joaquin Blaum[†], Claire Lelarge[‡], Michael Peters[§]

September 2014

Abstract

Trade in intermediate inputs raises firm productivity as it enables producers to access both better and novel inputs of production. The question is: by how much? This paper uses a general framework of import demand to estimate the productivity gains from trade both at the firm and aggregate level in the French manufacturing sector. First, we show that in any model where domestic and foreign inputs are combined in a CES fashion the productivity gains of importing at the firm level are given by $s_D^{\gamma/(1-\varepsilon)}$, where s_D is the observable firm's domestic expenditure share in material spending, γ is the share of materials in the production function and ε is the elasticity of substitution between domestic and foreign varieties. In particular, this expression does not require any information on quality differences across countries, the nature of product market competition or which countries firms actually import from. For the population of French importers, we find modest productivity gains at the micro-level in that the median importer is 5% more productive relative to autarky. To map these micro gains into the aggregate effect of trade, we embed our framework in a general equilibrium economy and calibrate it to the microdata. As bigger firms have a higher import intensity, the aggregate gains from importing are larger and range from 16% to 47% depending on the strength of interlinkages across firms. We show that the micro-data on domestic spending plays an important role in identifying the aggregate gains from trade. Calibrating the model to aggregate statistics biases the answer to the aggregate gains from importing by 10-20%.

*We thank Costas Arkolakis, Lorenzo Caliendo, Arnaud Costinot, Jonathan Eaton, Pablo Fajgelbaum, Amit Khandelwal, Sam Kortum, Espen Moen, Andrés Rodríguez-Clare, Peter Schott, Jonathan Vogel, David Weinstein, Daniel Xu and seminar participants at Brown, Columbia, LSE, Yale and the BI Norwegian Business School.

[†]Brown University. eMail: joaquin_blaum@brown.edu

[‡]Centre de Recherche en Économie et Statistique (CREST). eMail: claire.lelarge@ensae.fr

[§]Yale University. eMail: m.peters@yale.edu

1 Introduction

A large fraction of world trade is accounted for by firms sourcing foreign inputs. Trade in intermediates reduces firms' unit costs, or increases their productivity, by providing access to both novel and higher quality inputs. A natural question is: what is the magnitude of these productivity gains? In this paper, we use theory and microdata to answer this question. More specifically, we estimate the productivity gains at the firm-level, as well as the aggregate gains from importing intermediate inputs for the economy as a whole. While the former, which we refer to as "micro-gains", measure how much productivity a particular importing firm would lose if forced to source all inputs domestically, the latter, which we refer to as "macro-gains", measures the aggregate TFP and welfare consequences of shutting down trade in intermediate inputs taking general equilibrium adjustments into account.

Our first main result concerns the micro-gains. In particular we derive a powerful sufficiency result that can be implemented using readily available micro-data. We consider a general class of firm-based models of importing where import demand arises from love-of-variety effects. We show that any model that imposes a CES production function between domestic and foreign inputs features the property that the firm-level productivity gains from importing are simply given by the domestic share of intermediate spending raised to the power of $\gamma/(1-\varepsilon)$, where γ is the elasticity of output to intermediate inputs and ε is the elasticity of substitution between domestic and international varieties. Intuitively, the static gains from trade at the firm level are fully summarized by firms facing lower prices for their chosen input bundle. Conditional on a demand system for imported intermediates, firms' import demand can be simply inverted to read off the change in prices. Such change in prices turns out to be precisely given by the change in domestic spending raised to $\gamma/(1-\varepsilon)$.

A remarkable property of this result is its generality. The formula for the firm-level gains from trade, which is akin to a firm-level analogue of Arkolakis et al. (2012), is derived in a general firm-based framework of importing under minimal assumptions. In the model, firms produce using intermediate inputs which can be sourced domestically or from a set of foreign countries. The different varieties of an input are imperfect substitutes and may differ in quality. Thus, import demand arises from complementarities as well as quality differences across inputs. In this context, the micro gains estimator relies only on the assumption of a CES demand system across domestic and foreign varieties. No further assumptions on other structural primitives of the model are required. Notably, we do not require any restrictions on the underlying distribution of qualities and prices across potential sourcing countries or innate productivity across firms and we can allow for firm productivity and input quality to be complements, giving rise to non-homothetic demand. Additionally, we neither have to take a stand on the structure of output markets or the pattern of demand faced by firms at home, nor on *how* firms end up with their set of trading partners - that is, the specifics of the extensive margin are left unrestricted. Hence, our estimates of the micro gains from trade are consistent regardless of whether firms find their trading partners on a spot market, in which case importing might be limited through the presence of fixed costs, a process of network formation or through costly search.

This robust and easy-to-implement sufficient statistic for the micro gains from trade is useful for two reasons. From the point of view of applied researchers, it provides a convenient way to analyze episodes of trade liberalization (or other changes in firms' import environment), without having to fully specify and solve a full structural model of import behavior: the change in firms' share in domestic spending associated with a particular shock is the correct measure of firms' change in endogenous productivity taking all adjustments, like the switching of supplying countries, and complementarities across different input producers into account. Identifying an exogenous source of variation in policy and data on firms' domestic input spending is therefore sufficient to fully characterize the distribution of the partial equilibrium productivity changes induced by firms' outsourcing decisions. In a similar vain, the statistic is also useful for quantitative firm-based models of import-trade, in that it provides a tight benchmark against which different models can be compared. Precisely because any model with a CES production function will have the exact same implications for the micro gains from trade, different models of importing or different calibration strategies can have different positive implications (e.g. on the number of sourcing countries or whether or not import behavior is hierarchical), but they will agree on the productivity consequences at the firm-level.

We apply the sufficient estimator of the micro gains to the population of importing manufacturing firms in France. Because expenditure shares are readily available in the data, we only need to estimate the output elasticity of material spending (γ) and elasticity of substitution between domestically sourced and imported inputs (ε) . We estimate these elasticities with a procedure akin to production function estimation. In particular, we estimate γ using standard proxy methods as in De Loecker and Warzynski (2012) and ε with an instrumental variable strategy, whereby we exploit changes in the world supply of particular varieties as an instrument for firms' import spending as in Hummels et al. (2011). We find that the micro productivity gains from importing are moderate - they amount to 5% for the median French firm. The gains are larger for exporters, members of a foreign group and larger firms, which is expected as these firms are likely to gain more from the participation in international input markets and hence bias their intermediate expenditure shares towards foreign inputs. The reason why the firm-level gains are limited resonates well with the aggregate results of Arkolakis et al. (2012): many importers simply do not import enough for the gains to be substantial. As this number does not rely on any assumptions about the microstructure of trade, *any* model that combines domestic and imported inputs in a CES fashion will predict exactly the same static firm-level gains given the micro-data. Hence, if one thinks that international sourcing leads to higher productivity gains at the firm-level one would need to argue that there are important dynamic gains through e.g. higher innovation incentives or technology adoption.[1]

While the micro gains estimator can be a powerful instrument to analyze trade policy and to discipline quantitative models, there are limitations. First, the sufficient statistic is not useful to study counterfactual policies precisely because it conditions on the observable data. To analyze counterfactual changes in the environment, one needs to predict the changes in firms' domestic expenditure shares which requires specifying an extensive margin mechanism. Secondly, there is no direct link between the micro and the macro gains from trade, i.e. aggregate TFP and welfare. The micro estimator does not take into account general equilibrium effects, e.g. productivity spillovers that arise under input-output linkages across firms. This should come as no surprise since the

[1] See for example Boler et al. (2014), who show that R&D and international sourcing are complementary activities.

estimator requires no information on preferences or the structure of output markets. Studying the aggregate implications of trade in inputs requires fully specifying the macro side of the economy, namely the structure of interlinkages across firms and the interaction between firms and consumers, as well as an extensive margin mechanism for firms to select their sourcing countries.

To study the macro gains, we embed our model of firm importing into a general equilibrium aggregate framework. We assume that firms engage in monopolistic competition and incorporate a simple structure of roundabout production to allow for the productivity gains to spread from trading to domestic firms. We first show that the information contained in the micro-data is not sufficient to determine the aggregate gains from trade with reduced form methods. Even in partial equilibrium, aggregate TFP depends on the joint distribution between observable domestic expenditure shares and unobservable firm physical productivity. Intuitively, one needs to know which firms are the heavy importers to gauge the aggregate consequences of trade in intermediate inputs.

To make progress we need to specify a mechanism by which firms select their sourcing countries. We assume that firms maximize profits subject to the presence of fixed costs of importing. While this problem is in general quite involved, our assumptions are sufficiently strong to make the characterization of firms' import behavior tractable. In particular, we assume that there is a continuum of countries and that quality is Pareto distributed, and show that firms' import demand is parametrized by a single auxiliary parameter that captures how the the import price index reacts to changes in the number of countries sourced. While this parameter depends on a set of underlying structural parameters governing the distribution of quality and prices across countries and the complementarities across inputs, it captures the firms' extensive margin adjustment and is therefore all we need to predict changes in firms' import behavior. Furthermore, we can estimate this structural elasticity from an optimality condition that relates domestic expenditure shares with the number of varieties sourced using simple reduced-form regression analysis.

We calibrate the macro model to the French micro data. As stressed above, aggregate productivity depends on the joint distribution of domestic expenditure shares and innate physical productivity. While the latter is unobserved in the data, we can discipline this distribution by relying on the joint distribution of domestic shares and sales. To capture the lack of a perfectly negative correlation between these variables seen in the data, we allow for two possibly-correlated dimensions of heterogeneity at the firm level, physical productivity and fixed costs of importing. We find that the aggregate gains from trade are - depending on the strength of interlinkages - between 16% and 47% and hence are substantially higher than the median gains at the firm-level. This is due to fact that innate productivity and domestic spending are negatively correlated, i.e. bigger firms benefit more from international trade, which is beneficial for aggregate efficiency.

An important takeaway from our quantitative exercise is the crucial role played by the micro-data on domestic expenditure shares in identifying the aggregate gains from trade. In particular, the dispersion of domestic shares and their correlation with sales provide important information for the computation of aggregate productivity. We perform alternative calibrations of the model where these moments are dropped and find that this results in biases in our estimates of the macro gains of about 15-20%. Intuitively, the dispersion in domestic shares helps identify the dispersion in firms' effective productivity, while the correlation between domestic shares and sales helps identify the relation between innate physical productivity and the endogenous productivity gains associated with trade.

Related literature. This paper is closely related to Arkolakis et al. (2012) and Costinot and Rodriguez-Clare (2014) in that our sufficient statistic for firm productivity is related to their sufficient statistic for aggregate welfare. In particular we also show that conditional on the micro data and a "trade elasticity", a wide class of models will imply the exact same productivity gains, albeit at the firm-level.

Recently, a number of papers have focused on measuring the effect of imported inputs on firm productivity.[2] One strand of the literature provides reduced-form evidence by studying trade liberalization episodes. Amiti and Konings (2007) use micro data from Indonesia in the 1990s and show that reductions in input tariffs were associated to increases in firm productivity. Goldberg et al. (2010, 2009) and Khandelwal and Topalova (2011) report similar findings for the Indian trade liberalization in the 1990. Our results are consistent with this literature in that we provide an explicit structural interpretation to these reduced form regressions and show that firms' domestic expenditure share are in fact the correct sufficient statistic for the static productivity gains from trade.[3] Kasahara and Rodrigue (2008) use micro data from Chile and explicitly incorporate the domestic expenditure share as an additional input. Our approach is different in various respects. First, we show the sufficiency property of the domestic share in a much broader environment and hence derive the production function for a much more general class of models.[4] Second, our main interest lies not in the estimation of firms' production function, but we use firms' actual expenditure shares, together with our estimate of the trade elasticity, ε, to measure the productivity gains of importing for the population of importers. Finally, we explicitly show how to aggregate these firm-level productivity gains and devise a method to perform counterfactual analysis.[5]

There is also a literature that takes a more structural approach. Halpern et al. (2011) use Hungarian micro data to estimate a closely related framework of importing.[6] The main difference with our approach is that they do not feature firms' domestic expenditure share as the sufficient statistic for firms' productivity gains. Hence, they have to estimate the entire structural model simultaneously to identify all the structural parameters. Because the firm's extensive margin problem is tractable only under particular assumptions and one has to specify the entire market structure and demand parameters on output markets, their estimating procedure relies on these specifications. By expressing firms productivity in terms of the readily observable domestic expenditure share, we

[2]See also De Loecker and Goldberg (2013) for a recent survey about firms in international markets.

[3]Amiti and Konings (2007) have one specification where the use the domestic expenditure share as a measure of import behavior. However, they do not offer a structural interpretation.

[4]Kasahara and Rodrigue (2008) consider a setting where importing amounts to a discrete increase in the number of varieties. We allow firms to choose their varieties from a set of foreign inputs in an unrestricted way.

[5]Additionally, our procedure for estimating the production function is different, as we rely on exogenous shocks to the firm's input supply to identify the coefficient for domestic expenditure shares, while they use a standard OP-type proxy approach.

[6]In fact, our framework nests the one in Halpern et al. (2011).

obtain a production function equation that is valid regardless of how firms determine their extensive margin. Hence, we can obtain estimates of the gains from importing that are valid in a wide class of models and do not depend on the structure of output markets. Gopinath and Neiman (2014) use a related structural model to measure the aggregate productivity losses during the Argentine crisis. While they also do not feature the sufficiency property of domestic expenditure shares and hence have to simulate the entire structural model (including the structure on output markets), our characterization of the aggregate welfare consequences shares some similarities.

On a more technical level, our paper builds on a recent literature (Blaum et al., 2013; Antràs et al., 2014) that stresses that complementarities across inputs of production make the import problem very different from the better known export problem in that firms' extensive margin of trade is - in general - harder to characterize. On the export side[7], Eaton et al. (2004, 2011); Arkolakis (2010) have recently studied firms' entry behavior into different markets and developed quantitative models that can come to terms with the evidence. Arkolakis and Muendler (2011); Bernard et al. (2010) have studied multi-product firms and their entry decisions into different country-product markets. In contrast, theories that can quantitatively account for the pattern of entry into different import markets are far less developed. A notable exception is the recent contribution by Antràs et al. (2014), who analyze a firm-based model of importing in the spirit of Eaton and Kortum (2002) and embed it into a general equilibrium framework. They show that firms' sourcing strategies are predicted to be hierarchical as long as firms' profit function has increasing differences in the number of trading partners. This is an important result because it allows them to adapt the estimation procedure by Jia (2008) to bring the model to the data. They estimate their structural model using US firm-level data on import behavior and are able to quantitatively account for the number of importers across different sourcing countries and the distribution of imports by country of origin. As predicted by our main result, their model has the implications that firms' domestic shares are a sufficient statistic for firm-level productivity gains. Hence, conditional on the observed distribution of firm-level spending, the productivity gains from input trade do not depend on firms' extensive margin of trade. Their framework is, however, to the best of our knowledge the first one to explicitly allow for counterfactual policy analysis in a firm based model of importing taking general equilibrium considerations into account.

The remaining structure of the paper is as follows. Section 2 lays out the general framework of importing and Section 3 derives the main result, namely the sufficient statistic for firms' productivity gains of importing. Section 4 spells out the aggregate macroeconomic environment, while Section 5 deals with the extensive margin problem of importers. Section 6 contains an application of the firm-level productivity estimator to the French data and Section 6.3 calibrates the macroeconomic model and provides our estimates of the aggregate gains from trade. Section 7 concludes.

[7]The literature on exports is too vast to discuss here. Hence, we refer to the reader to Bernard et al. (2007) and Bernard et al. (2012), which are two recent surveys of the literature.

2 The Basic Framework

We consider a framework of import demand that nests most of the existing models and is arguably the most relevant model for applied empirical work. We model firms' import behavior as the solution to a static profit maximization problem. Firms' demand for imported inputs arises from a standard love-of-variety channel and quality differences between domestic and international inputs and is limited by the presence of fixed costs. Firms can source their inputs from multiple, heterogeneous import partners, who may differ in quality and fixed costs. Furthermore, firms are heterogeneous in their physical productivity and their fixed costs of sourcing and we explicitly allow for complementarities between firm productivity and input quality.

2.1 The Environment

Formally, we consider an economy populated by a set of firms that have access to the following production structure:

$$\begin{aligned} y &= \varphi f(l,k,x) = \varphi l^{1-\alpha-\gamma} k^{\alpha} x^{\gamma} && (1) \\ x &= \left(x_D^{\frac{\varepsilon-1}{\varepsilon}} + x_I^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} && (2) \\ x_D &= \eta(q_D,\varphi) z_D \\ x_I &= \left(\int_{c\in\Sigma} \left(\eta(q_c,\varphi) z_c\right)^{\frac{\rho-1}{\rho}} dc \right)^{\frac{\rho}{\rho-1}}. && (3) \end{aligned}$$

That is, intermediate inputs (x) are combined with capital (k) and labor (l) in a Cobb-Douglas way to produce output (y). Firms are heterogeneous in their productivity φ. Intermediate inputs are a CES composite of a domestic variety (x_D) and a foreign import bundle (x_I). The foreign bundle is in turn a CES aggregate of a continuum of foreign varieties.[8] The efficiency of the variety from country c is given by the product of physical units z_c sourced and a firm-specific quality flow $\eta(q,\varphi)$, where q is country quality. The function $\eta(q,\varphi)$ allows for complementarities between firm productivity and country quality, which makes firms' import demand non-homothetic.[9] The homothetic case is the one where $\eta(q,\varphi) = q$ and quality flows are simply proportional to qualities q_c. Without loss of generality we can also normalize the quality flow of the domestic variety as $\eta(q_D,\varphi) = q_D$.

Finally, there is an important *endogenous* object in the definition of the production structure, namely the firm's sourcing strategy Σ, which is the set of foreign inputs the firm has access to. The sourcing strategy Σ is the solution to the firms' extensive margin problem, i.e. from which countries

[8] For now we do not distinguish between products and varieties, i.e. imports of a given product stemming from different countries, in the theory. It will be clear below that we do not need to take stand on this distinction for our main result. In our empirical application we will mostly focus on the country dimension by including product fixed effects. Hence, we will sometimes refer to a foreign input as a variety directly.

[9] In particular, there is quality-productivity complementarity when h is log-supermodular in (q,φ). If h is log-submodular, firm productivity and product quality are substitutes. Kugler and Verhoogen (2011) provide evidence for the case of Colombian producers of a positive correlation between plant size and input prices. This evidence is consistent with the presence of a complementarity between input quality and firm productivity. Blaum et al. (2013) provide evidence for French manufacturing firms that is also consistent with such complementarity.

to buy its inputs from. We think of an economy where international sourcing is subject to fixed costs, which we denominate in units of labor. In particular, we assume that to source an input from country c the firm needs to pay a fixed cost f_c. It is precisely the existence of such fixed costs which makes firms not source from all countries in the world: the optimal Σ is determined by balancing love of variety effects vs the fixed costs. Crucially, variation in fixed costs across firms will map into variation in sourcing strategies: some firms will endogenously import from more markets than others. Conditional on accessing a country c, firms are assumed to be price-takers and face a price p_c. Note that prices include trade costs and are allowed to be correlated with country quality. Finally, we do not put (yet) any restrictions on the structure on domestic output markets and hence do not make assumptions on whether firms produce a homogeneous or differentiated final good and how they compete. Assumption 1 below formally defines the sources of heterogeneity in our economy.

Assumption 1. *Countries are heterogeneous in 3 dimensions: quality (q_c), prices (p_c) and fixed costs (f_{ci}). Furthermore, firms are heterogeneous in their physical productivity (φ_i) and their fixed costs of sourcing (f_{ci}). Denote by $G(q)$ and $F(\varphi)$ the marginal distributions of quality and productivity and by $H^f(f|q,\varphi)$ and $H^p(p|q)$ the conditional distribution of fixed costs and prices. At this point we do not impose any particular structure on these distributions.*

We consider this environment as a very general description of a standard firm-based model of trade. In particular, it nests most of the existing contributions directly. Gopinath and Neiman (2014) for example assume that demand is homothetic ($\eta(q,\varphi) = q$), that sourcing countries are identical ($q_c = q,\ p_c = p,\ f_c = f$) and that output markets are monopolistically competitive with isoelastic demand. Halpern et al. (2011) also impose homothetic demand, consider only a single sourcing country but allow for firm-specific fixed costs (f_i), which are uncorrelated with firm productivity.[10] They also assume output markets with monopolistic competition. Hence, all our theoretical results also apply to their models.

2.2 Import Demand

Firms choose their size and sourcing strategy - i.e. the set of varieties to import - as well as the quantities demanded of all inputs to maximize profits. It is convenient to split the firm's problem into a cost-minimization problem given a sourcing strategy, Σ, and the choice of the optimal firm size y and sourcing strategy Σ *given* the cost function for material services. Formally,

$$\pi \equiv \max_{\Sigma,y,l,k} \left\{ py - \Gamma(\Sigma, y, \varphi, l, k) - Rk - wl - w\left(\int_{c\in\Sigma} f_c dc + f^I I(\Sigma) \right) \right\}, \tag{4}$$

where

$$\Gamma(\Sigma, y, \varphi, l, k) \equiv \min_z \left\{ \int_{c\in\Sigma} p_c z_c dc \text{ s.t. } \varphi f(l,k,x) \geq y \right\}, \tag{5}$$

[10] More precisely, Halpern et al. (2011) allow for multiple *products* $\left(x = \prod_k x_k^{B_k}\right)$, each of which is produced according to (2). We explicitly show in the Appendix how our results apply in such a multi-product environment.

is the firm's cost function. Here p denotes the demand function the firm faces, Σ is the firm's sourcing strategy, w and R denotes the wage and interest rate, which the firm takes as given. Furthermore, $I(\Sigma)$ is an indicator of the firms import status, i.e. $I(\Sigma) = 1$ whenever the firm sources *any* foreign variety. As we do not need to take a stand on the nature of competition or market structure on the output side, the demand function p is unrestricted.

The aim of this paper is to take this framework to the data. In particular, we are interested in using the solution to (4) to ask how high are the productivity gains from importing, i.e. how much worse off in terms of productivity would a firm be if we were to force it to operate in autarky? Albeit conceptually easy, the answer to this question is not straightforward. The reason is that - in general - (4) is a hard problem. More precisely, while the cost-minimization problem given the firm's extensive margin of trade (5) is extremely tractable, actually solving for the optimal sourcing strategy is difficult unless we impose strong assumptions on the joint distribution of qualities and fixed costs and specify the nature of output market competition.

As shown more formally in Blaum et al. (2013) and Antràs et al. (2014), the usual intuition of Melitz-type models of the exporting literature suggests that the extensive margin of import demand should satisfy a sorting condition with respect to firm productivity φ, i.e. not only import status but also the number of products and the number of varieties sourced should be positively correlated with firm productivity so that international sourcing should be hierarchical. This intuition, however, is incorrect. The reason is the interdependence between the different choices on the extensive margin. International sourcing on the input side is a vehicle to reduce the variable cost of production. Hence, a particular variety is imported whenever the reduction in the average production costs outweighs the incurred fixed costs. As long as there is some complementarity across imported varieties, i.e. as long as the production function features some form of "love for variety", these cost reductions depend on the *entire* sourcing strategy Σ. Thus, it might be that unproductive firms source multiple varieties with low fixed costs and low quality flows and high productivity firms concentrate on few fixed cost expensive varieties which yield high quality flows. This interdependence renders the characterization of the extensive margin of importing much harder than for the case of exports. For exporting firms, the (outward) sourcing strategy can essentially be solved "market by market" - at least as long as production subject to constant returns to scale (see for example Eaton et al. (2011)). For imports, however, interdependencies in production are likely to be crucial. The intuition from the exporting literature that firms' extensive margin of trade features a hierarchy has therefore no direct counterpart for firms' importing decisions, unless more restrictions are imposed.[11]

The main insight of this paper is that we do *not* have to solve for the optimal sourcing strategy to answer a host of questions which are arguably of major importance. In particular, using standard

[11] Note, however, that this does not imply that general results concerning the extensive margin cannot be derived. If for example the demand elasticity exceeds unity and fixed costs are not firm-specific, more productive firms import and more productive firms adopt a sourcing strategy that leads to lower unit costs, i.e. $\gamma(\Sigma(\varphi')) \leq \gamma(\Sigma(\varphi))$ if $\varphi' > \varphi$. Hence, similar to the exporting intuition, more productive firms sell more and thus have a higher incentive to reduce their marginal costs by incurring the fixed costs of importing additional products/varieties. However, again this does *not* imply that more productive firms source *more* varieties or products. [THIS WE NEED TO CHECK] Antràs et al. (2014) show in a related model that sourcing can be shown to be hierarchical as long as the profit function function has increasing differences.

micro-data we can consistently identify the distribution of firm-level productivity gains of trade relative to autarky because there is a simple observable sufficient statistic for firms' sourcing strategy Σ. The same holds true for any evaluation of past policies which are observed in the micro data. This is important for two reasons. First of all, precisely because the extensive margin is hard to compute, it is convenient to not have to solve that problem if one was interested in measuring the firm-level productivity gains. Secondly, to solve for firms' extensive margin we would need to put more structure. Not only would we need to impose particular functional forms for the heterogeneity encapsulated in Assumption 1, but we would also need to specify the structure of interactions in output markets. Our results show that such issues can be sidestepped in that the implications for firm productivity will be invariant to such choices as long as the model is consistent with the microdata. Hence, reassuringly, the firm-level gains from importing do not hinge on such modeling assumptions. It is only when we want to conduct counterfactual policy experiments or when we want to make statements about aggregate outcomes such as welfare and aggregate TFP that we need to solve for the extensive margin problem in (4).

3 Measuring the gains from trade at the firm-level

In this section we measure the gains from trade at the firm-level stemming from (4) and (5). Our main result is that we only need to solve firms' *intensive* margin problem (5) to measure the productivity gains through the lens of the model. To see this, consider a firm with productivity φ that sources from a set Σ of countries. Given its sourcing strategy, the fixed costs are irrelevant for firms' import demand. It is easy to see from (1)- (3) that firms care only about price-adjusted qualities

$$\xi_c(\varphi) \equiv \frac{\eta(q_c,\varphi)}{p_c}. \tag{6}$$

Note that price-adjusted qualities depend on firm productivity if there are complementarities in the $\eta(\cdot)$ function. Conditional on the sourcing strategy Σ, the firm's expenditure shares follow the usual formula

$$s_c(\Sigma,\varphi) = \frac{\xi_c(\varphi)^{\rho-1}}{\int_{c\in\Sigma} \xi_c(\varphi)^{\rho-1}\,dc}, \tag{7}$$

where we explicitly denote the dependence on the endogenous sourcing strategy Σ. Letting m_I be total import *spending*, standard manipulations imply that total import services x_I are given by $x_I = A(\Sigma,\varphi)\, m_I$, where $A(\Sigma,\varphi)$ is a *firm-specific* import price index

$$A(\Sigma,\varphi) \;\;=\;\; \left(\int_{c\in\Sigma} \xi_c(\varphi)^{\rho-1}\right)^{\frac{1}{\rho-1}}. \tag{8}$$

This price index will play an important role because it is exogenous *given* the sourcing strategy Σ. Note in particular that A depends only on Σ when demand is homothetic (i.e. $\eta(q,\varphi) = q$).

Given $A(\Sigma,\varphi)$ we can easily solve the trade-off between domestic and foreign varieties at the firm-level. Letting p_D be the price of domestic intermediates and m (m_D) be total (domestic) spending on material inputs, we get that total material services x are related to material spending m via $x = Q(\Sigma, \varphi) m$, where $Q(\Sigma, \varphi)$ is again akin to a *firm-specific* price index given by

$$Q(\Sigma, \varphi) = \left((q_D/p_D)^{\varepsilon-1} + A(\Sigma, \varphi)^{\varepsilon-1} \right)^{\frac{1}{\varepsilon-1}}. \tag{9}$$

Hence, as before, given Σ, Q is exogenous from the point of view of the firm. Note also that firms' domestic expenditure shares are simply

$$s_D(\Sigma, \varphi) \equiv \frac{m_D}{m} = \frac{(q_D/p_D)^{\varepsilon-1}}{(q_D/p_D)^{\varepsilon-1} + (A(\Sigma, \varphi))^{\varepsilon-1}} = \left(\frac{q_D/p_D}{Q(\Sigma, \varphi)} \right)^{\varepsilon-1}. \tag{10}$$

Crucially, (10) gives us a simple relation between the *observed* domestic shares s^D and the *unobserved* firm-specific prices $A(.)$, which recall depend on the entire unobserved distribution of qualities and prices and on the firms' sourcing strategy Σ. It is this simple observation which implies the following powerful result.

Proposition 1. *Consider the setup described above. Firms' production functions are then given by*

$$y = \vartheta l^{1-\alpha-\gamma} k^{\alpha} m^{\gamma},$$

where firm effective productivity ϑ *is given by*

$$\vartheta \equiv \underbrace{\varphi \times \left(\frac{q_D}{p_D} \right)^{\gamma}}_{\text{Exogenous productivity}} \times \underbrace{(s_D(\Sigma, \varphi))^{\frac{\gamma}{1-\varepsilon}}}_{\text{Endogenous productivity gains from trade}}. \tag{11}$$

Hence, $s_D^{\frac{\gamma}{1-\varepsilon}}$ *is the productivity gain relative to autarky holding prices fixed.*

Proposition 1 is powerful because it says that we can measure the endogenous productivity gains from trade at the firm-level simply from the observed domestic shares $[s_D]$ and the structural parameters γ and ε, which can be estimated. Hence, the *observed* domestic shares s^D are sufficient statistics for the endogenous productivity gains from trade. Put differently: conditional on s^D, neither the extensive margin of trade Σ nor any other underlying structural parameters such as the distribution of import quality q, the distribution of fixed costs or the precise functional form $h(q, \varphi)$ are required to estimate the endogenous gains from trade at the firm level. *Any* model that imposes a CES demand system across domestic and international varieties will therefore have the exact same answer for the implied gains from trade given firm-level data on domestic spending shares and parameters ε and γ.[12]

[12] Additionally, there are other models, which satisfy the Proposition 1. Antràs et al. (2014) for example consider a model of importing in the spirit of Eaton and Kortum (2002). In their model, firms' total productivity is proportional to $\vartheta = \varphi s_D^{-1/\theta}$, where φ also denotes firms' exogenous productivity and θ is the parameter of the Frechet distribution, where suppliers' efficiency levels are dawn from. Hence, as in Arkolakis et al. (2012), the CES parameter $\varepsilon - 1$ changes to the heterogeneity parameter θ, but the basic result of Proposition 1 remains intact: conditional on the *observed*

Deriving (11) is easy but still insightful to understand the productivity effects of trade at the firm level. Using $x = Q(\Sigma, \varphi) m$, we get that

$$y = \varphi l^{1-\alpha-\gamma} k^{\alpha} x^{\gamma} = (\varphi Q(\Sigma, \varphi)^{\gamma}) k^{\alpha} l^{1-\alpha-\gamma} m^{\gamma}. \tag{12}$$

Substituting (10) yields (11). Hence, participation in international markets lowers production costs relative to the domestic numeraire. The effective price for firm i therefore depends on the price index Q^i which it "chooses" through its sourcing strategy Σ_i. Instead of directly evaluating Q from (9), we can simply invert the firm's demand function using (10) and relate its price Q^i to its share of domestic spending. As s^D is observable, we therefore *never* have to calculate Q to identify the productivity consequences of trade. One major reason for the usefulness of Proposition 1 is precisely that it is hard to solve for firms' sourcing strategy Σ. To do so we would need to specify the entire environment, e.g. the demand structure on output markets. This is the route taken by Halpern et al. (2011), Gopinath and Neiman (2014) and Antràs et al. (2014). For the question "How high are the productivity gains from importing?" however, it is *not* required to actually solve for firms' extensive margin of importing. Proposition 1 shows that we can sidestep all these issues and simply "read off" firm productivity gains from the micro-data. [Another reason why Proposition 1 is useful is that even if the sourcing strategy was easy to compute, the price adjusted qualities are not observable].

This discussion should also make clear that Proposition 1 holds regardless *how* the extensive margin is determined as we simply took Σ as given. For concreteness of exposition we considered a model with fixed costs, but as far as Proposition 1 is concerned, we never used this structure. In particular, Proposition 1 would be equally valid if we had considered a model where firms were characterized by an unrestricted joint distribution of productivity φ and sourcing strategies Σ. One can then consider various models of how these sourcing strategies came about. Besides the one considered here, we could have for example also considered a model of dynamic network formation as in Chaney (2013) or Oberfield (2013). Regardless of how firms meet their potential set of trading partners, as long as the intensive margin of trade is generated by a CES demand system, Proposition 1 characterizes the gains from trade at the firm level. Finally, note that Proposition 1 also did not use the CES structure across foreign varieties. As long as we can link import service flows x_I and import spending m_I via a price index $A(\Sigma, \varphi)$ (as in (8)), Proposition 1 applies. Hence, we can replace (3) by any (potentially firm-specific) constant-return production function.

Proposition 1 is not only useful to measure the firm-level gains from trade relative to autarky. We can also use it to analyze the productivity effects of trade policy, e.g. an episode of past trade liberalization. One object of immediate interest is the distribution of *changes* of firm-productivity through the access of better or complementary foreign inputs. Holding innate productivity φ fixed,

domestic shares and a value of θ, the productivity gains of importing are fully determined. The material share γ is equal to unity in Antràs et al. (2014) as their importing firms do only use intermediary products as an input to production. In our empirical exercise we will estimate ε directly from (11) using exogenous variation in firms' domestic spending. Hence, conditional on our exclusion restrictions of our instrument, our estimates of ε will also be consistent for θ.

the firm-level productivity gains from such policies are given by:

$$\left.\frac{\vartheta^{post}}{\vartheta^{pre}}\right|_{\varphi} = \left(\frac{s^D\left(\Sigma^{post},\varphi\right)}{s^D\left(\Sigma^{pre},\varphi\right)}\right)^{\frac{\gamma}{1-\varepsilon}}. \tag{13}$$

Hence, knowledge of the change in the domestic shares is sufficient to analyze the direct, static consequences of trade reform. For concreteness, suppose one was interested in analyzing the productivity effects of an episode of trade liberalization (e.g. Chile in 1980s (Pavcnik, 2002), Indonesia in the late 1980s and early 1990s (Amiti and Konings, 2007) or India in the 1990s (De Loecker et al., 2012)). One can then use (13) to estimate the *direct* effect of improved access to international inputs on firm productivity. In particular, (13) contains both the exogenous change in foreign prices due to lower trade barriers and tariffs and the endogenous change in firms adjusting their sourcing pattern. In particular, (13) is the correct structural measure of changes in firm productivity in any model that satisfies our general assumptions above. Conditional on the observed micro-data on domestic spending, one does not have to estimate a structural model to evaluate the productivity effects of trade reform.[13]

We view (11) as the firm-level analogue of Arkolakis et al. (2012). Broadly speaking, they show that in an aggregative model the domestic expenditure share at the *country level* is a sufficient statistic for welfare as long as the demand system is of the CES form. By analogy, (11) states that - at the firm level - firms' spending on domestic intermediaries raised to an appropriate trade elasticity, $s_D^{\frac{\gamma}{1-\varepsilon}}$, is a sufficient statistic for firm productivity. In the same vain as consumers gain purchasing power by sourcing cheaper or complementary products abroad, firms can lower the effective price of intermediate purchases by tapping into foreign input markets.

While Proposition 1 is powerful, there are limitations. The first relates to the partial equilibrium nature of the exercise. As seen from (11), $s_D^{\frac{\gamma}{1-\varepsilon}}$ is akin to a partial equilibrium productivity gain holding prices p_D constant (see (11)). Hence, $s_D^{\frac{\gamma}{1-\varepsilon}}$ answers the question: "How much productivity would a firm lose if it (and only it) was excluded from international markets?" The observed distribution of $s_D^{\frac{\gamma}{1-\varepsilon}}$ does *not* necessarily inform us about the aggregate consequences of trade if domestic intermediary prices p_D are endogenous. If for example domestic firms use the output of importers as an input to production, the aggregate gains from openness might be higher than suggested by the empirical distribution of $s_D^{\frac{\gamma}{1-\varepsilon}}$. More generally, to aggregate these micro gains we need to specify both the demand and competitive structure on output markets and take a stand on the interlinkages across firms to gauge the importance of round-about production. Secondly, precisely because it conditions on the observed micro-data, Proposition 1 is not directly amenable to answer counterfactual questions, e.g. how much firm productivity would change in response to a trade reform. Hence, to answer such counterfactual questions and quantify the aggregate welfare effects of international

[13] Of course, opening up to trade might induce firms' to engage in other productivity enhancing activities like R&D, in which case innate productivity φ would also increase. Such increases in complimentary investments are not encapsulated in (13), which only estimates the direct gains from trade holding productivity fixed. If one wanted to disentangle these indirect gains from trade from the direct gains from trade, more structure (and data) is required. See for example Eslava et al. (2014).

input sourcing, we both have to put additional structure and cannot rely on the data to indirectly infer firms' sourcing strategies but we actually have to solve firms' extensive margin problem. This is where we turn to now.

4 The Aggregate Gains From Trade

Up to now, we have entirely focused on the individual firm. In this section we turn to study the aggregate effects of international trade. This requires us to put more structure than was necessary until now. Importantly, we have to fully describe the macroeconomic structure to provide a link from the distribution of firm-level productivity to aggregate allocations and welfare. To quantify the aggregate gains from trade, we hence embed the structure from above into the following economy. There is a unit mass of workers providing their labor inelastically and a measure one of firms who produce using labor and intermediate inputs.[14] Their output is aggregated into the final good according to the usual CES aggregator

$$Y = \left(\int_0^1 y(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}. \tag{14}$$

As before intermediate inputs are produced from a domestic variety and many differentiated foreign varieties. Foreign varieties are in perfectly elastic supply at prices $[p_c]_c$. To allow for the possibility of round-about production, suppose there is a representative domestic intermediate good firm that produces the domestic intermediate product using both labor and a composite of final goods from the final good producers. In particular, suppose that the production of domestic intermediaries is given by

$$Y_D = M l_D^{\phi} Z^{1-\phi}, \tag{15}$$

where Z denotes the usage of the final good, so that ϕ measures the importance of interlinkages as in Jones (2011). We assume that the market for intermediate products is monopolistically competitive[15] and that trade is balanced. For simplicity, we assume that the economy exports the final good in exchange for imported inputs. Given this environment, we can formally define an equilibrium in this economy.

Definition 1. *An equilibrium is a set of prices* $(w, [p(i)]_i, p_D, P)$*, labor allocations* $([l(i)]_i, [l_F(i)]_i, l_D)$*, differentiated product supplies* $([y(i)]_i)$*, domestic and international input demands* $\left([z_D(i)]_i, [z_{c,I}(i)]_{ci}\right)$*, supply and demands for the final good* (Y, C, Z, Y_{ROW})*, supply of domestic intermediates* Y_D *and sourcing strategies* $([\Sigma(i)]_i)$ *such that*

1. *Prices* $[p(i)]_i$*, sourcing strategies* $[\Sigma(i)]_i$ *and input demands* $\left([l(i), z_D(i), [z_{c,I}(i)]]_{ci}\right)$ *solve firms' maximization problem, i.e.* (4) *and* (5)*,*

[14] As we are only interested in the static allocations, we abstract from capital to simplify the notation.

[15] While this (by construction) shuts down any issues of imperfect pass-through, we would need data on firm-specific prices to put discipline on the extent of pass-through. In principle one could consider different specifications of output market competition. See also Dhingra and Morrow (2012) and Fabinger and Weyl (2012) for recent contributions on incomplete pass-through in international trade.

2. *Consumers maximize utility given* (14),

3. *Intermediate good producers maximize profits given* (15),

4. *The aggregate price index P is consistent with $[p(i)]_i$,*

5. *The labor used for fixed costs $l_F(i)$ is consistent with firms' sourcing strategies $\Sigma(i)$,*

6. *Trade is balanced, i.e. $PY_{ROW} = \int p_c z_{c,I}(i)\, di$,*

7. *Markets clear, i.e. $L = \int l(i)\, di + \int l_F(i)\, di + l_D$, $Y = C + Y_{ROW} + Z$ and $Y_D = \int z_D(i)\, di$.*

Crucially, an equilibrium requires firms' sourcing strategies $\Sigma(i)$ to be optimal given the exogenous parameters of the environment (e.g. fixed costs f_{ci}, input quality q_c or prices p_c) and other endogenous variables, e.g. the price of the domestic input (p_D), which depends on *all* firms' sourcing strategies, as these affect their marginal costs. Proposition 1 however suggests that firms' domestic expenditure shares $s_D(i)$ are a sufficient statistic for firms' productivity and hence marginal costs of production. In particular, from above we know that (see (8), (9) and (10)) firm i's domestic share is fully determined from its sourcing strategy.[16] Hence, given the *equilibrium* sourcing strategies $[\Sigma(i)]_i$, there are unique accompanying *equilibrium* domestic shares $s_D(i)$. This property will prove useful to calibrate the model to the micro-data, and also provides a convenient characterization of two aggregate outcomes we particularly care about: the equilibrium real wage and consumer welfare.

Proposition 2. *Let $[\Sigma(i)]_i$ be firms' equilibrium sourcing strategies, $L_F = \int l_F(i)\, di$ be the accompanying equilibrium number of workers employed by firms to cover the fixed costs of sourcing and $s_D(i)$ the domestic expenditure share induced by $\Sigma(i)$. Then the following is true:*

1. *Real wages in the economy are given by*

$$\frac{w}{P} = \Gamma_{w/p} \times \left(\int_{i=0}^{1} \left[\varphi(i)\, s_D(i)^{\frac{\gamma}{1-\varepsilon}} \right]^{\sigma-1} di \right)^{\frac{1}{\sigma-1} \frac{1}{1-(1-\phi)\gamma}}, \tag{16}$$

where $\Gamma_{w/p}$ is a constant.

2. *Consumer welfare is given by*

$$U = \left(1 + \frac{1}{\sigma-1} \left[1 - \gamma \left(1 - \phi \int_{i=0}^{1} \frac{s_D(i) \left[\varphi(i)\, s_D(i)^{\gamma/(1-\varepsilon)} \right]^{\sigma-1}}{\left(\int \left[\varphi(i)\, s_D(i)^{\gamma/(1-\varepsilon)} \right]^{\sigma-1} di \right)} di \right) \right]^{-1} \right) \times \frac{w}{P} \times (L - L_F). \tag{17}$$

Proof. See Online Appendix. □

[16]Note that the converse is *not* true, i.e. there might be different sourcing strategies leading to the same domestic share.

Proposition 2 provides the link between the micro gains $\left(s_D{}^{\frac{\gamma}{1-\varepsilon}}\right)$ and the macro gains. Consider first the case of real wages (or TFP) in (16). Given a distribution of domestic shares in the population of firms, real wages depend only on the joint distribution of exogenous productivity and endogenous domestic shares. Hence, as before, firms' domestic share summarize all relevant information regarding the structure of international markets. That is, (16) holds regardless of how foreign countries differ in quality and prices or how firms' sourcing strategies Σ came about - given the observed domestic shares, the actual sourcing strategies are irrelevant as far as the real wage is concerned.

However, in contrast to Proposition 1, the observable *marginal* distribution of domestic shares s_D^i is *not* sufficient to gauge the aggregate effects of trade - it is the joint distribution of domestic shares and exogenous productivity that matters. While at the firm level we can simply take the observed domestic expenditure shares from the data to evaluate backward looking policies, the aggregate effects of those micro-changes now crucially depend on how such changes in domestic spending (and hence endogenous productivity increases) are correlated with innate productivity φ_i.[17] Intuitively, the economy as a whole gains substantially if the most productive firms are the most active participants in international trade. Conversely, if relatively unproductive firms are the most active importers the aggregate gains from trade will be small.

Reduced form methods are therefore not enough to link the microdata to the aggregate effects - one needs to have a structural model of import behavior to know how domestic shares are actually determined or more specifically how sourcing strategies $\Sigma(i)$ depend on firm efficiency $\varphi(i)$. If firms differ only in innate productivity φ_i, more productive firms will have higher import shares. Hence, there will be a perfect correlation between s_D^i and φ_i in the cross-section. This will be an environment where the aggregate gains from trade are large. If, on the other hand, firms differ substantially in their fixed costs of sourcing or in their efficiency of meeting international trading partners, firms' domestic shares will only be partially correlated with firm physical productivity. In that case, the aggregate gains from trade are likely to be smaller. Hence, to evaluate (16) we need to specify a full structural model and solve for firms' extensive margin of trade.

The need for the full structural model is even more clearly seen when we consider welfare. As seen from equation (17), welfare depends on three terms - all of which depend on firms' exposure to international trade. Clearly, welfare depends on real spending power, i.e. the real wage. In addition, welfare needs to take into account the loss of resources through the fixed costs of sourcing, as the effective amount of production labor is only $L - L_F$. Finally, the first term reflects that producers earn profits which accrue to the representative consumer. If mark-ups are zero (which is the case when $\frac{1}{\sigma-1}$ goes to zero), the first term disappears. To evaluate (17) a structural model is required where domestic shares are endogenously determined. In particular, we need to take a stand how much firms "pay" in terms of their fixed costs.[18] This is where we turn now.

[17]This point is also stressed by Gopinath and Neiman (2014).

[18]Note in particular how (16) and (17) nest the economy of Arkolakis et al. (2012) as a special case. If profits are zero (or not rebated to domestic consumers), firms do not have to pay fixed costs of sourcing ($f_{ci} = 0$) and demand is homothetic ($\eta(q,\varphi) = q$) , two results follow. First of all, welfare simply equal to the real wage. Secondly, firms will have identical sourcing strategies ($\Sigma_i = \Sigma$) and - because of homothetic demand - equalized domestic shares ($s_{D,i} = s_D$). Hence, (16) implies that $\frac{w}{P} \propto \left(\int_{i=0}^{1} \varphi(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}\frac{1}{1-(1-\phi)\gamma}} \times s_D{}^{\frac{\gamma}{1-\varepsilon}\frac{1}{1-(1-\phi)\gamma}}$, where the latter are simply the "ACR"-

5 The Extensive Margin of Importing

To generate the endogenous distribution of innate productivity and domestic shares, we now need to solve firms' extensive margin problem (4). To do so we need to impose more structure on the economy. Not only do we now need to consider particular functional forms for the distribution of foreign quality and prices and the distribution of fixed costs, but we also have to impose additional restrictions on the degree of heterogeneity to make the model tractable.

Assumption 2. *Consider the environment above and assume the following:*

1. *The fixed cost of sourcing varies across firms but is constant across sourcing countries, i.e.* $f_{ci} = f_i$ *where* f_i *and* φ_i *follow some arbitrary joint distribution.*
2. *The fixed cost of importing* f^I *is constant across firms.*
3. *Foreign prices* p_c *are dependent on quality and are given by* $p_c = \alpha q_c^{\nu}$*.*
4. *The distribution of quality is Pareto, i.e.* $G(q) = \Pr(q_c \leq q) = 1 - \left(\frac{q_{\min}}{q}\right)^{\theta}$ *where* θ *satisfies* $(\rho - 1)(1 - \nu) < \theta$*.*
5. *Import demand is homothetic, i.e.* $\eta(q, \varphi) = q$*.*

Assumption 2 contains sufficient restrictions for a tight characterization of firms' extensive margin. The essential assumption is the first one. When fixed costs do not vary by country, the firm selects its sourcing countries purely based on (price-adjusted) quality, for a given mass of countries sourced. More precisely, if country c with (price-adjusted) quality q_c/p_c is an element of Σ_k so are all countries c' with $q_{c'}/p_{c'} > q_c/p_c$. Thus the firm's sourcing strategy reduces from an entire set to a scalar: Σ_k can be summarized by a quality cutoff. See also Blaum et al. (2013) where this point is discussed in more detail, and Antràs et al. (2014) who offer a very insightful analysis in a related model.

The remaining four restrictions in Assumption 2 could be dispensed with and replaced by alternative specifications. While we could allow for firm-heterogeneity in f^I, this extra degree of freedom (in addition to the firm heterogeneity in f) does not seem to be essential. Our restriction on prices is a tractable way to link the distribution of qualities to the distribution of price-adjusted qualities (which is what importing firms care about). That qualities are distributed Pareto is a convenient parametric form for the distribution - we could have considered other specifications. Finally, we consider the homothetic model. We do so not only because this is the benchmark model in the literature, but also because disciplining the functional form for $\eta(q, \varphi)$ is not straight-forward.[19]

Endowed with Assumption 2 and the market structure laid out above, we now turn to the characterization of firms' extensive margin of trade. The discussion above showed that we can fully summarize importers' sourcing strategies by a quality-cutoff $\overline{q}$ such that all countries with quality

gains from trade in an economy with roundabout production.

[19] We note that for a given $\eta(q, \varphi)$ function, solving the non-homothetic model is conceptually and computationally simple.

$q > \overline{q}$ are sourced.[20] Hence, the number of countries firm i sources from is simply given by

$$n_i = P\left(q \geq \overline{q}_i\right) = \left(\frac{q_{\min}}{\overline{q}_i}\right)^{\theta}. \tag{18}$$

Firm i's sourcing strategy Σ_i is therefore simply given by the number of countries n_i.

While Proposition 1 only relied on firms' cost-minimization problem, n_i is the solution to the profit-maximization problem. Given the aggregate environment above, firms face an isoelastic demand and set a constant mark-up. Hence, variable profits before fixed costs are given by

$$\pi^V = (p - MC)\, y = \frac{1}{\sigma} py = \frac{1}{\sigma} D \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left(\frac{1}{MC}\right)^{\sigma-1}, \tag{19}$$

where MC are the marginal costs of production and D is the usual demand scalar containing aggregate quantities, which the firm takes as given ("aggregate spending"). From above, it is easy to see that firms' marginal costs are given by

$$MC\left(\varphi, n\right) = \frac{1}{\varphi}\left(\frac{w}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma}\frac{1}{Q\left(n\right)}\right)^{\gamma}, \tag{20}$$

where

$$Q\left(n\right) = \left(\left(q_D/p_D\right)^{\varepsilon-1} + A\left(n\right)^{\varepsilon-1}\right)^{1/(\varepsilon-1)},$$

and A is the price index of imported varieties given in (8). Note that we explicitly denote firm's sourcing strategy Σ by the mass of countries n and exclude φ from the definition of the price index as our assumption of homothetic demand implies the firm productivity does not affect the price index conditional on the sourcing strategy. Using (19) and (20), the profit maximization problem (4) can then be written as

$$\pi\left(\varphi, f\right) \equiv \max_n \left\{\frac{1}{\sigma} D \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\frac{1}{MC}\right)^{\sigma-1} - w\left(fn + f^I I\left(n > 0\right)\right)\right\}. \tag{21}$$

Note that we explicitly index firms' profits by (φ, f), which are the two sources of heterogeneity at the firm level.

Under our functional form assumptions in Assumption 2, (21) has a tractable characterization. The reason is that the firm specific price index A has a convenient closed form expression.

Proposition 3. *Consider the setup above and let Assumption 2 hold true. The import price index is then given by*

$$A\left(n\right) = z n^{\eta}, \tag{22}$$

where z and η are functions of exogenous parameters governing import prices (α, ν), the distribution

[20] Note that this result relies on the parametric restriction $\nu < 1$ so that $q/p = \frac{1}{\alpha} q^{1-\nu}$ is increasing in quality. Otherwise, one could simply reorder as countries as low-quality countries had the highest price-adjusted qualities. We will estimate ν in our micro-data and we can comfortably reject that $\nu \geq 1$.

of quality $(E[q],\theta)$ *and the elasticity of substitution of foreign varieties* θ

$$z = \frac{1}{\alpha}E[q]^{(1-\nu)}\left(\frac{\theta-1}{\theta}\right)^{(1-\nu)}\left(\frac{\theta}{\theta-(1-\nu)(\rho-1)}\right)^{\frac{1}{\rho-1}} \quad (23)$$

$$\eta = \frac{1}{\rho-1}-\frac{1-\nu}{\theta}. \quad (24)$$

Proof. See Appendix. □

Proposition 3 has two distinct virtues. First of all, it provides us with a very tractable expression for the firms' import price index $A(n)$, in that it has a simple power form where "total factor productivity" z and the "returns to scale to international varieties" η are closed-form expressions of the deep structural parameters.[21] Having this simple expression makes it easy to solve for firms' optimal extensive margin from (21). More importantly, Proposition 3 makes the calibration of the theory extremely transparent and parsimonious. As seen in (21) and (20): the underlying structure of the import environment *only* matters for the firm's problem through the price index A. But A is fully determined from z and η so that the two parameters governing the distribution of country quality, $E[q]$ and θ, together with the elasticity of substitution between foreign varieties, ρ, and the parameter controlling the price-quality correlation, ν, are all summarized into two "auxiliary" parameters, z and η, which we can directly estimate from the micro-data. Hence, we do *not* need to estimate the structural parameters $(\alpha, E[q], \theta, \rho, \nu)$ to solve for the behavior of firms on the extensive margin, but rather (z,η) suffice. This in particular implies that neither detailed information on the distribution of spending across sourcing partners nor independent information about import prices is required.

In fact, this sufficiency result can be strengthened. Using (10) and (3), we get that

$$s_D(\Sigma,\varphi) = s_D(n) = \frac{(q_D/p_D)^{\varepsilon-1}}{(q_D/p_D)^{\varepsilon-1}+A(n_i)^{\varepsilon-1}} = \frac{1}{1+\left[\frac{p_D}{q_D}zn^{\eta}\right]^{\varepsilon-1}}. \quad (25)$$

Hence, given z and η and a solution for firm's extensive margin n, firms' equilibrium domestic shares are fully determined (given the equilibrium price p_D, an estimate of the "trade" elasticity ε and a normalization for q_D). This implies that given (z,η) the underlying deep parameters $(\alpha, E[q], \theta, \rho, \nu)$ are not only irrelevant from the firms' point of view, but also for *all* aggregate outcomes, which - as shown in Proposition 2 - depend only on the distribution of firms' domestic shares.

This however, does not mean that the degree of quality heterogeneity (θ) or the substitutability of inputs (ρ) do not have a well defined role in shaping firms' import demand. As in Melitz and Redding (2012), the microstructure does matter in the sense of entertaining comparative static exercises with respect to these structural parameters. In our setup these are contained in the following Proposition.

Proposition 4. *Consider the setup above, let* z *and* η *be defined as in Proposition 3. Then*

1. *Diversity increases import productivity* z*, i.e.* $\frac{\partial z(E[q],\theta,\rho)}{\partial\theta} < 0$*, iff* $(\rho-1)(1-\nu) > 1$*,*

[21] Note that $\eta > 0$ because of Assumption 2.

2. *Diversity and substitutability are complements, i.e.* $\frac{\partial^2 z(E[q],\theta,\rho)}{\partial\theta\partial\rho} < 0$*, iff* $(\rho-1)(1-\nu) > 1$*,*

3. *Substitutability increases import productivity* z*, as* $\frac{\partial z(E[q],\theta,\rho)}{\partial\rho} > 0$*,*

4. *Diversity and substitutability both decrease the returns to importing as* $\frac{\partial\eta}{\partial\theta} > 0$ *and* $\frac{\partial\eta}{\partial\rho}<0$*.*

Proof. See Online Appendix. □

To see the intuition for Proposition 4, note that import productivity A satisfies

$$A^{\rho-1} = \int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)} dG(q).$$

When $(\rho-1)(1-\nu) > 1$, the firm is effectively *risk loving* and values diversity. As importing is effectively an option (as only the best countries are selected as sourcing countries), more variance in the unconditional distribution (holding mean equality $E[q]$ fixed!) will increase the benefit of importing. Such gains from diversity however are only available, if inputs are sufficiently substitutable, i.e. the higher ρ the more can firms leverage such quality differences. To understand the fourth part, note that if quality is very unequal, the marginal country is of much lower quality relative to the ones already sourced. Hence, import quality flattens out quickly, which is akin to low returns to sourcing. The same is true for substitutes: as the dispersion of expenditure shares increases in ρ, quality differences are leveraged intensely if ρ is high. The marginal country, which is by construction also the worst country the firm sources from, receives only a very small expenditure share and thus does not change the total quality of imports very much. Hence, import quality or the technology, which with imports are combined, do affect import demand. However, for a given micro-data set, and hence estimates of η and z, they do not change the researchers' conclusion on firms' import demand or the aggregate gains from trade.

Using this functional form for A, we can easily characterize the solution of firms' extensive margin of trade.

Proposition 5. *Consider the setup above and suppose that*

$$\eta(\varepsilon-1) < 1 \text{ and } \eta\gamma(\sigma-1) < 1. \tag{26}$$

Then, the optimal number of countries firms import from is given by

$$n(\varphi,f) = \begin{cases} n^* & \text{if } f^I < \left[\left(1+\left(\frac{p_D}{q_D}z(n^*)^{\eta}\right)^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} - 1\right](q_D/p_D)^{\gamma(\sigma-1)}\Upsilon\frac{D}{w^{1+(1-\gamma)(\sigma-1)}}\varphi^{\sigma-1} - fn^* \\ 0 & \text{otherwise} \end{cases}$$

where Υ is a constant[22] *and n^* is implicitly defined by*

$$\left((q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1}(n^*)^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} \eta z^{\varepsilon-1}(n^*)^{\eta(\varepsilon-1)-1} = \frac{1}{\gamma(\sigma-1)} \frac{w^{1+(1-\gamma)(\sigma-1)}}{\Upsilon D} \frac{f}{\varphi^{\sigma-1}}. \quad (27)$$

In particular, n^* is increasing in productivity (φ), decreasing in the fixed costs of sourcing (f) and increasing in average import quality (z). Furthermore, n^* is increasing in the price-adjusted domestic quality (q_D/p_D) if and only if $\gamma(\sigma-1) > (\varepsilon-1)$.

Proof. See Section 9.2 in the Appendix. □

The characterization of firms' optimal import behavior is intuitive. As importing is subject to fixed costs, firms face the usual trade-off: is the import-induced reduction in variable costs sufficient to cover the fixed costs. (27) implicitly characterizes the firms' optimal number of sourcing countries.[23] Note that it stresses the general equilibrium interlinkages of firms' import behavior. As the price of domestic inputs (p_D), the equilibrium wage (w) and consumer spending (D) are affected by firms effective productivity $\vartheta = \varphi s_D^{\gamma/(1-\varepsilon)}$, firms' import decisions are interrelated. Our quantitative analysis will explicitly take these interlinkages into account. The parametric restriction in (26) ensures that the firm's profit function is concave so that (27) is indeed necessary and sufficient. Furthermore, (26) implies that the LHS of (27) is decreasing in n, which immediately implies the comparative statics with respect to productivity φ and fixed costs f. That firms increase their international sourcing if z increases is also intuitive: z and n are complements in generating import quality (see (22)) so that the marginal returns to sourcing increase if z increases. That the relationship between domestic sourcing potential (q_D/p_D) and firms' international activities is ambiguous is due to the fact that there are two counteracting forces. If domestic varieties provide more quality per dollar spent, it both reduces firms' marginal cost of production and leads to a substitution towards domestic varieties an elasticity of substitution above unity implies that increases in (q_D/p_D) are biased towards domestic spending. While the first effect will *increase* the demand for foreign sourcing for the simple reason that productivity (regardless of its source) and import behavior are complements, the latter effect will *reduce* firms' incentives to source from abroad. The first effect is strong if increases in efficiency translate into large increases in market size, i.e. if σ is high. The second effect on the other hand dominates if there are limited technological complementarities, i.e. if ε is large. Hence, the parametric restriction $\gamma(\sigma-1) > (\varepsilon-1)$ exactly captures the trade-off between these two forces.

With Proposition 5 we now have all the ingredients in place to not only estimate the partial equilibrium productivity gains from importing but to also calculate their aggregate effects on welfare and TFP. According to Proposition 2 the crucial endogenous object is the joint distribution between innate productivity φ and domestic expenditure shares $s_D(n)$, say $G(\varphi, s_D)$. This joint distribution

[22] It is given by $\Upsilon = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}(1-\gamma)^{(1-\gamma)}\gamma^{\gamma}\right)^{\sigma-1}$.

[23] To be absolutely precise, n^* is bounded from above by unity. Hence, the optimal number of countries is in fact $\min\{n^*, 1\}$. He abstract from this case for ease of exposition. In our calibration we will find parameters, where this constraint is not binding.

however is fully determined by Proposition 5 as we can use (25) to go back and forth between s_D and n. Note that (27) stresses the importance of firm-specific fixed costs - it productivity was the only source of heterogeneity, domestic and productivity were (by construction) perfectly negatively correlated, which puts tight restrictions on the endogenous distribution $G(\varphi, s_D)$.

6 Empirical Implementation

In this Section we take the framework laid out above to data on French manufacturing firms to estimate the productivity gains from importing at the firm level as well as in the aggregate. First, we rely on our sufficiency result about firms' partial equilibrium gains from trade, Proposition 1 of Section 2 above, to consistently estimate the distribution of firm-level productivity gains in the population of French importers. As firms' domestic shares are directly observed, we only need to estimate the "trade elasticity" ε which here is simply the elasticity of substitution between imported and domestic varieties, as well as the elasticity of intermediate inputs γ in firms' production function. While we estimate γ using standard proxy-methods for the estimation of production functions (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; De Loecker and Warzynski, 2012), we identify ε using an instrumental variables strategy given an estimate of firms' effective productivity ϑ.

We then turn to the aggregate effects of trade. As shown in Proposition 2 above, aggregate productivity depends on the joint distribution of domestic shares and physical productivity. Because the latter is not observable, we rely on the macroeconomic model laid out in Section 4 to endogenously generate such joint distribution. We take a calibration approach and proceed in two steps. We first estimate a crucial parameter that governs the extensive margin of importing: the returns to scale parameter η - see the optimality condition (27) in Section 5 above. Given this parameter, we then calibrate our economy - in particular the underlying distribution of physical productivity and fixed costs - to different moments in the French microdata to gauge the aggregate gains from trade.

6.1 Data

In this subsection we provide a general overview of the dataset.[24] A detailed description of how the data is constructed is contained in the Appendix. Because we are interested in the demand for inputs, we restrict the analysis to manufacturing firms. We observe import flows for every manufacturing firm in France from the official custom files. Manufacturing firms account for 31% of the population of French importing firms and 56% of total import value in 2001. Overall, French firms trade with a total 226 countries. The flows are classified at the 8-digit (NC8) level of aggregation, which means that the product space consists of roughly 9,500 products. Using unique firm identifiers we can match this dataset to fiscal files, which contain detailed information on firm characteristics. The final sample consists of an unbalanced panel of roughly 260,000 firms which are active between 2001

[24] This dataset is not new and has been used in the literature before (e.g. Mayer et al. (2010); Eaton et al. (2004, 2011)). However, these contributions focused almost exclusively on the export side. We have also used this data in our earlier work (Blaum et al., 2013).

and 2006, 30.000 of which are importers. Table 10 in the Appendix contains some basic descriptive statistics of our data.

6.2 Firm-Level Productivity Gains from Importing

We now estimate the partial equilibrium productivity gains from importing (relative to autarky) for the population of importing manufacturing firms in France. According to Proposition 1, these are given by

$$\frac{\vartheta^{Trade}}{\vartheta^{Aut}} = \frac{\varphi \times s_D\left(\Sigma, \varphi\right)^{\gamma/(1-\varepsilon)}}{\varphi} = s_D\left(\Sigma, \varphi\right)^{\gamma/(1-\varepsilon)}. \tag{28}$$

As firms' domestic expenditure shares are directly observed in the microdata, we only need to estimate γ and ε. Recall also that (28) did *not* require the restrictions imposed in Assumption 2. As explained above, *any* trade model satisfying (1) and (2) will lead to (28).

To estimate γ and ε we employ a proxy method from the production function estimation literature, augmented to allow for foreign intermediate inputs. For the theory, we did not need to put any restrictions on product markets as (28) could be derived from firms' cost-minimization problem. In our data we do not see firms' physical output, or rather we do not observe firm-specific prices. Hence, we have to impose assumptions on the demand side, to infer physical quantities from observed revenues. For consistency with our quantitative exercise, we assume that firms' face an isoelastic demand curve with demand elasticity σ. This yields our estimating equation

$$ln\left(S\right) = \delta + \tilde{\alpha} ln\left(k\right) + \tilde{\beta} ln\left(l\right) + \tilde{\gamma} ln\left(x\right) + ln\left(\omega\right), \tag{29}$$

where $\tilde{\gamma} = \frac{\sigma-1}{\sigma}\gamma$, $\tilde{\alpha} = \frac{\sigma-1}{\sigma}\alpha$ and $\tilde{\beta} = \frac{\sigma-1}{\sigma}\left(1-\alpha-\gamma\right)$ and S denotes firm sales. Furthermore, the theory implies that productivity ω is given by

$$ln\left(\omega\right) \quad = \quad \frac{\sigma-1}{\sigma} ln\left(\vartheta\right) = \frac{1}{1-\varepsilon}\tilde{\gamma} ln\left(s_D\right) + \frac{\sigma-1}{\sigma} ln\left(\varphi\right). \tag{30}$$

Our coefficients of interest are γ and ε, which we can estimate using equations (29) and (30) in the following two-step procedure. First, we estimate equation (29) using the usual proxy methods for production function estimation. In particular, we follow the procedure in De Loecker and Warzynski (2012) to arrive at estimates of the vector of coefficients $\left(\tilde{\alpha}, \tilde{\gamma}, \tilde{\beta}\right)$ and an estimate of $ln\left(\omega\right)$ for each firm. Second, using the estimated coefficient $\tilde{\gamma}$ and values for $ln\left(\omega\right)$, we estimate ε from equation (30).[25] Clearly, we cannot estimate (30) via OLS, as the required orthogonality restriction clearly

[25] This approach is related to Kasahara and Rodrigue (2008), who use plant level data from Chile and estimate a production function equation similar to (29)-(30). Using the proxy method of Olley and Pakes (1996) and Levinsohn and Petrin (2003) augmented with import status as an additional state variable, they obtain a positive coefficient for the domestic share and conclude that foreign intermediate inputs increase firm productivity. Their point estimates imply that decreasing the domestic share by 100% increases firm productivity by 5.8-27%, depending on the estimator used. Our approach is different in two respects. First, we identify the coefficient for the domestic share from exogenous variation in export supplies. Second, we use the estimated coefficient to back out the trade elasticity, ε, and then use this elasticity together with the *observed* domestic expenditure shares to measure the productivity gains from trade relative to autarky for every firm.

fails: s^D is not orthogonal to innate productivity φ. In particular, more productive firms are likely to sort into more and different sourcing countries and this variation in the extensive margin of trade (which is correlated with productivity) will induce variation in firm-specific price indices and hence domestic shares. Hence, we estimate ε from (30) using an instrumentals variable strategy. In particular, we follow Hummels et al. (2011) and instrument s^D with shocks to world export supplies. More precisely, we construct the instrument

$$z_{it} = \sum_{ck} \Delta WES_{ckt} \times s_{cki}^{pre}, \tag{31}$$

where $\Delta WES_{ckt} = WES_{ckt} - WES_{ckt-1}$ denotes the change in total exports for product k of county c between year t and $t-1$ to the entire world (excluding France) and s_{cki}^{pre} is firm i's import share on product k of county c *prior* to our sample. Hence, z_{it} can be viewed as a firm-specific index of shocks to the supply of a firm's input bundle. Movements in this index should induce variation in firms' domestic shares, which are plausibly orthogonal to firm productivity. Intuitively: if we see China's exports in product k increasing in year t, French importers that source product k from China will be relatively more affected by this positive supply shock, and they should increase their import activities. Using this source of variation of import prices at the firm-level, we can identify the elasticity of substitution ε. In particular we estimate (30) in first differences using (31) to instrument the domestic share.

The results of this exercise are reported below. We first estimate firms' production function, in particular the share of material inputs (γ). More specifically, we estimate (29) separately for each sector s, i.e. allow for sector-specific output elasticities. As the residual is correlated with static factor demands, we follow the large literature on the estimation of production functions using proxy methods (for our application in particular De Loecker and Warzynski (2012)). As our paper does not add any methodological insights, we discuss our estimation procedure in detail in the Appendix. There we also discuss in how far the identifying assumptions on the law of motion for productivity ϑ_{ist} are consistent with the endogenous choice of import status and we also report a robustness check where we move away from a Cobb Douglas specification and consider a more general translog production function.[26] Allowing for this added flexibility does not change the estimated coefficients γ and ε in any material way. The results are contained in Table 11 in the Appendix.

Given the estimates of the production function and the series for firms' endogenous productivity, we now turn to the second step, i.e. the estimation of the elasticity of substitution between foreign and domestic varieties, ε. In particular, we use the estimated coefficients $\tilde{\gamma}_s$ to build the scaled domestic shares $\tilde{\gamma}_s ln\left(s^D\right)$ and estimate ε from (30) using the instrument in (31). More specifically,

[26] In particular we estimate (29) in each sector s as

$$ln\left(S_{ist}\right) = \delta_t + \alpha_s ln\left(k_{ist}\right) + \alpha_s^{kk} ln\left(k_{ist}\right)^2 + \beta_s ln\left(l_{ijt}\right) + \beta_s^{ll} ln\left(l_{ijt}\right)^2 + \psi ln\left(k_{ist}\right) ln\left(l_{ist}\right) + \gamma_s ln\left(x_{ist}\right) + \vartheta_{ist} + u_{ist},$$

i.e. we continue to assume a constant output elasticity for intermediate inputs but allow for second-order terms in capital and labor.

we consider the specification

$$\Delta ln\left(\hat{\vartheta}_{ist}\right) = \delta_s + \delta_t + \frac{1}{1-\varepsilon} \times \Delta\tilde{\gamma}_s ln\left(s^D_{ist}\right) + x'_{ist}\zeta + u_{ist}, \tag{32}$$

where δ are again the respective fixed effects, x_{ist} are firm-level controls and $\Delta ln\left(\hat{\vartheta}_{ist}\right)$ and $\Delta\tilde{\gamma}_s ln\left(s^D_{ist}\right)$ are the changes in firm productivity and firms' domestic shares respectively, which are instrumented by (31). For our baseline results, we define products at the 6-digit level and consider all importing firms in our sample, taking their respective first year as an importer to calculate the pre-sample expenditure shares s^{pre}_{cki}. The results of this exercise are contained in Table 1 below.

In column (1) we report the first stage relationship between changes in aggregate supplies and firms' domestic shares. As expected there is a strong negative correlation between international supply shocks and firms' domestic expenditure shocks. Column (2) contains the IV estimate of $\frac{1}{1-\varepsilon}$ for our baseline specification. The implied elasticity of substitution ε is given by 2.35 with a standard error of 0.648. Hence, imported inputs are not essential ($\varepsilon > 1$) but there are non-trivial complementarities between imported and domestic inputs. Columns (3) and (4) contain two alternative specification. In column (3) we only consider firms that were already importers in the first year of our sample 2001. Hence, the sample of firms in column (3) have a common year at which the initial pre-sample shares s^{pre} are calculated. While the standard error of the estimate increases, the point estimate is similar. In column (4) we finally focus on a balanced panel of firms that were already importers in 2001 and remain importers for the full 5 years until 2006. Again, we estimate an elasticity of substitution of roughly 2.5.[27]

Given our estimates of ε and γ and the micro data on firms' domestic shares we can now implement (28) and hence consistently estimate the partial equilibrium gains from trade for the population of French manufacturing firms. We depict these gains in Figure 1 and summarize them in Table 2. It is clearly seen that (i) the direct gains from trade are limited and (ii) that these are highly concentrated. While for the average firm productivity in the observed trade equilibrium is 12% higher than in autarky, for the median firm the gains amount to only 5%. Hence, the median firm would merely lose 5% in effective productivity if it were prevented from tapping into foreign import markets. The reason why these gains seem small is akin to the reason why the *aggregate* gains from trade in Arkolakis et al. (2012) also seem small: most importing firms in France simply do not import very much. As we did not have to assume anything about the distribution of fixed costs, the underlying heterogeneity on the country level or firms' import demand structure, *any* model with a CES demand system between domestic and imported varieties will arrive at exactly the same numbers for the gains from trade at the micro-level if the model matches Figure 1 and utilizes the same values for γ and ε.

We also want to stress that the necessity to estimate productivity $ln\left(\omega\right)$ and to use the instrumental variables approach in (30) arises only because we aimed to estimate γ and ε. If one had

[27]This result is consistent with Antràs et al. (2014). In their Eaton and Kortum (2002)-style model of importing, they estimate the Frechet parameter of the efficiency distribution θ. Above, we showed that - as far as the firm-level productivity consequences are concerned - their model has the same implications as the "love-for-variety" models, where $\theta = \varepsilon - 1$. They estimate $\theta \in [1, 1.8]$.

outside information about these values, one could estimate the distribution of gains simply from the observed domestic expenditure share s^D. Hence, our procedure is applicable in a variety of settings and provides an easy to implement tool to quantify the gains from importing without having to rely on any substantial assumptions besides the CES production structure and without having to estimate a structural model of importing.[28]

Having the endogenous gains from trade at hand, we can learn about firm characteristics that are correlated with such gains. In particular, we run regressions of the form:

$$-\frac{\sigma}{\sigma-1}\frac{\tilde{\gamma}}{\varepsilon-1}ln\left(s_{it}^{D}\right)=\delta_s+\delta_t+\beta x_{it}+u_{it}, \tag{33}$$

where δ are industry and time fixed effects and x_{it} are different firm characteristics. To interpret β, consider for example the homothetic model, which implied that firm-characteristics *only* matter via the extensive margin of trade. As the import quality A is "increasing" in the extensive margin of trade in that a larger set Σ implies a higher productivity of the import bundle A, we can interpret the partial correlations in (33) as reflecting the equilibrium relationship between different firm characteristics, the accompanying sourcing strategy and the resulting productivity effects. The results are contained in Table 3 and very intuitive. Bigger firms as measured by either value added or employment see higher gains, because they are more likely to endogenously select into more import markets. Quantitatively, the estimated elasticity implies, that an increase in firm size by one standard deviation will endogenously increase productivity through importing by roughly 0.8 percent. While seemingly small, this number is consistent with the fact that (a) the endogenous gains from trade are limited and (b) firm size is only an imperfect predictor of import intensity.[29] In column (3) we add two firm characteristics, which we expect to be positively related to firms' ability to reap productivity gains from international sourcing. As expected, both exporters and members of a foreign group have 2.3% and 8% higher productivity though their import activities.[30] An important observable variable of these gains is if course the number of international varieties sourced. In the special case of our model discussed above, A would in fact only depend on n. Columns (4) and (5) show that there is a strong positive correlation between firms' extensive margin of importing and the resulting productivity gains and that other firm characteristics shrink in importance once the number of varieties is controlled

[28] Halpern et al. (2011) use a related framework and estimate it on Hungarian micro data. They derive a production function equation analog to (29)-(30), as well as an import demand equation analog to Proposition 5. The main difference with our approach is that they obtain the parameters of their structural model, namely the trade elasticity (analog to ε) and the quality of foreign varieties (analog to z), by *simultaneously* estimating the production function and import demand equations. Because both of these estimating equations are derived after solving for the extensive margin of trade (i.e. n), they hold under rather restrictive assumptions, which are akin to our Assumption 2. In contrast, we identify ε not within the structural model but by using exogenous variation in input supplies. This allows to estimate ε without having to impose additional assumptions and without having to take a stance how the extensive margin of trade is determined. Halpern et al. (2011) find a much bigger elasticity of substitution between the domestic and foreign variety of 7.3.

[29] The correlation between firm size and import intensity will be an important moment we target in our quantitative exercise below.

[30] It is interesting to note that the coefficient on firm size is now negative. This is intuitive. A large firm, which decides to neither be an exporter nor a member of a foreign group, is likely to be disadvantaged on international markets, e.g. through a large fixed cost draw.

for.

6.3 The Aggregate Gains from Trade

In this section we take the aggregate environment laid out in Section 4 above to the data. We keep the assumptions of Section 5 to solve for firms' extensive margin of importing. As stressed above, aggregate productivity depends on the joint distribution of domestic expenditure shares and innate productivity. While the latter is not directly observable, we put discipline on this distribution by relying on the observable joint distribution of domestic shares and sales. Empirically, there is very little correlation between sales and domestic shares conditional on importing. A model with a single source of heterogeneity (productivity) will predict a perfectly negative correlation. To match the micro-data, we therefore explicitly allow for two dimensions of firm heterogeneity, namely innate productivity (φ) and the fixed costs of sourcing (f). We identify the joint distribution of these variables from properties of the observed joint distribution of sales and domestic shares. We then use the calibrated model to compute the aggregate TFP and welfare gains of moving from autarky to the current trade equilibrium. We find that the aggregate gains range between 16-47% and are therefore substantially larger than the typical firm-level gains measured in Section 6.2 above. To highlight how important it is to exploit the information contained in firms' domestic shares, we also contrast our results with alternative calibration strategies and show that ignoring the micro-data on domestic expenditure shares results in estimates of the gains from trade that can be substantially biased.

Our benchmark strategy is as follows. We parametrize the distribution of innate productivity and fixed costs as a joint log-normal distribution

$$\left(\begin{array}{c} ln\left(\varphi\right) \\ ln\left(f\right) \end{array}\right) \sim \mathcal{N}\left(\left(\begin{array}{c} \mu_{\varphi} \\ \mu_{f} \end{array}\right), \left(\begin{array}{cc} \sigma_{\varphi}^{2} & \rho\sigma_{\varphi}\sigma_{f} \\ \rho\sigma_{\varphi}\sigma_{f} & \sigma_{f}^{2} \end{array}\right)\right), \tag{34}$$

where ρ controls the correlation between productivity and fixed costs. We normalize μ_{φ} and calibrate the rest of the parameters in (34) to match the dispersion of sales and of domestic expenditure shares, as well as their correlation. Most of the remaining parameters that are needed to solve the model can be transparently seen in the optimality condition characterizing the domestic expenditure share[31]:

$$\left(s_D\right)^{1-\frac{\gamma(\sigma-1)}{\varepsilon-1}}\left(\frac{s_D}{1-s_D}\right)^{\frac{1-\eta(\varepsilon-1)}{\eta(\varepsilon-1)}} = \underbrace{\frac{1}{\gamma\left(\sigma-1\right)\Upsilon}\frac{1}{\eta}}_{\text{Parameters}}\underbrace{\left(\frac{q_D}{p_D}z\right)^{1/\eta}\frac{w^{1+(1-\gamma)(\sigma-1)}}{D}\left(\frac{p_D}{q_D}\right)^{\gamma(\sigma-1)}}_{\text{GE Variables}}\underbrace{\frac{f}{\varphi^{\sigma-1}}}_{\text{Heterogeneity}}. \tag{35}$$

First, note that we have already estimated ε and γ directly from the micro-data in Section 6.2 above. Second, we need the parameter η, which determines the import-price index (22) and parametrizes the demand for foreign varieties. We will estimate η outside of the calibration with data on firms' expenditure shares and the number of countries sourced. Note that solving the model in principle entails finding a fixed point of (35), as the general equilibrium variables depend on the domestic

[31] See Section 4 in the Online-Appendix for the derivation.

shares of all other firms which in turn depend on the general equilibrium variables. The structure of (35), however, suggests a calibration approach which bypasses this computation and which we describe in detail in the Appendix. The intuition is as follows: as (35) shows, we can calibrate a normalized version of the fixed costs, where these are scaled by an appropriate transformation of the general equilibrium variables. Because the general equilibrium variables depend on firms' import behavior only via domestic shares, which are itself a calibration target, we can compute all prices after the normalized calibration and thus back out the underlying true fixed costs. This not only reduces the computational burden substantially (as we do not have to solve for a fixed point), but also implies that the parameter z is not required for the calibration.

As for the remaining parameters, we first choose the fixed cost of being an importer f_I to match the share of importers in the French population. Finally, there are two parameters for which we do not have sufficient information in our data. For the demand elasticity σ we take a standard value from the literature to match an average mark-up of 30% (see e.g. Hsieh and Klenow (2009), Broda and Weinstein (2006)). An important parameter is the strength of round-about production, ϕ, which is embedded in the general equilibrium variables. Instead of choosing a particular value, we report results for a range of admissible values.

Estimation of η. Instead of estimating η within the calibration, we estimate it directly from the microdata. The required information stems from the cross-sectional relationship between firms' extensive margin of trade and their domestic shares. In particular, (25) implies that

$$s_D = \left(1 + \left[\frac{p_D}{q_D} z n^{\eta}\right]^{\varepsilon-1}\right)^{-1}$$

so that given our estimate of ε we can estimate η from the cross-sectional relationship between firms' domestic share and their number of sourcing countries. To implement this regression empirically, we have to take a stand what a variety is. In the data, importers import many products and have many trading partners for each of these products. For this paper, we identify η solely from the variation stemming from the number of countries *per product*. This is not only in line with the notion of varieties in the literature (see e.g. Broda and Weinstein (2006) and Goldberg et al. (2010)), but it also conforms with firms' economic rationale of multiple sourcing in the model. While the number of products sourced might be determined to a large degree by technological considerations, the demand for multiple suppliers within a given product category might plausibly stem from love-for-variety effects. In the data, we measure products at the 8-digit level.

To estimate η, we note that (25) implies that $\hat{z} n^{\eta} = \left(\frac{1-s_D}{s_D}\right)^{1/(\varepsilon-1)}$ and hence consider the regression

$$\frac{1}{\varepsilon-1} ln\left(\frac{1-s_{D,ist}}{s_{D,ist}}\right) = \delta_s + \delta_t + \delta_{NK} + \eta ln\left(n_{ist}\right) + u_{ist}, \tag{36}$$

where n_{ist} denotes the average number of countries per products sourced and δ_{NK} contains a set of fixed effects for the number of products sourced. The elasticity η is therefore identified from firms sourcing the same number of product from a different number of supplier countries.

For our baseline results, we estimate (36) on the subsample of firm-product pairs which source their respective products from at least two supplier countries. There is a sizable number of firm-product pairs in our data which are sourced from a single country and we are concerned that such single-variety interactions may not credibly identify the extensive margin of varieties but rather pick-up other variation across firms. The results are contained in Table 4.

Column 1 contains our baseline results of (36), where we add additional firm level controls that can affect firms' import behavior conditional on the number of varieties sourced. The implied value of η is 0.253 and it is precisely estimated. The remaining columns contain robustness checks. Once we include firm-product pairs that are sourced from a single trading partner the estimated elasticity η increases, as single-variety importers have very high domestic shares in the data. Columns three and four show that this estimate is unaffected by additional firm-level controls. Finally, the last column shows that it is important to control for the number of products sourced as import-intensive firms source both more varieties per-product and more products on international markets - without the product fixed effects, the estimated η increases substantially, reflecting the fact that the number of products sourced and the number of trading partner per product are positively correlated.[32] For our quantitative analysis we take column (1) as a natural benchmark but also present results for different values η in the Appendix.

Calibration. To calibrate the five remaining structural parameters $(\mu_f, \sigma_f, \sigma_\varphi, \rho, f_I)$ we target the following five moments: (i) the aggregate domestic share of the French manufacturing sector[33], (ii) the share of importing firms, (iii) the dispersion of importers' domestic shares, (iv) the dispersion of importers' sales and (v) the correlation between sales and domestic expenditure shares for importers. While all parameters are calibrated jointly, the average level of fixed costs (μ_f) controls mostly the aggregate domestic share, the fixed cost of importing (f_I) is mostly identified from the share of importers and the dispersion in fixed costs (σ_f) and productivity (σ_φ) from the dispersion in domestic shares and sales, respectively. Importantly, the correlation between productivity and fixed costs (ρ) is disciplined by the correlation between sales and domestic spending.

We focus on the economy's aggregate domestic share because, as explained in Arkolakis et al. (2012), this is the correct welfare statistic in aggregative environments. This allows us to compare our results to those of an aggregative approach where the moments from the micro-data are not used. We come back to this in Section 6.3.1 below.[34]

[32]Recall that the parameter η is a combinations of different structural parameters of the economy. While η is indeed sufficient to characterize the aggregate gains from trade, one might be interested to decompose the returns to international sourcing into the the elasticity of substitution across varieties ρ, the dispersion in input quality θ, and the elasticity of input prices with respect to quality ν. Using our estimate of η, we need two additional moments for identification. It turns out that this can be done in a very tractable way using reduced form methods. The two crucial additional pieces of information required are import prices (to identify ν) and data on firms' expenditure shares across trading partners (to identify θ). In the Appendix we perform this decomposition and identify ρ, θ and ν separately.

[33]This statistic is computed as the share of aggregate spending in materials that is accounted by domestic inputs.

[34]There we also show the sensitivity of the results with respect to different choices for this moment. While our baseline calibration strategy, which explicitly targets the micro-data on domestic shares, is relatively insensitive with

Table 5 lists the structural parameters of our model.[35] Table 6 contains the results of the calibration. As can be seen, the model can be calibrated to match the data accurately. Note in particular that the correlation between productivity and fixed costs is calibrated to be positive. This is necessary to match the observed low degree of correlation between sales and domestic shares.[36] Table 7 shows that the model is also able to match a set of non-targeted moments relatively well. The model does a particularly good job in matching the aggregate domestic share for importers[37] as well as the share of sales accounted by importers.

Figure 2 reports the marginal distribution of domestic shares and log sales for importers, both in the model and in the French data. We see that the model captures the overall shape of the marginal distribution of domestic shares, but tends to generate too many firms trading small amounts and too few firms trading intensively relative to the data. Thus, the model gets the average domestic share too high[38] - see Table 7. Note also that the model tends to capture well the normality of the log sales distribution. Figure 3 reports average domestic shares by sales quintile for the sample of importers. The model matches the small negative overall correlation by generating an inverted U-shaped relation: size leads to first greater and the lower average shares. The data displays a similar pattern but to a considerably smaller degree.

Finally, 3 depicts the positive relation between physical productivity and fixed costs by showing how the latter correlate with log sales. It turns out that this implication is directly testable in the micro-data. In particular, we can express the optimality condition for firms' extensive margin directly in terms of observables:[39]

$$wf = \frac{\eta\gamma\left(\sigma-1\right)}{\sigma} S\left(\varphi,n\right)\left(1-s_D\left(n\right)\right)s^m\left(n\right), \tag{37}$$

where $S\left(\varphi,n\right)$ denotes firm sales and $s^m\left(n\right)$ is the expenditure share on the *marginal* variety, i.e. the expenditure share on the variety with the lowest quality within the firms' sourcing strategy. Hence, bigger firms will have lower expenditure shares on their marginal variety holding the fixed costs of sourcing fixed, and firms with low fixed costs find it profitable to source from low quality countries (as they source from more countries) holding their size constant. (37) is attractive because it allows for a simple diagnostic of the cross-sectional variation in fixed costs. In particular, we can use (37) to identify firms' fixed costs up to scale. When we regress $ln\left(f\right)$ against proxies of firm-productivity like value-added (employment), we find an elasticity of 0.825 (0.857), both of which are highly significant.[40] Running the same regression in the model generated data yields an elasticity of

respect to which precise moment to target, this is not the case when we do not explicitly target the microdata on domestic spending.

[35] Note that these parameters satisfy the sufficient conditions of Proposition 5 as $\eta\left(\varepsilon-1\right) = 0.463 < 1$ and $\eta\gamma\left(\sigma-1\right) = 0.3188 < 1$.

[36] An increase in the correlation between physical productivity and fixed costs makes bigger firms trade relatively less.

[37] Note that the baseline calibration targets the aggregate domestic share for the full population of firms.

[38] For the purpose of computing aggregate productivity, however, the object of interest is not the marginal distribution of domestic shares per se but rather the joint distribution of domestic shares and productivity.

[39] See Section 6 in the Appendix.

[40] The full results of this exercise are contained in Table 13 in the Online-Appendix.

1.42.

Finally, while we have no data on spending in fixed costs, we note that the average firm spends about 3% of sales on fixed costs.

Aggregate Gains from Importing. Table 8 contains aggregate statistics from the calibrated model. The first two rows report the percentage change in aggregate TFP and welfare from moving from autarky to the trade equilibrium.[41] We report both numbers for different degrees of roundabout production. As expected, roundabout production acts like an investment-multiplier so that the gains from trade are increasing in ϕ. We see that the TFP gains range between 16-47% and the welfare gains range between 17-47%, depending on the degree of roundabout. To put this numbers in perspective, the welfare gains predicted by the formula in Arkolakis et al. (2012) using the aggregate import share are 10.5%.[42] The table also shows that the share of workers allocated to fixed cost production ranges between 0.2-6%.[43] The last row reports a "model-free" measure of the aggregate productivity gains, namely the sales weighted average of the firm-level productivity gains $s_D^{\gamma(1-\varepsilon)}$. Thus, this measure consists of the weighted average of the partial equilibrium gains depicted in Figure 1 above. We see that this measure is in general not an accurate statistic for the aggregate productivity gains as it overestimates or underestimates the true gains depending on the strength of roundabout.[44]

6.3.1 The Importance of Domestic Shares

In this subsection, we assess the value of the micro data on domestic shares for gauging the aggregate gains from trade. In other words, we ask: how would our estimates of the aggregate gains from trade would look like in the absence of the domestic share micro data? To answer this question, we perform the following exercise. We re-calibrate the model dropping the two moments associated with domestic expenditure shares, namely the dispersion of domestic shares and their correlation with sales. That is, we perform an alternative calibration where we match: (i) the share of importers, (ii) the aggregate import share and (iii) the dispersion in the distribution of sales. Accordingly, we set the dispersion in fixed costs and their correlation with physical productivity both to zero, i.e. $\sigma_f = \rho = 0$.

We report the results in Table (9) where the baseline calibration is also displayed for comparison. The calibrated parameters in the model without heterogeneity on fixed costs - henceforth NSD - imply aggregate gains from trade that are biased relative to those of the baseline: the NSD model predicts TFP (welfare) gains that are about 19% (14%) lower than the baseline. This follows from two types of biases associated with the NSD model. On the one hand, by relying on productivity as the single source of heterogeneity, the NSD model generates a perfectly negative correlation between physical productivity and the domestic share, which translates into a counterfactually low correlation between sales and domestic shares. This "correlation bias" is a force that leads to overestimating the aggregate gains as the better firms are the ones that trade more. On the other hand, the NSD model

[41] TFP and Welfare are computed using the expressions in Proposition 2.

[42] Our economy without roundabout predicts TFP gains of 13.6% and welfare gains of 14.6%.

[43] As roundabout becomes more important (ϕ decreases), the production of the domestic variety becomes more intensive in the final good and relies less on labor.

[44] This measure of aggregate TFP is used in Halpern et al. (2011).

generates too little dispersion in domestic shares and this leads to underestimating the aggregate gains. This “dispersion bias” arises because firms’ output are substitutes in the production of the final good and hence aggregate TFP is increasing in the underlying dispersion in effective productivity - see (16). The lower aggregate gains in the NSD model relative to the baseline arise because the latter effect dominates.

We can distinguish the dispersion from the correlation bias by performing an additional calibration. We calibrate a version of the model with heterogeneity in physical productivity and fixed costs that is uncorrelated - henceforth UH model. We use the same moments as in our baseline calibration except for the correlation between domestic shares and sales. Table 8 contains the results and includes for comparison the NSD and baseline calibrations. The dispersion effect can be understood as follows. Starting from the NSD model, introducing heterogeneity in fixed costs disciplined by the observed dispersion in domestic shares leads to an increase in the gains from trade of 39% (TFP) and 22% (welfare) - see UH model. The correlation effect, on the other hand, can be understood as follows. Starting from the UH model and allowing physical productivity and fixed to be correlated -so that the model matches the observed correlation between domestic shares and sales- reduces the aggregate gains by 20% (TFP) and 8.5% (welfare).

7 Conclusion

Firms engage in outsourcing, that is in acquiring production inputs from abroad, to increase productivity. This paper developed a framework to estimate the magnitude of these gains both at the firm and the aggregate level. Our main result showed that firms’ domestic expenditure share in material spending is a sufficient statistic for these productivity gains at the firm level. We showed that this statistic is robust under a variety of assumptions. In particular, we did not need to impose any assumptions on the underlying heterogeneity at the firm-level (e.g. innate productivity or the fixed costs of sourcing) or at the country level (e.g. quality, prices or variable trade costs). We also did not need to take a stand on the structure on output markets. Using micro data for the population of French importers we showed that the direct productivity gains from importing are moderate. The median importer gains only 5% of productivity by being allowed to source inputs internationally. The aggregate productivity gains, however, are substantially larger as bigger firms have a higher import intensity. We showed that the micro data on domestic expenditure shares is important to discipline the joint distribution of size and import intensity, a key object in assessing the aggregate gains from trade.

References

Amiti, M. and J. Konings: 2007, ‘Trade Liberalization, Intermediate Inputs and Productivity’. *American Economic Review* **97**(5), 1611–1638.

Antràs, P., T. C. Fort, and F. Tintelnot: 2014, 'The Marginal of Global Sourcing: Theory and Evidence from U.S. Firms'. Working Paper.

Arkolakis, C.: 2010, 'Market Penetration Costs and the New Consumer Margin in International Trade'. *Journal of Political Economy* **118**(6), 1151–1199.

Arkolakis, C., A. Costinot, and A. Rodriguez-Clare: 2012, 'New Trade Models, Same Old Gains?'. *American Economic Review* **102(1)**, 94–130.

Arkolakis, C. and M.-A. Muendler: 2011, 'The Extensive Margin of Exporting Products: A Firm-level Analysis'. Working Paper.

Bernard, A. B., J. B. Jensen, S. Redding, and P. Schott: 2007, 'Firms in International Trade'. *Journal of Economic Perspectives* **3**(21), 105–130.

Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott: 2012, 'The Empirics of Firm Heterogeneity and International Trade'. *Annual Review of Economics* **4**, 283–313.

Bernard, A. B., S. Redding, and P. Schott: 2010, 'Multi-Product Firms and Product Switching'. *American Economic Review* **100**(1), 70–97.

Blaum, J., C. Lelarge, and M. Peters: 2013, 'Non-Homothetic Import Demand: Firm Productivity and Quality Bias'. Working Paper.

Boler, E. A., A. Moxnes, and K. H. Ulltveit-Moe: 2014, 'R&D, International Sourcing and the Joint Impact on Firm Performance'. Working Paper.

Broda, C. and D. E. Weinstein: 2006, 'Globalization and the Gains from Varieties'. *Quarterly Journal of Economics* **121**(2), 541–585.

Chaney, T.: 2013, 'The Network Structure of International Trade'.

Costinot, A. and A. Rodriguez-Clare: 2014, 'Trade Theory with Numbers: Quantifying the Consequences of Globalization'. In: *Handbook of International Economics*.

De Loecker, J. and P. Goldberg: 2013, 'Firm Performance in a Global Market'.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik: 2012, 'Prices, Markups and Trade Reform'. Working Paper.

De Loecker, J. and F. Warzynski: 2012, 'Markups and Firm-Level Export Status'. *American Economic Review*.

Dhingra, S. and J. Morrow: 2012, 'Monopolistic Competition and Optimum Product Diversity Under Firm Heterogeneity'. Working Paper.

Eaton, J. and S. Kortum: 2002, 'Technology, Geography, and Trade'. *Econometrica* **70**(5), 1741–1779.

Eaton, J., S. Kortum, and F. Kramarz: 2004, 'Dissecting Trade: Firms, Industries, and Export Destinations'. *The American Economic Review* **Vol. 94**(2), 150–154.

Eaton, J., S. Kortum, and F. Kramarz: 2011, 'An Anatomy of International Trade: Evidence from French Firms'. *Econometrica* **79**(5), 1453–1498.

Eslava, M., A. C. Fieler, and D. Y. Xu: 2014, 'Trade, Skills, and Quality Upgrading: A Theory with Evidence from Colombia'. Working Paper.

Fabinger, M. and G. Weyl: 2012, '"Pass-Through and Demand Forms"'. Working Paper.

Goldberg, P., A. Khandelwal, N. Pavcnik, and P. Topalova: 2009, 'Trade Liberalization and New Imported Inputs'. *American Economic Review (P&P)* **99**(2), 494–500.

Goldberg, P., A. Khandelwal, N. Pavcnik, and P. Topalova: 2010, 'Imported Intermediate Inputs and Domestic Product Growth: Evidence from India'. *Quarterly Journal of Economics*.

Gopinath, G. and B. Neiman: 2014, 'Trade Adjustment and Productivity in Large Crises'. *American Economic Review* **104**(3).

Halpern, L., M. Koren, and A. Szeidl: 2011, 'Imported Inputs and Productivity'. Working Paper.

Hsieh, C.-T. and P. Klenow: 2009, 'Misallocation and Manufacturing TFP in China and India'. *Quarterly Journal of Economics* **124**(2), 771–807.

Hummels, D., R. Jorgenson, J. R. Munch, and C. Xiang: 2011, 'The Wage Effect of Offshoring: Evidence from Danish Matched Worker-Firm Data'. Working Paper.

Jia, P.: 2008, 'What Happens When Wal-Mart Comes to Town: An Empirical Analysis of the Discount Retailing Industry"'. *Econometrica* **76**(6), 1263–1316.

Jones, C.: 2011, 'Intermediate Goods and Weak Links in the Theory of Economic Development'. *American Economic Journal: Macroeconomics*.

Kasahara, H. and J. Rodrigue: 2008, 'Does the use of imported intermediates increase productivity? Plant-level evidence'. *Journal of Development Economics* **87**, 106–118.

Khandelwal, A. and P. Topalova: 2011, 'Trade Liberalization and Firm Productivity: The Case of India'. *The Review of Economics and Statistics* **93**(3), 995–1009.

Kugler, M. and E. Verhoogen: 2011, 'Prices, Plant Size, and Product Quality'. *Review of Economic Studies* (14418), 1–33.

Levinsohn, J. and A. Petrin: 2003, 'Estimating Production Functions Using Inputs to Control for Unobservables'. *Review of Economic Studies* **70**, 317–341.

Mayer, T., M. Melitz, and G. Ottaviano: 2010, 'Market size, Competition, and the Product Mix of Exporters'. Working Paper.

Melitz, M. J. and S. J. Redding: 2012, 'Heterogeneous Firms and Trade'. NBER Working Paper 18652.

Oberfield, E.: 2013, 'Business Networks, Production Chains, and Productivity: A Theory of Input-Output Architecture'. Working Paper.

Olley, S. and A. Pakes: 1996, 'The Dynamics of Productivity in the Telecommunications Equipment Industry'. *Econometrica* **64**(6), 1263–97.

Pavcnik, N.: 2002, 'Trade Liberalization, Exit and Productivity Improvements: Evidence from Chilean Plants'. *Review of Economic Studies* **69**(1), 245–276.

8 Tables and Figure

	First Stage	IV Estimate		
		Baseline	Importers in 2001	Balanced Panel
ΔWES	-0.010***			
	(0.003)			
$\gamma_s \Delta ln(s_D)$		-0.741**	-0.546*	-0.646**
		(0.356)	(0.324)	(0.282)
Implied ϵ		2.35	2.83	2.55
		(0.648)	(1.087)	(0.676)
N	67,696	67,696	58,027	48,480
R^2	0.00	0.265	0.266	0.267

Notes: Robust standard errors in parentheses with ***, ** and * respectively denoting significance at the 1%, 5% and 10% levels. DESCRIPTION

Table 1: Estimating the trade elasticity ε

Mean	Median	p25	p75	N
1.1201	1.0517	1.0137	1.1391	114,723

Notes: The Table reports moments of the empirical distribution of $s_{D,i}^{\frac{\gamma}{\varepsilon-1}}$.

Table 2: Distribution of Gains: Moments

	Dep. Variable: Gains from Importing ($\frac{\gamma}{1-\epsilon} ln(s_D)$)				
ln(va)	0.005***		-0.003***		-0.017***
	(0.000)		(0.000)		(0.000)
ln(l)		0.002***			
		(0.000)			
Exporter			0.023***		0.013***
			(0.001)		(0.001)
Foreign Group			0.079***		0.063***
			(0.002)		(0.002)
Num of varieties				0.083***	0.092***
				(0.001)	(0.001)
N	111,975	113,266	111,975	113,266	111,975
R^2	0.11	0.11	0.13	0.18	0.20

Notes: Robust standard errors in parentheses with ***, ** and * respectively denoting significance at the 1%, 5% and 10% levels.

Table 3: Variation in the gains from trade

	Dep. Variable: $\frac{1}{1-\epsilon}ln(\frac{1-s_D}{s_D}))$				
	Mutiple Varieties	Full Sample			
Num of varieties	0.253***	0.389***	0.402***	0.405***	0.714***
	(0.011)	(0.005)	(0.005)	(0.005)	(0.005)
Exporter	-0.111***		-0.209***	-0.205***	
	(0.016)		(0.008)	(0.008)	
Foreign Group	0.126***		0.084***	0.097***	
	(0.011)		(0.009)	(0.009)	
$ln(k/l)$				-0.040***	
				(0.003)	
Product fixed effects	Yes	Yes	Yes	Yes	No
N	34,621	114,723	114,723	114,723	114,723
R^2	0.23	0.36	0.36	0.36	0.23

Table 4: Estimating η

Set Exogenously	Symbol	Value
Demand Elasticity	σ	3
Strength of Linkages	ϕ	$\in (0,1)$
Estimated		
Elasticity of Substitution	ε	2.83
Returns to Scale of Importing	η	0.3
Material Share	γ	0.63
Calibrated		
Dispersion in Productivity	σ^2_φ	
Average Fixed Cost	μ_f	
Dispersion in Fixed Costs	σ^2_f	
Correlation Fixed Costs - Productivity	ρ	
Fixed Cost of Being Importer	f^I	

Table 5: Structural Parameters

Target Moments	French Data	Model	Parameter
Aggregate Domestic Share	0.71	0.71	$\mu_f = 9.05$
Dispersion in Domestic Shares	0.27	0.26	$\sigma_f = 3.26$
Dispersion in log Sales	1.63	1.62	$\sigma_\varphi = 1.01$
Correlation log Sales - Dom Shares	-0.01	-0.01	$\rho = 0.41$
Share of Importers	0.32	0.32	$f^I = 0.003$

Table 6: Calibration

Non-Targeted Moments	French Data	Baseline	No Micro Data
Agg Domestic Share (Importers)	0.62	0.63	0.68
Avg Domestic Share (Importers)	0.70	0.78	0.95
Dispersion log Sales (Population)	1.59	2.08	3.46
Share of Sales by Importers	0.79	0.79	0.99

Table 7: Non-Targeted Moments

	$\phi = 0$	$\phi = 0.25$	$\phi = 0.5$	$\phi = 0.75$	$\phi = 0.95$
TFP Gains (in %)	47.48	31.32	23.35	18.60	16.00
Welfare Gains (in %)	47.23	36.16	28.21	21.75	17.43
% of Labor in Fixed Cost Production	0.17	0.91	2.43	4.31	5.81
VA-weigthed Avg Gains (in %)	25.01	25.01	25.01	25.01	25.01

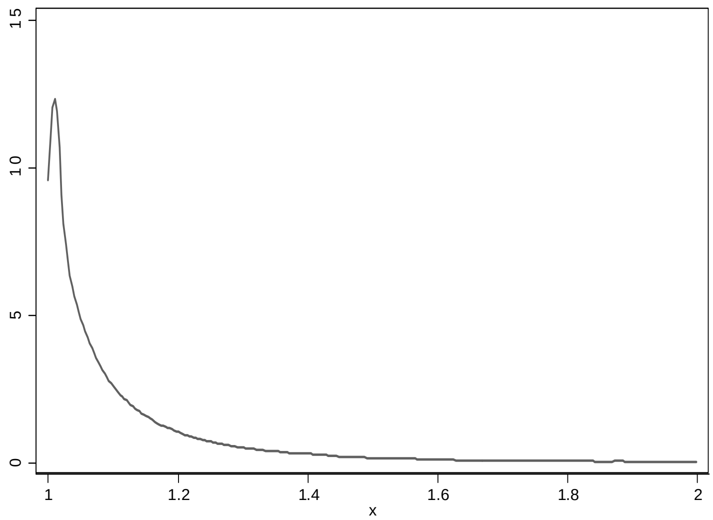
Table 8: Aggregate Effect of Trade

	Baseline		No s_D Data	
	Model	Parameter	Model	Parameter
Aggregate Domestic Share	0.71	$\mu_f = 9.05$	0.71	$\mu_f = 5.19$
Dispersion in log Sales	1.62	$\sigma_\varphi = 1.01$	1.63	$\sigma_\varphi = 1.72$
Share of Importers	0.32	$f^I = 0.003$	0.32	$f^I = 8.8e^{-05}$
Dispersion in Domestic Shares	0.26	$\sigma_f = 3.26$	0.09	$\sigma_f = 0$
Correlation log Sales - Dom Shares	-0.01	$\rho = 0.41$	-0.76	$\rho = 0$
TFP Gains	23.35 %		19.43%	Bias:16.76 %
Welfare Gains	28.21%		25.92%	Bias:8.10 %

Table 9: Calibration Without Domestic Shares

	Baseline		No s_D Data		Uncorrelated Heterogeneity	
	Model	Parameter	Model	Parameter	Model	Parameter
Agg Dom Share	0.71	$\mu_f = 6.75$	0.71	$\mu_f = 5.07$	0.71	$\mu_f = 22.71$
Dispersion Sales	1.63	$\sigma_\varphi = 1.05$	1.63	$\sigma_\varphi = 1.75$	1.63	$\sigma_\varphi = 0.80$
Share Importers	0.32	$f^I = 0.003$	0.32	$f^I = 1.4e^{-05}$	0.32	$f^I = 7e^{-08}$
Disp DomShares	0.27	$\sigma_f = 2.85$	0.09	$\sigma_f = 0$	0.27	$\sigma_f = 5.25$
Correlation	-0.01	$\rho = 0.48$	-0.76	$\rho = 0$	-0.39	$\rho = 0$
TFP Gains	20.05 %		16.63%		23.17%	
Welfare Gains	24.21%		22.32%		27.20%	

Note: Results are for $\varepsilon = 3$, $\eta = 0.30$ and $\gamma = 0.60$



Notes: The Figure displays a kernel density estimator based on the empirical distribution of $s_{D,i}^{\frac{\gamma}{\varepsilon-1}}$. Data is from year 2004. The value of ε is taken from the third column of Table 1.

Figure 1: Distribution of Productivity Gains

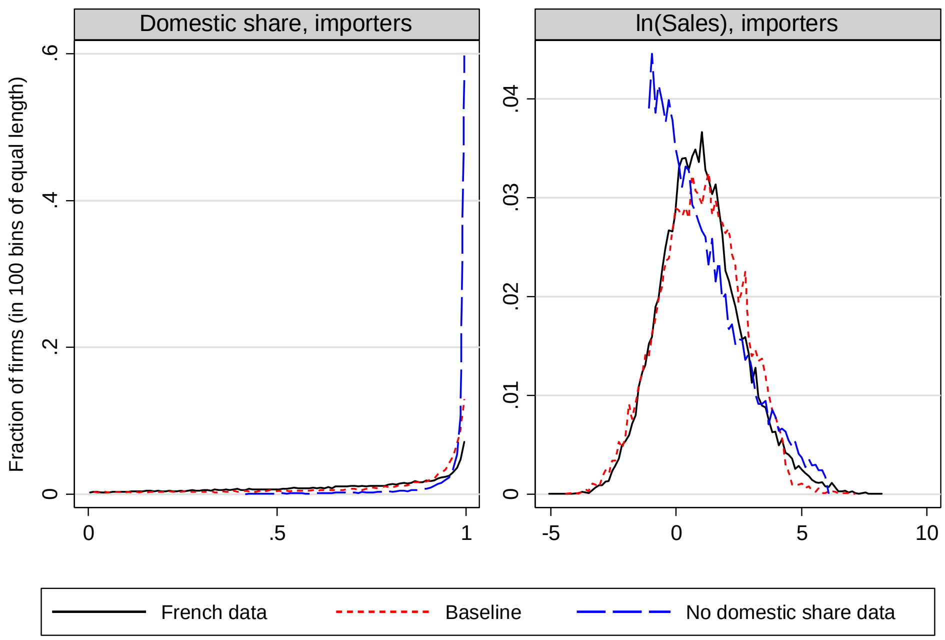


Figure 2: Marginal Distributions: Model and Data

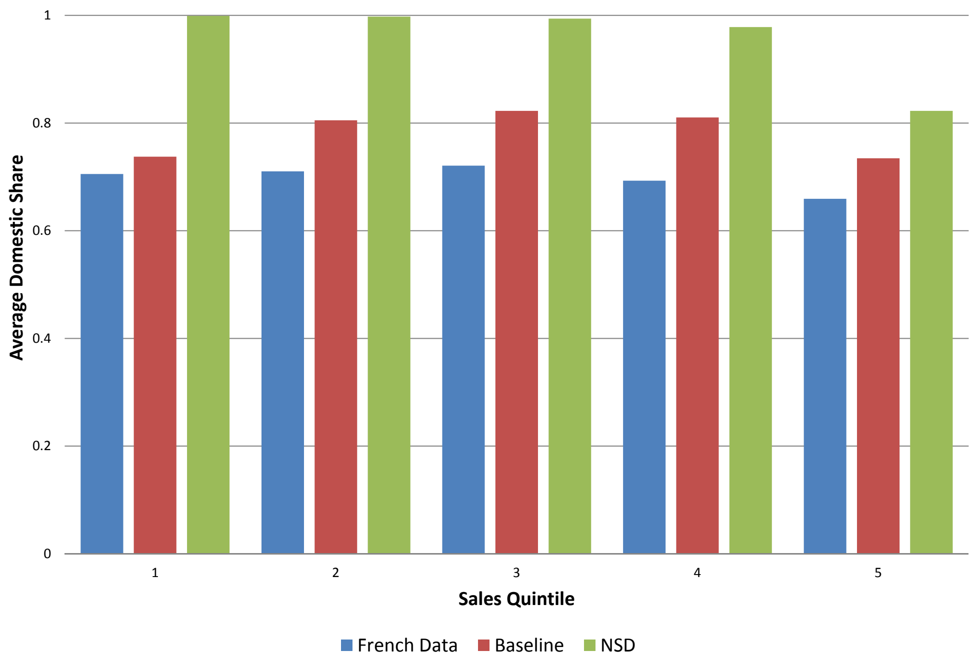


Figure 3: Correlation Structure: Model and Data

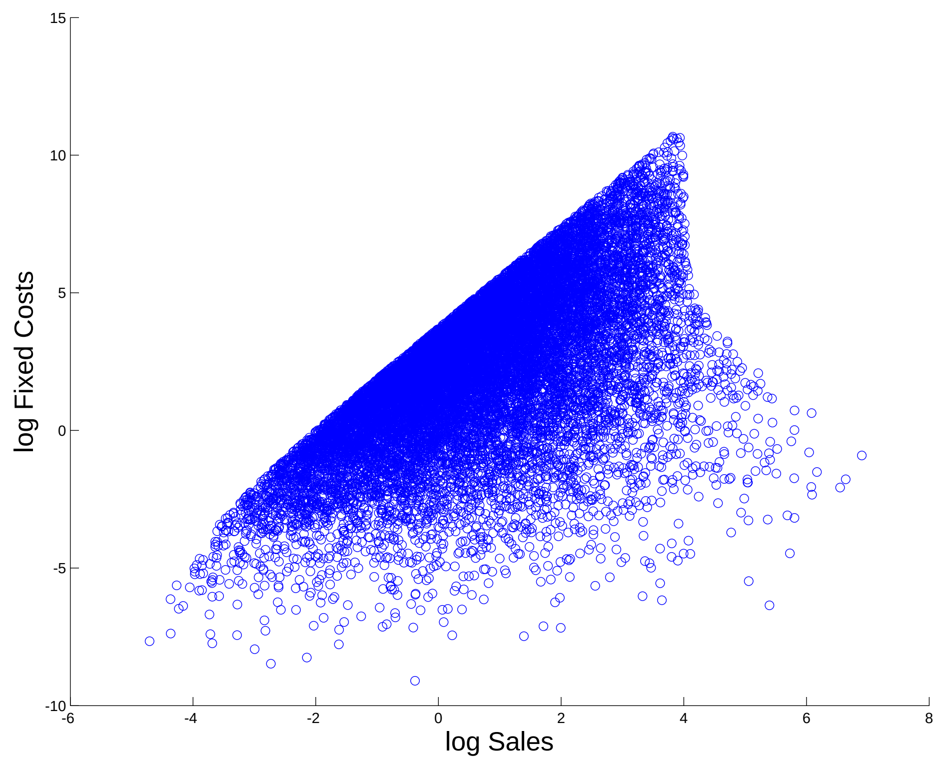


Figure 4: Correlation between Fixed Costs and log Sales

9 Appendix

9.1 Proof of Proposition 3

Under Assumption 2, the price index of the imported bundle (8) is given by

$$A(\overline{q},\varphi) = \left(\int_{\overline{q}}^{\infty} (q/p(q))^{\rho-1}\, dG(q)\right)^{\frac{1}{\rho-1}} = \frac{1}{\alpha}\left(\int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)} dG(q)\right)^{\frac{1}{\rho-1}}. \tag{38}$$

As quality follows a Pareto distribution, (38) implies

$$A(\overline{q},\varphi) = A(\overline{q}) = \frac{1}{\alpha}\left(q_{\min}^{\theta}\frac{\theta}{\theta-(1-\nu)(\rho-1)}\overline{q}^{(1-\nu)(\rho-1)-\theta}\right)^{\frac{1}{\rho-1}}. \tag{39}$$

The optimal number of varieties n is related to the chosen cutoff $\overline{q}$ via

$$n = P(q \geq \overline{q}) = \left(\frac{q_{\min}}{\overline{q}}\right)^{\theta}. \tag{40}$$

Substituting into (39) yields

$$\begin{aligned} A(n) &= \frac{1}{\alpha}\left(q_{\min}^{(1-\nu)(\rho-1)}\frac{\theta}{\theta-(1-\nu)(\rho-1)}n^{\frac{\theta-(1-\nu)(\rho-1)}{\theta}}\right)^{\frac{1}{\rho-1}} \\ &= \frac{1}{\alpha}(E[q])^{1-\nu}\left(\frac{\theta-1}{\theta}\right)^{1-\nu}\left(\frac{\theta}{\theta-(1-\nu)(\rho-1)}\right)^{\frac{1}{\rho-1}} n^{\frac{1}{\rho-1}-\frac{1-\nu}{\theta}}, \end{aligned}$$

which is the required expression.

9.2 Proof of Proposition 5

Suppose first that the firm's profit function conditional on importing is indeed concave. The necessary and sufficient condition for the optimal number of sourcing countries is then given by

$$\Gamma\frac{\gamma(\sigma-1)}{\varepsilon-1}\left((q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\eta(\varepsilon-1)\,z^{\varepsilon-1}n^{\eta(\varepsilon-1)-1}\varphi^{\sigma-1} = wf, \tag{41}$$

where $\Gamma = \Upsilon\frac{D}{w^{(1-\gamma)(\sigma-1)}}$. Rearranging terms yields the implicit definition of $n(\varphi, f)$ as

$$\left((q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1}n(\varphi,f)^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\eta z^{\varepsilon-1}n(\varphi,f)^{\eta(\varepsilon-1)-1} = \frac{1}{\gamma(\sigma-1)}\frac{w}{\Gamma}\frac{f}{\varphi^{\sigma-1}}.$$

Given this optimal sourcing strategy $n(\varphi, f)$, total profits are

$$\pi^{IM}(\varphi,f) = \Gamma\left((q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1}n(\varphi,f)^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\varphi^{\sigma-1} - w\left(fn(\varphi,f) + f^{I}\right).$$

If the firms was not importing, its profits were

$$\pi^D(\varphi, f) = \Gamma (q_D/p_D)^{\gamma(\sigma-1)} \varphi^{\sigma-1}.$$

Hence, the firm is an importer as long as $\pi^{IM}(\varphi, f) \geq \pi^D(\varphi, f)$, i.e.

$$\left[\left(1+\left(\frac{p_D}{q_D}zn(\varphi,f)^\eta\right)^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} - 1\right](q_D/p_D)^{\gamma(\sigma-1)}\frac{\Gamma}{w}\varphi^{\sigma-1} > fn(\varphi,f) + f^I.$$

Let us now prove that n is indeed unique. The marginal product per sourcing country is given in (41) as

$$\pi'(n) = \Psi\left((q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} z^{\varepsilon-1}n^{\eta(\varepsilon-1)-1} - wf, \tag{42}$$

where $\Psi = \Gamma\gamma(\sigma-1)\eta\varphi^{\sigma-1} > 0$. Hence,

$$\begin{aligned}
\pi''(n) &= \Psi z^{\varepsilon-1}\frac{\partial}{\partial n}\left\{\left((q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} n^{\eta(\varepsilon-1)-1}\right\} \\
&= \Psi z^{\varepsilon-1}\left((q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} \frac{n^{\eta(\varepsilon-1)-2}}{\eta(\varepsilon-1)-1} \times \\
&\quad \left\{1 + \frac{(\gamma(\sigma-1)-\varepsilon+1)\eta}{(\eta(\varepsilon-1)-1)}\frac{z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}{(q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}\right\}.
\end{aligned}$$

The optimal choice of n is unique if π is concave, i.e. if $\pi''(n) < 0$. Note that we can write $\pi''(n)$ as

$$\pi''(n) = \Psi(n) \times \frac{1}{(\eta(\varepsilon-1)-1)^2}\left\{(\eta(\varepsilon-1)-1) + (\gamma(\sigma-1)-\varepsilon+1)\eta\sigma(n)\right\}.$$

where $\Psi(n) > 0$ and $\sigma(n) = \frac{z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}{(q_D/p_D)^{\varepsilon-1}+z^{\varepsilon-1}n^{\eta(\varepsilon-1)}} \in [0,1]$. For π to be concave in n we therefore need that

$$(\eta(\varepsilon-1)-1) + (\gamma(\sigma-1)-\varepsilon+1)\eta\sigma(n) < 0. \tag{43}$$

A necessary and sufficient condition for this to be the case is $\eta(\varepsilon-1) < 1$ and $\eta\gamma(\sigma-1) < 1$ given in the Proposition. Sufficiency is trivial. To see that it is also necessary, suppose one the conditions was not satisfied. Then we could always find some γ, ε and $\sigma(n)$ such that (43) was not satisfied. This proves Proposition 5.

9.3 Data Description

Our main data set stems from the information system of the French custom administration (DGDDI) and contains the universe of import and export flows by French manufacturing firms. The data is collected at the 8-digit (NC8) level and a firm located within the French metropolitan territory must report this detailed information as long as the following criteria are met. Within EU imports, have to be reported as long as the firm's annual trade value exceeds 150,000 Euros. If that threshold is not met, firms can choose to report under a simplified scheme. However, in practice, many firms under that threshold report the detailed information. For imports from outside the EU, all shipments must be reported to the custom administration as this data is used to calculate the value added tax in all cases. The conditions are more stringent for exports. For within EU exports, all shipments must be reported to the custom administration. For exports outside the EU, reporting is required if the exported value to exceeds 1,000 Euros or weighs more than a ton.

The attractive feature of the French data is the presence of unique firm identifiers (the SIREN code), which is available in all French administrative files. Hence, various other datasets can be matched to the trade data at the firm level. To learn about the characteristics of the firms in our sample we employ fiscal files.[45] Sales are deflated using price indices of value added at the 3 digit level obtained from the French national accounts. To measure the expenditure on domestic inputs, we subtract the total import value from the total expenditure on wares and inputs reported in the fiscal files. Capital, used for the TFP estimation, was computed using a permanent inventory method. The series were initialized with the deflated value of assets reported in the first year of reported fiscal account (1995). We then used the reported investment expenditure, which we deflated with an investment price index available from the French national accounts. We assumed a depreciation rate of 10%.

Additionally we use the French business registers (SIRENE files), created by the Firm Demography Department of the French National Institute of Statistics (INSEE). The SIRENE files report the yearly creation and destruction of French firms and provide us with information about firm age and legal status. Finally we incorporate information on the ownership structure from the LIFI/DIANE (BvDEP) files. These files are constructed at INSEE using a yearly survey (LIFI) describing the structure of ownership of all of the French firms in the private sector whose financial investments in other firms (participation) are higher than 1.2 million Euros or having sales above 60 million Euros or more than 500 employees. This survey is complemented with the information about ownership structure available in the DIANE (BvDEP) files, which are constructed using the annual mandatory reports to commercial courts, and with the register of firms that are controlled by the State.

Using these datasets, we construct a non-balanced panel dataset spanning the period from 2001 to 2006. Some basic characteristics of importing and non-importing firms are contained in Table 10. For comparison, we also report the results for exporting firms. Expectedly, importers outperform domestic firm in essentially all dimensions we look at (see also Bernard et al. (2012)). Furthermore, import and export status are highly correlated.

[45] The firm level accounting information is retrieved from two different files: the BRN ("Bénéfices Réels Normaux") and the RSI ("Régime Simplifié d'Imposition"). The BRN contains the balance sheet of all firms in the traded sectors with sales above 730,000 Euros. The RSI is the counterpart of the BRN for firms with sales below 730,000 Euros. Although the details of the reporting differs, for our purpose these two data sets contain essentially the same information. Their union covers nearly the entire universe of all French firms.

	Full Sample	Importers	Non-Importers	Exporters	Non-Exporters
Employment	18.18	89.72	5.71	78.89	5.9
Sales	4342.35	23646.8	969.15	20842	1035.52
Sales / Worker	104.42	157.62	92.3	150.46	92.3
Capital / Worker	33.74	44.28	31.29	42.45	31.38
Inv. / Worker	2.9	4.13	2.6	3.91	2.61
Inputs (mat.)	0.2	0.3	0.18	0.28	0.18
Import share	0.04	0.3	0	0.2	0.01
Share of Importers	0.15	1	0	0.66	0.05
Share of Exporters	0.17	0.75	0.07	1	0
Firm Age	14.38	19.45	13.51	19.54	13.36
Foreign owned	0.02	0.13	0.01	0.11	0.01
Foreign Group	0.05	0.23	0.01	0.2	0.02
Labor Productivity	43.783	56.496	40.726	56.023	40.405
TFP-LP	28.135	30.255	27.788	29.23	27.999
TFP-OLS	25.412	26.519	25.251	26.61	25.189
Number of Firms	259,602	31,022	228,580	34,527	225,075

Notes: Sales, wages, expenditures on imports or exports are all expressed in 2000 prices using a 3-digit industry level price deflator. Our capital measure is the book value reported in firms' balance sheets ("historical cost"). We measure employees by occupation. Skilled workers are engineers, technicians and managers, workers of intermediate skills are skilled blue and white collars and low skilled workers are members of unskilled occupations. A firm is foreign owned, if the controlling entity is a foreign company. A firm is member of a foreign group if at least one affiliate or the headquarter is located outside of France.

Table 10: Characteristics of importers, exporters and domestic firms

9.4 Estimating the production functions

In Table below we report the results of estimating (29) using the procedure of De Loecker and Warzynski (2012).

9.5 A Simple Parametric Example

To see the importance of distribution of domestic shares, in particular the correlation with innate productivity and the dispersion, consider the following example. Suppose that domestic shares and productivity are distributed according to

$$(ln\left(\varphi\right), ln\left(s_D\right)) \sim N\left(\left(\begin{array}{c}\mu_P\\ \mu_{SD}\end{array}\right), \left(\begin{array}{cc}\sigma_P^2 & \rho\sigma_P\sigma_{SD}\\ \rho\sigma_P\sigma_{SD} & \sigma_{SD}^2\end{array}\right)\right). \tag{44}$$

Of course, s_D is an endogenous object so that both the correlation ρ and the dispersion σ_{SD} are endogenously generated though the interplay between productivity and fixed costs. Now consider the TFP aggregator

$$ln\left(TFP\right) = \frac{1}{\sigma-1}ln\left(\int\left(\varphi_i\left(s_D^i\right)^{\frac{\gamma}{1-\varepsilon}}\right)^{\sigma-1}di\right) = \frac{1}{\sigma-1}ln\left(E\left[\left(\varphi_i\left(s_D^i\right)^{\frac{\gamma}{1-\varepsilon}}\right)^{\sigma-1}\right]\right).$$

Sector	l	k	m
NACE 14	0.482	0.276	0.259
NACE 15	0.389	0.087	0.561
NACE 17	0.614	0.060	0.351
NACE 18	0.773	0.043	0.320
NACE 19	0.586	0.079	0.342
NACE 20	0.392	0.060	0.522
NACE 21	0.468	0.099	0.424
NACE 22	0.886	-0.002	0.200
NACE 24	0.380	0.080	0.580
NACE 25	0.410	0.085	0.477
NACE 26	0.866	0.021	0.281
NACE 27	0.297	0.049	0.605
NACE 28	0.510	0.099	0.363
NACE 29	0.641	0.045	0.352
NACE 30	0.615	0.073	0.385
NACE 31	0.520	0.058	0.430
NACE 32	0.672	0.059	0.288
NACE 33	0.591	0.078	0.325
NACE 34	0.483	0.095	0.445
NACE 35	0.911	0.036	0.196
NACE 36	0.580	0.057	0.393
NACE 37	0.411	0.181	0.457

Table 11: Production Function Coefficient Estimates, by 2-digit Sector

According to (44), φ and s_D are jointly lognormal so that

$$ln\left(\varphi^{\sigma-1}\left(s_D^i\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}\right)\sim N\left((\sigma-1)\mu_P+\frac{\gamma(\sigma-1)}{1-\varepsilon}\mu_{SD},(\sigma-1)^2\sigma_P^2+\left(\frac{\gamma(\sigma-1)}{1-\varepsilon}\right)^2\sigma_{SD}^2+2\rho\left(\frac{\gamma(\sigma-1)}{1-\varepsilon}\right)(\sigma-1)$$

Hence,

$$ln\,(TFP)=\mu_P+\frac{\gamma}{1-\varepsilon}\mu_{SD}+\frac{1}{2}(\sigma-1)\left(\sigma_P^2+\left(\frac{\gamma}{1-\varepsilon}\right)^2\sigma_{SD}^2+2\rho\left(\frac{\gamma}{1-\varepsilon}\right)\sigma_P\sigma_{SD}\right).$$

In autarky we have $s_D=1$ so that $\sigma_{SD}=\mu_{SD}=0$. Hence, the TFP gains from trade are given by

$$\begin{aligned} G &= ln\,(TFP)-ln\left(TFP^{Aut}\right) \\ &= \frac{\gamma}{1-\varepsilon}\mu_{SD}+\frac{1}{2}(\sigma-1)\left(\frac{\gamma}{1-\varepsilon}\right)^2\sigma_{SD}^2+(\sigma-1)\,\rho\left(\frac{\gamma}{1-\varepsilon}\right)\sigma_P\sigma_{SD}. \end{aligned} \tag{45}$$

Now consider the *aggregate* import-share in this economy. This aggregate share is given by

$$\begin{aligned} S_{ACR}^D=\frac{\text{Total Domestic Spending}}{\text{Total Spending on Intermediates}} &= \frac{\int_i s_D^i s_i di}{\int_i s_i di} \\ &= \int s_{D,i}\frac{\varphi_i^{\sigma-1}\left(s_{Di}\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}}{\int\varphi_i^{\sigma-1}\left(s_{Di}\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}di}di \end{aligned}$$

where the last equation follows from the fact, that total spending $s(i)$ is proportional to sales $p(i)\,y(i)$ (see (53) and (55)). Hence,

$$S^D_{ACR} \quad = \quad \frac{E\left[\varphi^{\sigma-1}\left(s_D\right)^{\frac{\gamma(\sigma-1)+(1-\varepsilon)}{1-\varepsilon}}\right]}{E\left[\varphi^{\sigma-1}\left(s_D\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}\right]}.$$

Now note that

$$E\left[\varphi^{\sigma-1}\left(s_D\right)^{\frac{\gamma(\sigma-1)+(1-\varepsilon)}{1-\varepsilon}}\right] = exp\left((\sigma-1)\,\mu_P + \frac{\gamma\,(\sigma-1)+(1-\varepsilon)}{1-\varepsilon}\mu_{SD} + \frac{1}{2}\left((\sigma-1)^2\,\sigma_P^2 + \left(\frac{\gamma\,(\sigma-1)+(1-\varepsilon)}{1-\varepsilon}\right)^2 \sigma\right.\right.$$

so that

$$\frac{E\left[\varphi^{\sigma-1}\left(s_D\right)^{\frac{\gamma(\sigma-1)+(1-\varepsilon)}{1-\varepsilon}}\right]}{E\left[\varphi^{\sigma-1}\left(s_D\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}\right]} \quad = \quad exp\left[\mu_{SD} + \frac{1}{2}\left(\left[2\frac{\gamma\,(\sigma-1)}{1-\varepsilon}+1\right]\sigma_{SD}^2 + 2\rho\,(\sigma-1)\,\sigma_P\sigma_{SD}\right)\right].$$

Hence,

$$ln\left(S^D_{ACR}\right) \quad = \quad \mu_{SD} + \frac{\gamma\,(\sigma-1)}{1-\varepsilon}\sigma_{SD}^2 + \frac{1}{2}\sigma_{SD}^2 + \rho\,(\sigma-1)\,\sigma_P\sigma_{SD}. \tag{46}$$

Using (46) in (45) we get that

$$G \quad = \quad \frac{\gamma}{1-\varepsilon}\left(ln\left(S^D_{ACR}\right) - \frac{1}{2}\left[\frac{\gamma\,(\sigma-1)}{1-\varepsilon}+1\right]\sigma_{SD}^2\right). \tag{47}$$

Online Appendix (not for publication)

1 General Equilibrium in the Aggregate Economy

There is a measure $\overline{L}$ of workers who value a unique final good. There is a measure 1 of firms. The final good is a composite of firms' output

$$Y = \left(\int y(i)^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}.$$

Firm i produces according to

$$y(i) = \varphi l^{1-\gamma} x^{\gamma}.$$

x denotes intermediary services. Intermediary services can come from domestic and international intermediate inputs. Letting q be the quality of the respective intermediary input, x is given by

$$x = \left((q_D z_D)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\left(\int_{\overline{q}}^{\infty} (q z_F(q))^{\frac{\rho-1}{\rho}} \, dq \right)^{\frac{\rho}{\rho-1}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The domestic intermediate is produced from a representative intermediary sector according to

$$X = M l_X^{\phi} y^{1-\phi}, \tag{48}$$

where l_X is the number of workers employed in intermediary production and y denotes the final good. If $\phi < 1$, there is roundabout production.

Definition 2. Let firms' sourcing strategy $[n(i)]_i$ be given. An equilibrium in this economy is a set of prices $[w, P_X, [p(i)]_i]$ and a set of allocations $\left[L_X, [l(i)]_i, \left[y^C(i)\right], \left[y^X(i)\right], [z_D(i)]_i\right]$ such that

- the intermediary producer maximizes profits taking prices $[w, p(i), P_X]$ as given
- monopolistic producers maximize profits, i.e. chose prices $[p(i)]$ and quantities $\left[l(i), z_D(i), y^C(i), y^X(i)\right]$ optimally given (w, P_X)
- consumers maximize utility, i.e. choose quantities $\left[y^C(i)\right]$ optimally given prices $[p(i)]$
- the RWO buys $\left[y^{ROW}(i)\right]$ units of differentiated varieties such that trade is balanced
- markets clear, i.e.
  - labor market clear

$$L = L_X + \int_{i=0}^{1} l(i) \, di + \int_{i=0}^{1} f^V n(i) \, di + \int_{i=0}^{1} f^I 1[n(i) > 0] \, di$$

– good markets clear

$$\begin{aligned} y(i) &= y(i)^C + y^X(i) + y^{ROW}(i) \\ X &= \int_{i=0}^{1} z_D(i)\, di \end{aligned}$$

1.1 Characterizing the equilibrium in two steps

We are going to solve for the equilibrium in 2 steps. First we are going to solve for the allocations across the heterogeneous producers in the monopolistically competitive sector. We let P, Y and S be the price level, total output and aggregate spending on the differentiated good sector. Then we are going to solve for the equilibrium aggregate quantities.

1.1.1 Aggregating the monopolistic sector given total spending S

Let total spending on the differentiated varieties be S. Hence,

$$S = S_C + S_X + S_{ROW},$$

where S_X, S_{ROW} and S_C denotes total spending of consumers, ROW and the intermediary sector. We know from cost-minimization on the respective demand side that

$$\begin{aligned} y^C(i) &= \left(\frac{p(i)}{P}\right)^{-\sigma} \frac{S_C}{P} \\ y^X(i) &= \left(\frac{p(i)}{P}\right)^{-\sigma} \frac{S_X}{P} \\ y^{ROW}(i) &= \left(\frac{p(i)}{P}\right)^{-\sigma} \frac{S_{ROW}}{P}, \end{aligned}$$

where P is the price index

$$P = \left(\int_{i=0}^{1} p(i)^{1-\sigma}\, di\right)^{\frac{1}{1-\sigma}}. \tag{49}$$

With monopolistic competition, we have that

$$p(i) = \frac{\sigma}{\sigma-1} MC(i), \tag{50}$$

where $MC(i)$ are the marginal costs.

Deriving firms' marginal costs To derive firm i's marginal costs $MC(i)$, let $\overline{q}(i)$, i.e. firm $i's$ extensive margin of trade, be given. Under our assumptions we can go back and forth from $n(i)$ to $\overline{q}(i)$ via

$$n(i) = P(q \geq \overline{q}(i)) = 1 - G(\overline{q}(i)).$$

From above, firm i's *import* services are given by

$$m_F(i)=\left(\int_{\overline{q}(i)}^{\infty}(qz_F(q))^{\frac{\rho-1}{\rho}}dq\right)^{\frac{\rho}{\rho-1}}=A(\overline{q}(i))\,s_F,$$

where s_F denotes total *spending* on imported inputs and

$$A(\overline{q}(i))=A(n(i))=zn(i)^{\eta},$$

where $z=\frac{1}{\alpha}E[q]^{(1-\nu)}\left(\frac{\theta-1}{\theta}\right)^{(1-\nu)}\left(\frac{\theta}{\theta-(1-\nu)(\rho-1)}\right)^{\frac{1}{\rho-1}}$ and $n(i)=\left(\frac{q_{min}}{\overline{q}(i)}\right)^{\theta}$. Hence, the service flow from intermediary products is given by

$$x=\left(\left(\frac{q_D}{P_X}\right)^{\varepsilon-1}+A(n(i))^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}s_X=Q(P_X,n(i))\,s_X,$$

where s_X denotes *total intermediary spending.* Not in particular, that total intermediary spending productivity Q depends on $n(i)$ in a monotone fashion. Given $n(i)$, the domestic share is equal to

$$s^D(i)=\frac{(q_D/P_X)^{\varepsilon-1}}{(q_D/P_X)^{\varepsilon-1}+A(n(i))^{\varepsilon-1}}=\left(\frac{q_D/P_X}{Q(P_X,n(i))}\right)^{\varepsilon-1},$$

which again gives us a relationship between $s^D(i)$ and $n(i)$ is a monotone fashion (given the aggregate price P_X). Hence, total intermediary services are given by

$$x=\frac{q_D}{P_X}\left(s_i^D\right)^{\frac{1}{1-\varepsilon}}s_X,$$

so that $\frac{q_D}{P_X}\left(s_i^D\right)^{\frac{1}{1-\varepsilon}}$ is firm i's productivity of intermediary spending. Firm i's production function is given by

$$\begin{aligned} y(i) &= \varphi l^{1-\gamma}\left(\frac{q_D}{P_X}\left(s_i^D\right)^{\frac{1}{1-\varepsilon}}s_X\right)^{\gamma}=\varphi\left(\frac{q_D}{P_X}\left(s_i^D\right)^{\frac{1}{1-\varepsilon}}\right)^{\gamma}l^{1-\gamma}s_X^{\gamma} \\ &\equiv \vartheta(i)\left(\frac{q_D}{P_X}\right)^{\gamma}l^{1-\gamma}s_X^{\gamma}. \end{aligned} \tag{51}$$

Again: given $n(i)$, productivity $\vartheta(i)$ is fully determined. Using (51), we can derive firm's marginal costs. Let total spending be

$$wl+s_X=s(i).$$

Because of Cobb-Douglas production we get

$$s_X=\gamma s(i)\ \text{ and } wl=(1-\gamma)\,s(i) \tag{52}$$

Hence,

$$y(i)=\vartheta(i)\left(\frac{q_D}{P_X}\right)^{\gamma}l^{1-\gamma}s_X^{\gamma}=\vartheta(i)\left(\frac{1-\gamma}{w}\right)^{1-\gamma}\left(\frac{\gamma q_D}{P_X}\right)^{\gamma}s(i), \tag{53}$$

so that

$$MC(i) = \frac{1}{\vartheta(i)} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \left(\frac{P_X}{\gamma q_D}\right)^{\gamma}. \tag{54}$$

Hence, (50) implies that

$$p(i) = \frac{\sigma}{\sigma-1} \frac{1}{\vartheta(i)} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \left(\frac{P_X}{\gamma q_D}\right)^{\gamma}, \tag{55}$$

which determines individual prices $[p(i)]$ as a function of parameters and aggregate prices (w, P_X).

Allocations in the differentiated sector The aggregate price index (49) is given by

$$\begin{aligned} P &= \left(\int_{i=0}^{1} p(i)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \left(\frac{P_X}{\gamma q_D}\right)^{\gamma} \left(\int_{i=0}^{1} \left[\frac{1}{\vartheta(i)}\right]^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}. \\ &= \frac{\sigma}{\sigma-1} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \left(\frac{P_X}{\gamma q_D}\right)^{\gamma} \left[\left(\int_{i=0}^{1} \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}\right]^{-1}. \end{aligned}$$

Total demand for firm i's products is given by

$$\begin{aligned} y^D(i) &= \left(\frac{p(i)}{P}\right)^{-\sigma} \frac{S}{P} \\ &= \left(\frac{\vartheta(i)^{-1}}{\left[\left(\int_{i=0}^{1} \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}\right]^{-1}}\right)^{-\sigma} \frac{S}{\frac{\sigma}{\sigma-1} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \left(\frac{P_X}{\gamma q_D}\right)^{\gamma} \left[\left(\int_{i=0}^{1} \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}\right]^{-1}} \\ &= \left(\frac{\vartheta(i)}{\left(\int_{i=0}^{1} \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma} \frac{S}{\frac{\sigma}{\sigma-1} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \left(\frac{P_X}{\gamma q_D}\right)^{\gamma} \left[\left(\int_{i=0}^{1} \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}\right]^{-1}}. \end{aligned} \tag{56}$$

The required labor for these products is given by

$$\begin{aligned} l^D(i) &= \frac{1-\gamma}{w} s(i) = \frac{1-\gamma}{w} \frac{1}{\vartheta(i) \left(\frac{1-\gamma}{w}\right)^{1-\gamma} \left(\frac{\gamma q_D}{P_X}\right)^{\gamma}} y^D(i) \\ &= \frac{1-\gamma}{w} \frac{\sigma-1}{\sigma} \left(\frac{\vartheta(i)}{\left(\int_{i=0}^{1} \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma-1} S. \end{aligned} \tag{57}$$

The required labor for the total fixed costs in the differentiated sector is given by

$$l^F(i) = n(i) f^V(i) + 1[n(i) > 0] f^I.$$

Given spending S, aggregate prices (w, P_X) and firms' extensive margin of trade $[n(i)]_i$, these equations fully determine the allocation in the differentiated sector.

1.1.2 Aggregate Allocations from the Differentiated Sector

Given the fictional price P we can define the aggregate bundles Y^C and Y^X by consumers and intermediary producers. In particular, by construction

$$\begin{aligned} Y^C &= \left(\int y^C(i)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} = \left(\int \left[\left(\frac{\vartheta(i)}{\left(\int_{i=0}^1 \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma} \frac{S_C}{P}\right]^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\ &= \frac{S_C}{P}\left(\int \frac{\vartheta(i)^{\sigma-1}}{\left(\int_{i=0}^1 \vartheta(i)^{\sigma-1} di\right)} di\right)^{\frac{\sigma}{\sigma-1}} = \frac{S_C}{P}. \end{aligned}$$

Similarly, $Y^X = \frac{S_X}{P}$ and $Y^{ROW} = \frac{S^{ROW}}{P}$. The aggregate demand for production labor is given by (see (57))

$$\begin{aligned} L^P &= \int l^D(i)\, di = \int \frac{1-\gamma}{w}\frac{\sigma-1}{\sigma}\left(\frac{\vartheta(i)}{\left(\int_{i=0}^1 \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma-1} S di \\ &= \frac{1-\gamma}{w}\frac{\sigma-1}{\sigma} S. \end{aligned} \tag{58}$$

The aggregate demand for fixed costs labor is given by

$$L^F = \int \left[n(i) f^V(i) + 1\left[n(i) > 0\right] f^I\right] di.$$

The aggregate demand for domestic intermediary inputs of production is given as follows. From $n(i)$ we can go to $s^D(i)$ and back and forth. Hence, we can treat $s^D(i)$ as given. Total spending on domestic intermediaries is given by

$$\begin{aligned} P_X X &= \int_{i=0}^1 \underbrace{s^D(i)\, s_X(i)}_{\text{Domestic share in material spending}} di \\ &= \int_{i=0}^1 s^D(i)\, \gamma s(i)\, di \\ &= \int_{i=0}^1 s^D(i)\, \frac{\gamma}{1-\gamma} wl(i)\, di \\ &= \frac{\gamma}{1-\gamma} w \int_{i=0}^1 s^D(i) \left[\frac{1-\gamma}{w}\frac{\sigma-1}{\sigma}\left(\frac{\vartheta(i)}{\left(\int_{i=0}^1 \vartheta(i)^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma-1} S\right] di \\ &= \gamma\frac{\sigma-1}{\sigma} S \int_{i=0}^1 \frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int_{i=0}^1 \vartheta(i)^{\sigma-1} di\right)} di. \end{aligned}$$

Hence,

$$X^D = \gamma \frac{\sigma - 1}{\sigma} \frac{S}{P_X} \int_{i=0}^{1} \left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int \vartheta(i)^{\sigma-1} di\right)} di \right), \tag{59}$$

where $\int_{i=0}^{1} \left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int \vartheta(i)^{\sigma-1} di\right)} di \right)$ is a constant (given $[n(i)]_i$). The value of imports is hence given by

$$\begin{aligned} \text{Imports} &= \int_{i=0}^{1} \left[1 - s^D(i)\right] \gamma s(i)\, di \\ &= \gamma \int_{i=0}^{1} s(i)\, di - \int_{i=0}^{1} s^D(i)\, \gamma s(i)\, di \\ &= \gamma \frac{\sigma - 1}{\sigma} S - P_X X \end{aligned} \tag{60}$$

1.2 Aggregate Equilibrium

We can now define the "aggregate" equilibrium for the two sectors.

Definition 3. An aggregate equilibrium in this economy is a set of prices $[w, P_X, P]$ and a set of allocations $\left[L_X, L_P, Y^C, Y^X, Y^{ROW}\right]$ such that

- The aggregate price index is consistent with the allocations in the differentiated sector

$$P = \frac{\sigma}{\sigma - 1} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \left(\frac{P_X}{\gamma q_D}\right)^{\gamma} \left[\left(\int_{i=0}^{1} \vartheta(i)^{\sigma-1} di \right)^{\frac{1}{\sigma-1}} \right]^{-1}$$

- the intermediary producer maximizes profits taking prices $[w, P, P_X]$ as given, i.e.
  - Profit maximization, i.e. P_X has to be equal to the marginal costs of production

$$P_X = \frac{1}{M} \left(\frac{w}{\phi}\right)^{\phi} \left(\frac{P}{1-\phi}\right)^{1-\phi}$$

  - Cost minimization, i.e. marginal products are equalized

$$P_X M L_X^{\phi} Y_X^{1-\phi} = \frac{1}{\phi} w L_X = \frac{1}{1-\phi} P Y_X$$

- markets clear, i.e.
  - labor market clear

$$L - L_F \equiv L^{NET} = \frac{1-\gamma}{w} \frac{\sigma - 1}{\sigma} (S_C + S_X + S_{ROW}) + L_X$$

– the market for intermediary products clears

$$ML_X^{\phi}Y_X^{1-\phi} = X^D = \gamma\frac{\sigma-1}{\sigma}\frac{(S_C+S_X+S_{ROW})}{P_X}\int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)$$

- trade is balanced (see 60), i.e.

$$S_{ROW} = \text{Imports} = \gamma\frac{\sigma-1}{\sigma}S - P_X X = \gamma\frac{\sigma-1}{\sigma}S - P_X ML_X^{\phi}Y_X^{1-\phi}.$$

Hence, the final equilibrium conditions for $\left[\frac{P_X}{w},\frac{P}{w},L_X,L_P,Y^C,Y^X,Y^{ROW}\right]$ are given by

$$\frac{P}{w} = \frac{\sigma}{\sigma-1}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{\frac{P_X}{w}}{\gamma q_D}\right)^{\gamma}\left[\left(\int_{i=0}^{1}\vartheta(i)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}\right]^{-1} \tag{61}$$

$$\frac{P_X}{w} = \frac{1}{M}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{\frac{P}{w}}{1-\phi}\right)^{1-\phi} \tag{62}$$

$$L^{NET} = (1-\gamma)\frac{\sigma-1}{\sigma}\frac{P}{w}\left(Y^C+Y^X+Y^{ROW}\right)+L_X \tag{63}$$

$$ML_X^{\phi}Y_X^{1-\phi} = \gamma\frac{\sigma-1}{\sigma}\frac{P/w}{P_X/w}\left(Y^C+Y^X+Y^{ROW}\right)\int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right) \tag{64}$$

$$L_X = \phi\frac{P_X}{w}ML_X^{\phi}Y_X^{1-\phi} \tag{65}$$

$$\frac{P}{w}Y_X = (1-\phi)\frac{P_X}{w}ML_X^{\phi}Y_X^{1-\phi} \tag{66}$$

$$\frac{P}{w}Y^{ROW} = \gamma\frac{\sigma-1}{\sigma}\frac{P}{w}\left(Y^C+Y^X+Y^{ROW}\right)-\frac{P_X}{w}ML_X^{\phi}Y_X^{1-\phi}. \tag{67}$$

Here, (61) and (62) are the two price indices for the final good the domestic intermediate, (63) is the labor market clearing condition, (64) is the market clearing condition for domestic inputs, (65) and (66) are the optimality conditions for labor and input demand from the intermediary sector and (67) is the aggregate trade balance equation.

Let us take labor as the numeraire, i.e. $w = 1$. Consider first (61) and (62) to solve for P and P_X. This yields

$$P = \frac{\sigma}{\sigma-1}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{\frac{1}{M}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{P}{1-\phi}\right)^{1-\phi}}{\gamma q_D}\right)^{\gamma}\frac{1}{\left(\int_{i=0}^{1}\vartheta(i)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}$$

$$P^{1-(1-\phi)\gamma} = \frac{\sigma}{\sigma-1}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{\frac{1}{M}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{1}{1-\phi}\right)^{1-\phi}}{\gamma q_D}\right)^{\gamma}\frac{1}{\left(\int_{i=0}^{1}\vartheta(i)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}$$

or

$$
\begin{aligned}
P &= \left[\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{\frac{1}{M}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{1}{1-\phi}\right)^{1-\phi}}{\gamma q_D}\right)^{\gamma}\frac{1}{\left(\int_{i=0}^{1}\vartheta\left(i\right)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right]^{\frac{1}{1-(1-\phi)\gamma}}. \\
&= \Gamma_P\left(\frac{1}{\left(\int_{i=0}^{1}\vartheta\left(i\right)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right)^{\frac{1}{1-(1-\phi)\gamma}} \qquad (68)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
P_X &= \frac{1}{M}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{1}{1-\phi}\right)^{1-\phi}\left[\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{\frac{1}{M}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{1}{1-\phi}\right)^{1-\phi}}{\gamma q_D}\right)^{\gamma}\frac{1}{\left(\int_{i=0}^{1}\vartheta\left(i\right)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right]^{\frac{1-\phi}{1-(1-\phi)\gamma}} \\
&= \left[\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma q_D}\right)^{\gamma}\right]^{\frac{1-\phi}{1-(1-\phi)\gamma}}\left[\frac{1}{M}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{1}{1-\phi}\right)^{1-\phi}\right]^{\frac{1}{1-(1-\phi)\gamma}}\left[\frac{1}{\left(\int_{i=0}^{1}\vartheta\left(i\right)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right]^{\frac{1-\phi}{1-(1-\phi)\gamma}} \\
&= \Gamma_X\left[\frac{1}{\left(\int_{i=0}^{1}\vartheta\left(i\right)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right]^{\frac{1-\phi}{1-(1-\phi)\gamma}}. \qquad (69)
\end{aligned}
$$

From (65) and (64) we get

$$
ML_X^{\phi}Y_X^{1-\phi}=\frac{1}{\phi}\frac{1}{P_X}L_X=\gamma\frac{\sigma-1}{\sigma}\frac{P}{P_X}\left(Y_C+Y_X+Y_{ROW}\right)\int_{i=0}^{1}\left(\frac{s^D\left(i\right)\vartheta\left(i\right)^{\sigma-1}}{\left(\int\vartheta\left(i\right)^{\sigma-1}di\right)}di\right),
$$

so that

$$
P\left(Y_C+Y_X+Y_{ROW}\right)=\frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}}\frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}L_X. \qquad (70)
$$

Hence, (63) implies that

$$
\begin{aligned}
L^{NET} &= \left(1-\gamma\right)\frac{\sigma-1}{\sigma}\frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}}\frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}L_X+L_X \\
&= \left[\frac{\left(1-\gamma\right)}{\gamma\phi}\frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}+1\right]L_X,
\end{aligned}
$$

so that

$$L_X = \frac{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{(1-\gamma)}{\gamma\phi}}L^{NET}. \tag{71}$$

Then (58) and (70) gives

$$\begin{aligned} L_P &= (1-\gamma)\frac{\sigma-1}{\sigma}(S_C+S_X) = (1-\gamma)\frac{\sigma-1}{\sigma}P(Y_C+Y_X) \\ &= \left(\frac{1-\gamma}{\phi\gamma}\right)\frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}L_X \\ &= \frac{\frac{(1-\gamma)}{\gamma\phi}}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{(1-\gamma)}{\gamma\phi}}L^{NET}. \end{aligned} \tag{72}$$

Hence, total spending is given by

$$\begin{aligned} S &= P(Y_C+Y_X+Y_{ROW}) = \frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}}\frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}L_X \\ &= \frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}}\frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{(1-\gamma)}{\gamma\phi}}L^{NET}. \end{aligned} \tag{73}$$

Now we can solve for the individual components of demand, S_X, S_{ROW} and S_C. Note first that (65) and (66) imply that

$$\frac{L_X}{PY_X} = \frac{\phi}{1-\phi},$$

so that (71) implies that

$$PY_X = \frac{1-\phi}{\phi}L_X = \frac{1-\phi}{\phi}\frac{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{(1-\gamma)}{\gamma\phi}}L^{NET}. \tag{74}$$

Using (67) and (64) we get that

$$
\begin{aligned}
PY^{ROW} &= \gamma\frac{\sigma-1}{\sigma}P\left(Y^C+Y^X+Y^{ROW}\right)-P_X M L_X^{\phi}Y_X^{1-\phi} \\
&= \gamma\frac{\sigma-1}{\sigma}P\left(Y^C+Y^X+Y^{ROW}\right)-\gamma\frac{\sigma-1}{\sigma}P\left(Y^C+Y^X+Y^{ROW}\right)\int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}\,di\right)}di\right) \\
&= \gamma\frac{\sigma-1}{\sigma}P\left(Y^C+Y^X+Y^{ROW}\right)\left[1-\int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}\,di\right)}di\right)\right] \\
&= \gamma\frac{\sigma-1}{\sigma}\frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}}\frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}L_X\left[1-\int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}\,di\right)}di\right)\right] \\
&= \frac{1}{\phi}\frac{\left[1-\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)\right]}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{(1-\gamma)}{\gamma\phi}}L^{NET}. \qquad (75)
\end{aligned}
$$

(73), (74) and (75) also imply that consumer spending is equal to

$$
\begin{aligned}
PY_C &= P\left(Y_C+Y_X+Y_{ROW}\right)-PY_X-PY_{ROW} \\
&= \left(\frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}}-\frac{1-\phi}{\phi}\int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}\,di\right)}di\right)-\frac{1}{\phi}\left[1-\int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}\,di\right)}di\right)\right]\right)\frac{L^{NET}}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)} \\
&= \left(\frac{1-\gamma\frac{\sigma-1}{\sigma}}{\phi\gamma\frac{\sigma-1}{\sigma}}+\int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}\,di\right)}di\right)\right)\frac{L^{NET}}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{(1-\gamma)}{\gamma\phi}} \\
&= \frac{\sigma}{\sigma-1}\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}+\phi\gamma\frac{\sigma-1}{\sigma}\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}{\phi\gamma\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+(1-\gamma)}\right)L^{NET} \\
&= \left(\frac{\frac{\sigma}{\sigma-1}-\gamma+\phi\gamma\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)}{\phi\gamma\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+(1-\gamma)}\right)L^{NET} \\
&= \left(1+\frac{\frac{1}{\sigma-1}}{\phi\gamma\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+(1-\gamma)}\right)L^{NET}.
\end{aligned}
$$

This fully characterizes the equilibrium $(P, P_X, Y_C, Y_X, Y_{ROW}, L_P, L_X)$.

In particular, the equilibrium real wage is given in (68) as

$$
\frac{w}{P}=\frac{1}{P}=\Gamma_P\left(\left(\int_{i=0}^{1}\vartheta(i)^{\sigma-1}\,di\right)^{\frac{1}{\sigma-1}}\right)^{\frac{1}{1-(1-\phi)\gamma}}. \qquad (77)
$$

Similarly, consumer welfare, i.e. real consumption is given by

$$Y^C = \frac{S^C}{P} = \left(\frac{w}{P}\right)\left(1 + \frac{\frac{1}{\sigma-1}}{\phi\gamma\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+(1-\gamma)}\right)L^{NET}. \tag{78}$$

(77) and (78) are the required expressions for Proposition 2.

1.3 Firm-level allocations

Given the aggregate equilibrium, we can use the relations from firm-level to back out the allocations at the micro-level. Firm level sales are given (see (73))

$$\begin{aligned}
p(i)\,y^D(i) &= \left(\frac{\vartheta(i)}{\left(\int_{i=0}^{1}\vartheta(i)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma}\frac{p(i)}{P}S \\
&= \left(\frac{\vartheta(i)}{\left(\int_{i=0}^{1}\vartheta(i)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma-1} S \\
&= \frac{1}{1-\gamma}\frac{\sigma}{\sigma-1}\left(\frac{\vartheta(i)}{\left(\int_{i=0}^{1}\vartheta(i)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma-1}\frac{\frac{1-\gamma}{\phi\gamma}}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{(1-\gamma)}{\gamma\phi}}L^{NET}.
\end{aligned} \tag{79}$$

Firm level employment is given by (see (57))

$$\begin{aligned}
l(i) &= \frac{1-\gamma}{w}\frac{\sigma-1}{\sigma}\left(\frac{\vartheta(i)}{\left(\int_{i=0}^{1}\vartheta(i)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}}\right)^{\sigma-1}(PY_C+PY_X) \\
&= \frac{1-\gamma}{w}\frac{\sigma-1}{\sigma}p(i)\,y(i)\,.
\end{aligned}$$

Finally it can checked that consumers indeed satisfy their budget constraint, i.e.

$$\begin{aligned}
PY_C = S_C = \text{Income} &= \overline{L} + \int_i \pi(i) \\
&= \overline{L} + \int_i \left[p(i)\,y(i) - s(i) - l^F(i)\right]di \\
&= L^{NET} + \frac{1}{\sigma}\int_i p(i)\,y(i)\,di.
\end{aligned}$$

This concludes the characterization of the equilibrium given sourcing strategies (or domestic shares) $[n(i)]$ (or $\left[s^D(i)\right]$).

1.4 The Closed Economy in Autarky

A special case is of course the economy in autarky. If the economy is closed, we have

$$s_i^D = 1 \text{ and } \vartheta(i) = \varphi(i).$$

Clearly we also have

$$f\left(s^D(i)\right) = f(1) = 0,$$

so that $L^{NET} = \overline{L}$. Hence, (72) and (71) imply that

$$L_X = \frac{1}{1+\frac{(1-\gamma)}{\gamma\phi}}\overline{L} = \frac{\gamma\phi}{1-(1-\phi)\gamma}\overline{L} \quad (80)$$

$$L_P = \frac{1-\gamma}{1-(1-\phi)\gamma}\overline{L}. \quad (81)$$

Nominal consumer spending is given from (76) as

$$PY^C = \left(1+\frac{\frac{1}{\sigma-1}}{1-(1-\phi)\gamma}\right)\overline{L}.$$

The aggregate price index is

$$P = \Gamma_P \times \left[\frac{1}{\left(\int_{i=0}^{1}\varphi(i)^{\sigma-1}\,di\right)^{\frac{1}{\sigma-1}}}\right]^{\frac{1}{1-(1-\phi)\gamma}}.$$

1.5 The Economy "without fixed costs", i.e. equalized domestic shares

If all the firms have equal domestic shares

$$s_i^D = s^D$$

and there are no fixed costs, we get that consumer spending (76) is given by

$$PY^C = \left(1+\frac{\frac{1}{\sigma-1}}{1-\gamma(1-\phi s^D)}\right)\overline{L},$$

which is decreasing in s_D. The price index is

$$P = \Gamma_P \times \left[\frac{1}{\left(\int_{i=0}^{1}\varphi(i)^{\sigma-1}\,di\right)^{\frac{1}{\sigma-1}}}\right]^{\frac{1}{1-(1-\phi)\gamma}} s_D^{-\frac{\gamma}{1-e}\frac{1}{1-(1-\phi)\gamma}},$$

i.e. prices are increasing in s_D. Real consumption is given by

$$C = \frac{PY^C}{P} = \frac{\overline{L}}{\Gamma_P}\left[\left(\int_{i=0}^{1}\varphi\left(i\right)^{\sigma-1}di\right)^{\frac{1}{\sigma-1}}\right]^{\frac{1}{1-(1-\phi)\gamma}} \times\left(1+\frac{\frac{1}{\sigma-1}}{1-\gamma\left(1-\phi s^D\right)}\right)\times s_D^{\frac{\gamma}{1-e}\frac{1}{1-(1-\phi)\gamma}},$$

which is of course decreasing in s^D, as spending is decreasing and prices are increasing.

2 Proof of Proposition 4

Consider the expression z in (23) given as

$$z = \frac{1}{\alpha}E\left[q\right]^{(1-\nu)}\left(\frac{\theta-1}{\theta}\right)^{(1-\nu)}\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\right)^{\frac{1}{\rho-1}}.$$

As $z>0$ we will focus on the transformation

$$ln\left(z\right) = \text{const} + \frac{1}{\rho-1}\left(\left(1-\nu\right)\left(\rho-1\right)\left[ln\left(\theta-1\right)-ln\left(\theta\right)\right]+\left[ln\left(\theta\right)-ln\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)\right]\right).$$

Hence,

$$\begin{aligned}
\frac{\partial ln\left(z\right)}{\partial\theta} &= \frac{1}{\rho-1}\left[\left(1-\nu\right)\left(\rho-1\right)\left(\frac{1}{\theta-1}-\frac{1}{\theta}\right)+\left(\frac{1}{\theta}-\frac{1}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\right)\right]\\
&= \frac{\left(1-\nu\right)}{\left(\theta-1\right)\theta}-\frac{\left(1-\nu\right)}{\theta\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)}\\
&= \frac{\left(1-\nu\right)}{\theta}\left[\frac{1}{\left(\theta-1\right)}-\frac{1}{\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)}\right]\\
&= -\frac{\left(1-\nu\right)}{\theta}\frac{\left(1-\nu\right)\left(\rho-1\right)-1}{\left(\theta-1\right)\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)},
\end{aligned}$$

which shows that $\frac{\partial ln(z)}{\partial\theta}<0$ as long as $\left(1-\nu\right)\left(\rho-1\right)>1$.

Similarly,

$$\begin{aligned}
\frac{\partial ln\left(z\right)}{\partial\rho} &= -\frac{\left[ln\left(\theta\right)-ln\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)\right]}{\left(\rho-1\right)^2}+\frac{1}{\rho-1}\frac{\left(1-\nu\right)}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\\
&= \left(\frac{1}{\rho-1}\right)^2\left(\frac{\left(1-\nu\right)\left(\rho-1\right)}{\theta-\left(1-\nu\right)\left(\rho-1\right)}-ln\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\right)\right)\\
&= \left(\frac{1}{\rho-1}\right)^2\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}-1-ln\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\right)\right)\\
&= \left(\frac{1}{\rho-1}\right)^2\left(x-1-ln\left(x\right)\right)>0,
\end{aligned}$$

where $x=\frac{\theta}{\theta-(1-\nu)(\rho-1)}>1$.

Also

$$\begin{aligned}\frac{\partial^2 z\left(E\left[q\right],\theta,\rho\right)}{\partial\theta\partial\rho} &= \frac{\partial}{\partial\rho}\left[\frac{\partial z\left(E\left[q\right],\theta,\rho\right)}{\partial\theta}\right]=\frac{\partial}{\partial\rho}\left[\frac{\partial ln\left(z\left(E\left[q\right],\theta,\rho\right)\right)}{\partial\theta}z\left(E\left[q\right],\theta,\rho\right)\right]\\ &= \frac{\partial}{\partial\rho}\left[\frac{\left(1-\nu\right)}{\theta}\frac{1-\left(1-\nu\right)\left(\rho-1\right)}{\left(\theta-1\right)\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)}z\left(E\left[q\right],\theta,\rho\right)\right]\\ &= \text{const}\times\frac{\partial}{\partial\rho}\left[\underbrace{\frac{1-\left(1-\nu\right)\left(\rho-1\right)}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\right)^{\frac{1}{\rho-1}}}_{g(\rho)}\right].\end{aligned}$$

Hence, let us focus on

$$ln\left(g\left(\rho\right)\right)=ln\left(1-\left(1-\nu\right)\left(\rho-1\right)\right)-ln\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)+\frac{1}{\rho-1}\left(ln\left(\theta\right)-ln\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)\right),$$

so that

$$\begin{aligned}\frac{\partial ln\left(g\left(\rho\right)\right)}{\partial\rho} &= \frac{-\left(1-\nu\right)}{1-\left(1-\nu\right)\left(\rho-1\right)}+\frac{\left(1-\nu\right)}{\theta-\left(1-\nu\right)\left(\rho-1\right)}-\left(\frac{1}{\rho-1}\right)^2\left(ln\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\right)\right)+\frac{1}{\rho-1}\frac{1 \ldots}{\theta-(1- \ldots}\\ &= \left(1-\nu\right)\left(\frac{1}{\theta-\left(1-\nu\right)\left(\rho-1\right)}-\frac{1}{1-\left(1-\nu\right)\left(\rho-1\right)}\right)+\left(\frac{1}{\rho-1}\right)^2\left(\frac{\left(1-\nu\right)\left(\rho-1\right)}{\theta-\left(1-\nu\right)\left(\rho-1\right)}-\left(ln\left(\frac{\ldots}{\theta-(\ldots}\right.\right.\right.\\ &= \left(1-\nu\right)\left(\frac{1-\theta}{\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)\left(1-\left(1-\nu\right)\left(\rho-1\right)\right)}\right)+\left(\frac{1}{\rho-1}\right)^2\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}-1-\left(ln\left(\ldots\right.\right.\right.\\ &= \left(1-\nu\right)\left(\frac{\theta-1}{\left(\theta-\left(1-\nu\right)\left(\rho-1\right)\right)\left(\left(1-\nu\right)\left(\rho-1\right)-1\right)}\right)+\left(\frac{1}{\rho-1}\right)^2\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}-1-\left(ln\left(\ldots\right.\right.\right.\\ &> 0\end{aligned}$$

Hence

$$\frac{\partial g\left(\rho\right)}{\partial\rho}=\frac{\partial ln\left(g\left(\rho\right)\right)}{\partial\rho}g\left(\rho\right)<0$$

as $g\left(\rho\right)<0$ if $\left(1-\nu\right)\left(\rho-1\right)>1$.

Finally, it is obvious from (24) that $\frac{\partial\eta}{\partial\theta}>0$ and $\frac{\partial\eta}{\partial\rho}$<0.

3 Calibrating the Model

Our calibration strategy is as follows: In Section 1 above, we derived all allocation and market-clearing prices as a function of parameters and the joint distribution of productivity and domestic shares $G\left(\varphi,s_D\right)$. To calibrate the model, we will now only have to make sure that (a) the distribution of domestic shares is consistent with firms' profit maximization problem, (b) that it is consistent with moments in the data and (c) that it satisfies all equilibrium conditions. We are going to process in two steps. First we will show that that firms' policy functions for their domestic expenditure shares depends on general equilibrium prices only through two aggregate statistics. Then we will show that we can calibrate the model in "normalized" parameters and then use the general equilibrium prices to back out the underlying structural parameters. This will ensure that all equilibrium conditions are satisfied even though we never have to solve for the equilibrium as a fixed point of firms' optimal behavior.

3.1 Solving for firms' domestic shares

Consider the firms' profit maximization problem. Optimal pricing implies that firms' variable profits are given by

$$\begin{aligned}\pi^V &= \frac{1}{\sigma}p(i)\,y(i) = \frac{1}{\sigma}\left(\frac{p(i)}{P}\right)^{1-\sigma} S = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}MC(i)\right)^{1-\sigma} P^{\sigma-1}S \\ &= \frac{1}{\sigma}\left(\frac{1}{\varphi(i)}\frac{1}{s^D(i)^{\frac{\gamma}{1-\varepsilon}}}\left(\frac{w}{1-\gamma}\right)^{1-\gamma}\left(\frac{P_X}{\gamma q_D}\right)^{\gamma}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} P^{\sigma-1}S \\ &\equiv \varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}} \times \Psi \times J,\end{aligned}$$

where

$$\Psi = \frac{1}{\sigma}\left(\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma q_D}\right)^{\gamma}\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \tag{82}$$

is a constant and

$$J = \left(\frac{P_X}{w}\right)^{\gamma(1-\sigma)}\left(\frac{P}{w}\right)^{\sigma-1} S \tag{83}$$

contains the influence of GE variables, which firms take as parametric. Hence, the firm solves

$$\pi\left(\varphi, f^V, f^I; J\right) \equiv \max_{s^D,n}\left\{\varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}} \times \Psi \times J - n f_i^V w - 1[n>0]\, f^I w\right\}, \tag{84}$$

where n denotes the *share* of countries the firm buys inputs from. From the price index $A = zn^{\eta}$, we get that n and s^D are related via

$$n = \left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}}\left(\frac{1}{z}\right)^{1/\eta}\frac{1}{(P_X/q_D)^{1/\eta}}.$$

Hence, we can express (84) in terms of s^D as the maximization problem

$$\max_{s^D}\left\{\varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}\Psi J - \left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}}\left(\frac{1}{z}\right)^{1/\eta}\frac{1}{(P_X/q_D)^{1/\eta}} f_i^V w - 1\left[s^D<1\right] f^I w\right\}.$$

An interior solution for s^D solves the optimality condition

$$\begin{aligned} 0 &= \frac{\gamma(\sigma-1)}{1-\varepsilon}\varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}-1}\Psi J - \frac{1}{\eta}\frac{1}{\varepsilon-1}\left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}-1}\frac{-s^D-\left(1-s^D\right)}{\left(s^D\right)^2}\left(\frac{1}{z}\right)^{1/\eta} f_i^V \frac{w}{(P_X/q_D)^{1/\eta}} \\ &= \frac{\gamma(\sigma-1)}{1-\varepsilon}\varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}-1}\Psi J + \frac{1}{\eta}\frac{1}{\varepsilon-1}(1-s_D)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}-1} s_D^{1-\frac{1}{\eta}\frac{1}{\varepsilon-1}-2}\left(\frac{1}{z}\right)^{1/\eta} f_i^V \frac{w}{(P_X/q_D)^{1/\eta}} \\ &= -\gamma(\sigma-1)\eta\varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}-1}\Psi J + (1-s_D)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}-1} s_D^{-1-\frac{1}{\eta}\frac{1}{\varepsilon-1}}\left(\frac{1}{z}\right)^{1/\eta} f_i^V \frac{w}{(P_X/q_D)^{1/\eta}}. \end{aligned}$$

Rearranging terms yields

$$\begin{aligned} (1-s_D)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}-1} s_D^{-1-\frac{1}{\eta}\frac{1}{\varepsilon-1}}\left(\frac{1}{z}\right)^{1/\eta} f_i^V \quad \frac{w}{(P_X/q_D)^{1/\eta}} &= \gamma(\sigma-1)\eta\varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}-1}\Psi J \\ \left(\frac{1}{z}\right)^{1/\eta} f_i^V \frac{1}{\Psi J}\frac{1}{\gamma(\sigma-1)\eta}\frac{w}{(P_X/q_D)^{1/\eta}} &= \varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}-1} s_D^{1+\frac{1}{\eta}\frac{1}{\varepsilon-1}}(1-s_D)^{1-\frac{1}{\eta}\frac{1}{\varepsilon-1}} \\ \left(\frac{1}{z}\right)^{1/\eta}\frac{f_i^V}{\varphi(i)^{\sigma-1}}\frac{1}{\Psi(J/w)}\frac{1}{\gamma(\sigma-1)\eta}\frac{1}{(P_X/q_D)^{1/\eta}} &= s^D(i)^{\frac{1}{\varepsilon-1}\left(\frac{1}{\eta}-\gamma(\sigma-1)\right)}(1-s_D)^{1-\frac{1}{\eta}\frac{1}{\varepsilon-1}}, \end{aligned} \tag{85}$$

which uniquely determines a solution

$$\widehat{s_D} = s_D\left(\underbrace{\frac{1}{\Psi\gamma(\sigma-1)\eta}}_{\text{Constant}}\ \underbrace{\frac{f_i^V}{\varphi(i)^{\sigma-1}}}_{\text{Heterogeneity}}\ \underbrace{\left(\frac{w}{J}\right)\left(\frac{q_D}{P_X}\frac{1}{z}\right)^{1/\eta}}_{GE}\right), \tag{86}$$

with the function s^D being *increasing.*

Total profits of an importer who behaves optimally are given by

$$\pi^{Imp} = \varphi(i)^{\sigma-1}\left(\widehat{s(i)^D}\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}\Psi J - f_i^V\left(\frac{1-\widehat{s(i)^D}}{\widehat{s(i)^D}}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}}\left(\frac{1}{z}\right)^{1/\eta}\frac{w}{(P_X/q_D)^{1/\eta}} - wf^I$$

Hence, (86) is the correct solution as long as

$$
\begin{aligned}
0 &< \pi^{Imp} - \pi^{Dom} \\
&= \varphi(i)^{\sigma-1}\left(\widehat{s(i)^D}\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}} \Psi J - f_i^V \left(\frac{1-\widehat{s(i)^D}}{\widehat{s(i)^D}}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}} \left(\frac{1}{z}\right)^{1/\eta} \frac{w}{(P_X/q_D)^{1/\eta}} - wf^I - \varphi(i)^{\sigma-1}\Psi J \\
&= \left[\left(\widehat{s(i)^D}\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}} - 1\right] \varphi(i)^{\sigma-1}\Psi J - f_i^V \left(\frac{1-\widehat{s(i)^D}}{\widehat{s(i)^D}}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}} \left(\frac{1}{z}\right)^{1/\eta} \frac{w}{(P_X/q_D)^{1/\eta}} - wf^I \\
&= \left[\left(\widehat{s(i)^D}\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}} - 1\right] - \left(\frac{1}{z}\right)^{1/\eta} \frac{f_i^V}{\varphi(i)^{\sigma-1}\Psi(J/w)(P_X/q_D)^{1/\eta}} \left(\frac{1-\widehat{s(i)^D}}{\widehat{s(i)^D}}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}} - \frac{f^I}{\varphi(i)^{\sigma-1}\Psi J/w} \qquad (87) \\
&= \left[\left(\widehat{s(i)^D}\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}} - 1\right] - \left(\frac{q_D}{P_X}\frac{1}{z}\right)^{1/\eta} \left(\frac{w}{J}\right) \frac{f_i^V}{\varphi(i)^{\sigma-1}\Psi} \left(\frac{1-\widehat{s(i)^D}}{\widehat{s(i)^D}}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}} - \frac{w}{J}\frac{f^I}{\varphi(i)^{\sigma-1}\Psi}. \qquad (88)
\end{aligned}
$$

The optimal domestic share is therefore given by

$$
s_D^* = \begin{cases} \hat{s_D} \text{ if } (86) & \text{if } (87) \text{ is satisfied} \\ 1 & \text{otherwise} \end{cases}.
$$

Hence, to determine the optimal domestic share, we require two equilibrium variables $\frac{w}{J}$ and $\left(\frac{q_D}{P_X}\frac{1}{z}\right)^{1/\eta}$. Note also that z is a nominal variable, as it depends on the foreign price level (α). Hence, we can use labor as the numeraire and set $w = 1$.

3.2 Calibrating a normalized model

(86) and (87) show that firms' optimal import decisions depend on general equilibrium variables, which in turn depend on the import decisions of all other firms in the economy. Hence, in principle we have to determine firms' import choices in equilibrium as the solution to a fixed point problem. It turns out however, that we can calibrate the some normalized structural parameters directly to the *observed* distribution of domestic shares and then back out the underlying structural parameters using our closed form expressions. The procedure is as follows:

1. Define the two *endogenous* numbers

$$
\tilde{f}^I \equiv \frac{f^I}{\Psi J} \qquad (89)
$$

$$
\tilde{f}_i^V = \left(\frac{q_D}{zP_X}\right)^{1/\eta} f_i^V \frac{1}{\Psi J}\frac{1}{\gamma(\sigma-1)\eta}. \qquad (90)
$$

In equilibrium, i.e. for given "prices" J and P_X, these are just monotone transformations of the underlying structural parameters f^I and f^V. The two optimality conditions for s^D are given by (85), which now reads

$$\frac{\tilde{f}_i^V}{\varphi(i)^{\sigma-1}} = s^D(i)^{\frac{1}{\varepsilon-1}\left(\frac{1}{\eta}-\gamma(\sigma-1)\right)}(1-s_D)^{1-\frac{1}{\eta}\frac{1}{\varepsilon-1}},$$

which determines $\widehat{s^D} = \widehat{s^D}\left(\frac{\tilde{f}_i^V}{\varphi(i)^{\sigma-1}}\right)$ and (87), which now reads

$$s^D = \begin{cases} \widehat{s^D} & \text{if } \left[\left(\widehat{s^D}\right)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}-1\right]-\gamma(\sigma-1)\eta\left(\tilde{f}_i^V/\varphi^{\sigma-1}\right)\left(\frac{1-\widehat{s^D}}{\widehat{s^D}}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}}-\tilde{f}^I/\varphi^{\sigma-1}>0 \\ 1 & \text{otherwise} \end{cases}.$$

Hence, for given $\tilde{f}^I$ and given firm-type $\left(\varphi, \tilde{f}^V\right)$, s_D can directly be calculates.

2. Now let $\tilde{f}_i^V$ and φ be distributed according to a joint distribution

$$\begin{pmatrix} ln(\varphi) \\ ln\left(\tilde{f}^V\right) \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_P \\ \tilde{\mu_F} \end{pmatrix}, \begin{pmatrix} \sigma_P^2 & \tilde{\rho}\sigma_P\tilde{\sigma}_F \\ \tilde{\rho}\sigma_F\tilde{\sigma}_F & \tilde{\sigma}_F^2 \end{pmatrix}\right),$$

which is parametrized by 5 numbers $(\mu_P, \tilde{\mu}_F, \sigma_P, \tilde{\sigma}_F, \tilde{\rho})$. We can again set $\mu_P = -\frac{1}{2}\sigma_P^2$, so that mean productivity is normalized to unity. Hence, we can calibrate the five parameters $\left(\sigma_P, \tilde{\mu_F}, \tilde{\sigma}_F^2, \tilde{\rho}, \tilde{f}^I\right)$ to match the five moments

(a) the share of importers $\left(\tilde{f}^I\right)$
(b) the aggregate domestic share $(\tilde{\mu}_F)$
(c) the dispersion in domestic shares $(\tilde{\sigma_F})$
(d) the correlation between domestic shares and sales $(\tilde{\rho})$
(e) the dispersion of sales (σ_P).

3. Hence, the calibrated model give us data of the form

$$\left(\varphi_i, \tilde{f}_i^V, \tilde{f}^I, s_i^D\right),$$

where s_i^D is calibrated to match the micro data on domestic shares. We are now going to use our results from above, that we can identify the two "prices" J and P_X from this data. In particular, we can solve for P and P_X from

$$P = \left[\frac{\sigma}{\sigma-1} \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{\left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi}}{\gamma q_D M} \right)^{\gamma} \frac{1}{\left(\int_{i=0}^{1} \left[\varphi(i)\, s^D(i)^{\gamma/(1-\varepsilon)} \right]^{\sigma-1} di \right)^{\frac{1}{\sigma-1}}} \right]^{\frac{1}{1-(1-\phi)\gamma}} .$$

$$P_X = \left[\frac{\sigma}{\sigma-1} \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{1}{\gamma q_D}\right)^{\gamma} \right]^{\frac{1-\phi}{1-(1-\phi)\gamma}} \left[\frac{1}{M} \left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi} \right]^{\frac{1}{1-(1-\phi)\gamma}} \left[\frac{1}{\left(\int_{i=0}^{1} \left[\varphi(i)\, s^D(i)^{\gamma/(1-\varepsilon)} \right]^{\sigma-} \right.} \right.$$

The constants M and q_D are arbitrary so that we can simply normalize parameters such that[46]

$$\begin{aligned} P &= \left(\int_{i=0}^{1} \left[\varphi(i)\, s^D(i)^{\gamma/(1-\varepsilon)} \right]^{\sigma-1} di \right)^{-\frac{1}{\sigma-1} \frac{1}{1-(1-\phi)\gamma}} \\ P_X &= \left(\int_{i=0}^{1} \left[\varphi(i)\, s^D(i)^{\gamma/(1-\varepsilon)} \right]^{\sigma-1} di \right)^{-\frac{1}{\sigma-1} \frac{1-\phi}{1-(1-\phi)\gamma}} , \end{aligned}$$

[46]Note that this implies that we first chose q_D such that

$$\frac{\sigma}{\sigma-1} \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{\left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi}}{\gamma q_D M} \right)^{\gamma} = 1.$$

Hence, M has to satisfy

$$\frac{\sigma}{\sigma-1} \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{1}{\gamma q_D}\right)^{\gamma} = \left(\frac{M}{\left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi}} \right)^{\gamma},$$

so that

$$\begin{aligned} \Gamma_{P_X} &= \left[\frac{\sigma}{\sigma-1} \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{1}{\gamma q_D}\right)^{\gamma} \right]^{\frac{1-\phi}{1-(1-\phi)\gamma}} \left[\frac{1}{M} \left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi} \right]^{\frac{1}{1-(1-\phi)\gamma}} \\ &= \left[\frac{M}{\left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi}} \right]^{\frac{\gamma(1-\phi)}{1-(1-\phi)\gamma}} \left[\frac{1}{M} \left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi} \right]^{\frac{1}{1-(1-\phi)\gamma}} \\ &= \left[\frac{1}{M} \left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi} \right]. \end{aligned}$$

Hence, we can still choose M as

$$M = \left(\frac{1}{\phi}\right)^{\phi} \left(\frac{1}{1-\phi}\right)^{1-\phi}.$$

The implied value of q_D is

$$q_D^{\gamma} = \frac{\sigma}{\sigma-1} \left(\frac{1}{1-\gamma}\right)^{1-\gamma} \left(\frac{1}{\gamma}\right)^{\gamma}.$$

which we can calculate. Using (82) and our normalization (see footnote 46) we get

$$\Psi = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\gamma}\right)^{1-\gamma}\left(\frac{1}{\gamma q_D}\right)^{\gamma}\right)^{1-\sigma} = \frac{1}{\sigma}. \tag{91}$$

Hence, (89) and (90) and (83) and (P, P_X) imply that the true structural parameters f and f^V are given by

$$\begin{aligned} f^I &= \tilde{f}^I \Psi \times J = \underbrace{\tilde{f}^I \Psi P_X^{\gamma(1-\sigma)} P^{\sigma-1}}_{\alpha^I} \times S \equiv \alpha^I \times S \\ f_i^V &= \left(z\frac{P_X}{q_D}\right)^{1/\eta} \tilde{f}_i^V \gamma(\sigma-1)\eta\Psi \times J = \underbrace{\tilde{f}_i^V \gamma(\sigma-1)\eta\Psi P_X^{\gamma(1-\sigma)} P^{\sigma-1}}_{\alpha_i^V} \times \left(z\frac{P_X}{q_D}\right)^{1/\eta} \times S \equiv \alpha_i^V \left(z\frac{P_X}{q_D}\right)^{1/\eta} S, \end{aligned}$$

where α^I and α_i^V are now known (as P_X and P are).

4. Now we have to solve for total spending S. To do so we need to know how much labor is used in fixed costs. Using that

$$n = \left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}} \left(\frac{1}{z}\frac{q_D}{P_X}\right)^{1/\eta},$$

we get that

$$\begin{aligned} L^{NET} &= \overline{L} - L^F \\ &= \overline{L} - \int \left[n(i) f^V(i) + 1\left[n(i) > 0\right] f^I\right] di \\ &= \overline{L} - \int \left[\left(\frac{1-\widehat{s^D(i)}}{\widehat{s^D(i)}}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}} \left(\frac{1}{z}\frac{q_D}{P_X}\right)^{1/\eta} \alpha_i^V \left(z\frac{P_X}{q_D}\right)^{1/\eta} S + 1\left[\widehat{s^D(i)} < 1\right]\alpha^I S\right] di \\ &= \overline{L} - S\left\{\int \left[\left(\frac{1-\widehat{s^D(i)}}{\widehat{s^D(i)}}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}} \alpha_i^V + 1\left[\widehat{s^D(i)} < 1\right]\alpha^I\right] di\right\} \\ &\equiv \overline{L} - S \times A \end{aligned}$$

where A can now be calculated as α^V and α^I and $[s_D(i)]_i$ are known.

5. Using L^{NET} we can now solve for total spending S. In particular, we can use (70) and (71) to

get

$$
\begin{aligned}
S &= \frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}} \frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)} L_X. \\
&= \frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}} \frac{1}{\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{(1-\gamma)}{\gamma\phi}} L^{NET} \\
&= \frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}\int_{i=0}^{1}\left(\frac{s^D(i)\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}di\right)}di\right)+\frac{\sigma-1}{\sigma}(1-\gamma)} L^{NET}.
\end{aligned}
$$

Hence, let

$$
B = \int_{i=0}^{1}\left(\frac{s^D(i)\,\vartheta(i)^{\sigma-1}}{\left(\int\vartheta(i)^{\sigma-1}\,di\right)}di\right) = \int_{i=0}^{1}\left(\frac{\widehat{s^D(i)}\left(\widehat{s^D(i)}^{\frac{\gamma}{1-\varepsilon}}\varphi(i)\right)^{\sigma-1}}{\left(\int\left(\widehat{s^D(i)}^{\frac{\gamma}{1-\varepsilon}}\varphi(i)\right)^{\sigma-1}di\right)}di\right),
$$

which we can now calculate. Hence,

$$
\begin{aligned}
S &= \frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}B+\frac{\sigma-1}{\sigma}(1-\gamma)}\left(\overline{L}-AS\right) \\
&= \frac{1}{\phi\gamma\frac{\sigma-1}{\sigma}B+\frac{\sigma-1}{\sigma}(1-\gamma)+A}\overline{L}, \qquad (92)
\end{aligned}
$$

which we can now calculate up to scale. In particular,

$$
\begin{aligned}
L^{NET} &= \overline{L}-AS \\
&= \overline{L}\left(1-\frac{A_2}{\phi\gamma\frac{\sigma-1}{\sigma}B+\frac{\sigma-1}{\sigma}(1-\gamma)+A}\right) \\
&= \overline{L}\left(\frac{\phi\gamma\frac{\sigma-1}{\sigma}B+\frac{\sigma-1}{\sigma}(1-\gamma)}{\phi\gamma\frac{\sigma-1}{\sigma}B+\frac{\sigma-1}{\sigma}(1-\gamma)+A}\right).
\end{aligned}
$$

As everything is proportional to L^{NET}, we normalize $\overline{L}$ to unity. Hence,

$$
L^{NET} = \frac{\phi\gamma\frac{\sigma-1}{\sigma}B+\frac{\sigma-1}{\sigma}(1-\gamma)}{\phi\gamma\frac{\sigma-1}{\sigma}B+\frac{\sigma-1}{\sigma}(1-\gamma)+A}
$$

can now be calculated. With L^{NET} at hand, we can also back out all other allocations. In particular, we now have S (from (92)) and the number of workers employed in fixed cost production as

$$
L_F = 1-L^{NET} = \frac{A}{\phi\gamma\frac{\sigma-1}{\sigma}B+\frac{\sigma-1}{\sigma}(1-\gamma)+A}.
$$

The remaining labor allocations then follow from (71) and (72).

6. Given the allocations as can then also go back to the structural parameters at the firm-level. For the fixed costs of importing we get that

$$f^I = \alpha^I S,$$

where S is given in (92) and α^I is known from $\tilde{f}^I \Psi P_X^{\gamma(1-\sigma)} P^{\sigma-1}$. For the variety specific fixed costs, we note the following. We assumed that

$$ln\left(\tilde{f}_i^V\right) \sim N\left(\tilde{\mu_F}, \tilde{\sigma}_F^2\right).$$

Hence, the true fixed costs are

$$\begin{aligned} f_i^V &= \left(z\frac{P_X}{q_D}\right)^{1/\eta} \tilde{f}_i^V \gamma(\sigma-1)\eta\Psi J \\ &= \left(z\frac{P_X}{q_D}\right)^{1/\eta} \tilde{f}_i^V \gamma\left(\frac{\sigma-1}{\sigma}\right)\eta\left(\frac{P_X}{w}\right)^{\gamma(1-\sigma)}\left(\frac{P}{w}\right)^{\sigma-1} S, \end{aligned}$$

where the second equality follows from (83) and (91). Note that $\left(z\frac{P_X}{q_D}\right)^{1/\eta}$ is not known yet (as z is not). However, z is actually not required given the domestic shares and $\tilde{f}^V$. To see this, suppose we wanted to calculate the fixed costs at the firm-level. In particular, consider the total fixed costs relative to sales, which are

$$\begin{aligned} \frac{FC_i}{Sales_i} &= \frac{n_i f_i^V + f^I}{\sigma\left(\varphi(i)^{\sigma-1} s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}\Psi J\right)} \\ &= \frac{\left(\left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}}\left(\frac{q_D}{P_x}\frac{1}{z}\right)^{1/\eta}\right)\left(\eta\gamma(\sigma-1)\Psi J\left(\frac{P_X}{q_D}z\right)^{1/\eta}\tilde{f}_i^V\right)+\Psi J\tilde{f}^I}{\sigma\varphi(i)^{\sigma-1}s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}\Psi J} \\ &= \frac{\left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}}\eta\gamma(\sigma-1)\tilde{f}_i^V+\tilde{f}^I}{\sigma\varphi(i)^{\sigma-1}s^D(i)^{\frac{\gamma(\sigma-1)}{1-\varepsilon}}}. \end{aligned}$$

Hence, conditional on the calibrated values of $\tilde{f}_i^V$ and $\tilde{f}^I$, we can calculate the fixed cost usage at the firm-level without any knowledge of z. Hence

$$ln\left(f_i^V\right) = ln\left(\left(z\frac{P_X}{q_D}\right)^{1/\eta}\gamma\left(\frac{\sigma-1}{\sigma}\right)\eta\left(\frac{P_X}{w}\right)^{\gamma(1-\sigma)}\left(\frac{P}{w}\right)^{\sigma-1}S\right)+ln\left(f_i^V\right),$$

so that the mean of the structural fixed cost distribution is not meaningful. Note however that

$$sd\left(ln\left(f_i^V\right)\right) = \sigma_F = \tilde{\sigma}_F,$$

and

$$
\begin{aligned}
\rho &= corr\left(ln\left(f_i^V\right), ln\left(\varphi\right)\right) = \frac{cov\left(ln\left(f_i^V\right), ln\left(\varphi\right)\right)}{\sigma_F \sigma_P} \\
&= \frac{cov\left(ln\left(\tilde{f}_i^V\right), ln\left(\varphi\right)\right)}{\tilde{\sigma}_F \sigma_P} \\
&= \tilde{\rho}.
\end{aligned}
$$

Hence, given $(\sigma_P, \tilde{\mu}_F, \tilde{\rho}, \tilde{\sigma}_F)$ we get

$$
\begin{pmatrix} ln\left(\varphi\right) \\ ln\left(f^V\right) \end{pmatrix} \sim N\left(\left(\begin{array}{c} -\frac{1}{2}\sigma_P^2 \\ \tilde{\mu_F} + ln\left(\left(z\frac{P_X}{q_D}\right)^{1/\eta} \gamma\left(\frac{\sigma-1}{\sigma}\right)\eta\left(\frac{P_X}{w}\right)^{\gamma(1-\sigma)}\left(\frac{P}{w}\right)^{\sigma-1} S\right) \end{array}\right), \begin{pmatrix} \sigma_P^2 & \tilde{\rho}\sigma_P\tilde{\sigma}_f \\ \tilde{\rho}\sigma_P\tilde{\sigma}_f & \tilde{\sigma}_F^2 \end{pmatrix}\right).
$$

These structural parameters exactly ensure that

(a) the model's domestic shares equal the domestic shares in the data (i.e. the first two moments)

(b) the models' allocation are consistent with a general equilibrium with roundabout production.

Note in particular, that we do not need to know z for any of the questions we are interested in. The reason is the following: *Conditional* on s_D, we do not need to know z to compute any allocations. Hence, z is only important when want to calculate the fixed costs - either at the aggregate level or at the firm-level. But the total fixed cost bill at the firm-level do not depend on z, *conditional on* s_D *and the calibrated* $\tilde{f}^V$ *as*

$$
n_i f_i^V = \left(\left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}}\left(\frac{q_D}{P_x}\frac{1}{z}\right)^{1/\eta}\right)\left(\eta\gamma\left(\sigma-1\right)\Psi J\left(\frac{P_X}{q_D}z\right)^{1/\eta}\tilde{f}_i^V\right) = \left(\frac{1-s_D}{s_D}\right)^{\frac{1}{\eta}\frac{1}{\varepsilon-1}}\eta\gamma\left(\sigma-1\right)\Psi J\tilde{f}_i^V.
$$

Intuitively, z affects the mapping between the effective quality of the foreign bundle and the domestic quality. This relative quality however, is already fully determined through the domestic share. Hence, conditional s_D, changes in z will simply imply that the firm has gone to less countries and vice versa. However, conditional on the calibrated value of $\tilde{f}^V$, the change in the price per country f^V will exactly adjust to make up for the fact that know "less" countries have to be "visited" to generate a given relative price index A. Note that this also implies that the normalization of the size of the world to a unit mass is innocuous. If we had started with a world of size $\overline{N}$, the price index would have been given by

$$
\begin{aligned}
A_{\overline{N}}\left(\overline{q}, \varphi\right) &= \left(\int_{c\in\Sigma} \xi_c\left(\varphi\right)^{\rho-1} dc\right)^{\frac{1}{\rho-1}} = \left(\overline{N}\int_{\overline{q}}^{\infty}\left(q/p\left(q\right)\right)^{\rho-1} dG\left(q\right)\right)^{\frac{1}{\rho-1}} = \overline{N}^{\frac{1}{\rho-1}} A\left(\overline{q}, \varphi\right). \\
&= z\overline{N}^{\frac{1}{\rho-1}} n^{\eta}.
\end{aligned}
$$

As z is arbitrary, it is of course inessential to also normalize $\overline{N}^{\frac{1}{\rho-1}}$.

Taking Stock Note that the above procedure does *not* require us to solve a fixed point problem, even though each firms' extensive margin of trade depends on all other other firms' sourcing strategy though the round-about production and market size effects. In principle we could have started with the non-transformed model

1. Pick $(\sigma_P, \mu_F, \sigma_F, \rho_F)$
2. Guess P and P_X and S
3. Solve for the optimal strategy for each firm, find (P, P_X, S), check if you found a fixed point, iterate until you found one
4. At the fixed point: check if you match the data
5. If not: pick different $(\sigma_P, \mu_F, \sigma_F, \rho_F)$ and start at 1.

Using our procedure we can sidestep these issues and calibrate everything in one step. The key is that in equilibrium all firms take prices as parametric anyway and that firms' behavior only depends on terms of the form $\frac{f}{P}$. Hence, we can directly find the equilibrium-adjusted parameters $\tilde{f}$, make sure that we match the micro-data and then use the structure of the theory (which we can calculate in closed form) to decompose $\tilde{f}$ into the exogenous component f and the endogenous component P. The resulting micro-parameters are exactly the ones that - by construction - ensure that given those we match the microdata once we iterate firms' decision to the fixed point.

4 Derivation of Equation (??)

To express firms' optimal import behavior in terms of domestic expenditure shares, we can use that $s_D = \frac{1}{1+\left[\frac{p_D}{q_D} z n^\eta\right]^{\varepsilon-1}}$ (see (25)). Hence, $\left(\frac{p_D}{q_D} z n^\eta\right)^{\varepsilon-1} = \frac{1-s_D}{s_D}$. We can then express the firms' optimality condition (27) as

$$\begin{aligned}
\frac{1}{\gamma(\sigma-1)} \frac{w}{\Upsilon \frac{D}{w^{(1-\gamma)(\sigma-1)}}} \frac{f}{\varphi^{\sigma-1}} &= \left((q_D/p_D)^{\varepsilon-1} + z^{\varepsilon-1} n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} \eta z^{\varepsilon-1} n^{\eta(\varepsilon-1)-1} \\
&= \left(\frac{q_D}{p_D}\right)^{(\varepsilon-1)\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)} \left(1 + \left(\frac{p_D}{q_D} z n^\eta\right)^{(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} \eta \left(z n^\eta\right)^{(\varepsilon-1)} \frac{1}{n} \\
&= \left(\frac{q_D}{p_D}\right)^{\gamma(\sigma-1)} \left(\frac{1}{s_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} \eta \left(\frac{1-s_D}{s_D}\right) \left(\frac{p_D}{q_D} z\right)^{1/\eta} \left(\frac{s_D}{1-s_D}\right)^{\frac{1}{\eta(\varepsilon-1)}} \\
&= \eta \left(\frac{p_D}{q_D} z\right)^{1/\eta} \left(\frac{q_D}{p_D}\right)^{\gamma(\sigma-1)} (s_D)^{1-\frac{\gamma(\sigma-1)}{\varepsilon-1}} \left(\frac{s_D}{1-s_D}\right)^{\frac{1-\eta(\varepsilon-1)}{\eta(\varepsilon-1)}},
\end{aligned}$$

which is the required equation (35).

5 Identifying Structural Parameters

According to (24) we have that

$$\eta = \frac{1}{\rho - 1} - \frac{1-\nu}{\theta}. \tag{93}$$

To identify (ρ, θ, ν) we need two further conditions. To identify ν we focus on the cross-sectional relationship between the import prices and country quality. In particular, note that average import prices are related to firms' number of varieties by

$$ln\left(\overline{p}\left(n\right)\right) = \text{const } - \frac{\nu}{\theta} ln\left(n\right), \tag{94}$$

where $\overline{p}\left(n\right)$ is the expenditure-share-weighted average price a firm with n trading partners pays. Hence, $\frac{\nu}{\theta}$ can directly be estimated from a regression of average prices on the number of trading partners. Intuitively: as the marginal variety sourced is of lower quality, variations in the extensive margin of trade generate cross-sectional variation in quality. Together with information on prices, this variation can be exploited to identify the sensitivity of prices to quality. To derive (94) note that the average price paid by a firm for its imports is given by

$$p\left(\overline{q}\right) \quad = \quad \int_{\overline{q}}^{\infty} p\left(q\right) s\left(q\right) dG\left(q\right) = \alpha \left(\frac{\int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)+\nu} \frac{dG(q)}{1-G(\overline{q})}}{\int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)} \frac{dG(q)}{1-G(\overline{q})}} \right).$$

Hence, the numerator and the denominator is the expected value of a random variable q^b, where q is pareto on $\overline{q}$ with shape θ. Hence, q^b is pareto on $\overline{q}^b$ with shape $\frac{\theta}{b}$ so that

$$p\left(\overline{q}\right) \quad = \quad \alpha \frac{\theta - (1-\nu)(\rho-1)}{\theta - (1-\nu)(\rho-1) + \nu} \overline{q}^{\nu}.$$

As $n = q_{min}^{\theta} \overline{q}^{-\theta}$, we get that

$$ln\left(p\left(n\right)\right) = \text{const } - \frac{\nu}{\theta} ln\left(n\right),$$

which is (94).

Additionally, the within-firm expenditure share on the top κ percent of varieties sourced is given by

$$ln\left(s_i^{\kappa}\right) = ln\left(\kappa\right) \left(\frac{(1-\nu)(\rho-1) - \theta}{\theta} \right). \tag{95}$$

Strikingly, the expenditure share on the top κ *percent* of sourcing countries does not depend on how many trading partners a firm has.[47] Intuitively, the more concentrated qualities are, i.e. the lower θ, the larger the quality difference between the average and the marginal sourcing partner. Hence, $ln\left(s_i^{\kappa}\right)$ is decreasing in θ. To derive (95), recall that firms import according to a cutoff rule on quality. Hence, the quality of countries firms actually import from is pareto on $[\overline{q}, \infty]$ with shape θ.

[47] This stems from our assumption of country quality being pareto and firms' sourcing strategies being a simple cut-off rule.

ρ	θ	ν
1.91	1.35	0.045

Table 12: Structural parameters (NOT YET RECALCULATED) - SHOULD PUT IN THE ONLINE APPENDIX

Expenditure shares are increasing in q. Hence, the top $\kappa\%$ countries are the countries with $q > q^{\kappa}$, where q^{κ} satisfies $\kappa^{-1} = P\left[q > q^{\kappa}(\varphi)\right] = \left(\frac{\overline{q}}{q^{\kappa}}\right)^{\theta}$. Total expenditure on these top $\kappa\%$ of countries is given by

$$s^{\kappa} = \left(\frac{A\left(q^{\kappa}\right)}{A\left(\overline{q}\right)}\right)^{\rho-1} = \left(\frac{q^{\kappa}}{\overline{q}}\right)^{(1-\nu)(\rho-1)-\theta},$$

where the last equality follows directly from (39). Hence,

$$\left(lns^{\kappa}\right) = \left(\frac{(1-\nu)(\rho-1)-\theta}{\theta}\right) \times ln\left(\kappa\right),$$

as required in (95).

Given our estimates of η, we can therefore use the three equations (93), (94) and (95) to solve for the structural parameters (ρ, θ, ν). The results are contained in Table 12 below.

6 The optimality condition in terms of observables

The firms' optimality condition from (41) is

$$\Gamma\eta\gamma\left(\sigma-1\right)\left(\left(q_D/p_D\right)^{\varepsilon-1} + z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} z^{\varepsilon-1}n^{\eta(\varepsilon-1)-1}\varphi^{\sigma-1} = wf.$$

Now note that firm sales are given by $S = \sigma\pi = \sigma\Gamma\left(\left(q_D/p_D\right)^{\varepsilon-1} + z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\varphi^{\sigma-1}$. Hence, we can rewrite the optimality condition as

$$\frac{\eta\gamma\left(\sigma-1\right)}{\sigma}S\left(\varphi,n\right)\frac{z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}{\left(q_D/p_D\right)^{\varepsilon-1} + z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}\frac{1}{n} = wf.$$

Now note that $\frac{z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}{\left(q_D/p_D\right)^{\varepsilon-1}+z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}$ is simply the forms' import share $s^{I}\left(n\right) = 1 - s^{D}\left(n\right)$. Finally, let $\overline{q}$ be the quality of the marginal variety, the firm sources from. The expenditure share on this variety is given by (see (7) and (8)) by

$$s\left(\overline{q},n\right) = \left(\frac{1}{\alpha}\frac{\overline{q}^{(1-\nu)}}{A\left(n\right)}\right)^{\rho-1} = \left(\frac{1}{\alpha}\frac{\left(q_{\min}n^{-\frac{1}{\theta}}\right)^{(1-\nu)}}{zn^{\eta}}\right)^{\rho-1} = \left(\frac{1}{\alpha}\frac{q_{\min}^{1-\nu}}{z}n^{-\left(\frac{1-\nu}{\theta}\right)-\eta}\right)^{\rho-1}.$$

	Dep. Variable: Fixed costs of sourcing			
	Full Sample	Mutiple Varieties		
ln(va)	0.825***		0.763***	0.933***
	(0.003)		(0.004)	(0.006)
ln(l)		0.857***		
		(0.003)		
Exporter			0.253***	0.371***
			(0.013)	(0.031)
Foreign Group			0.558***	0.412***
			(0.014)	(0.019)
N	113,407	114,723	113,407	34,356
R^2	0.45	0.42	0.46	0.60

Table 13: Cross-sectional variation in FC (97)

Using the definition of z and η (see (23) and (24)) we get that

$$\frac{1}{\alpha}\frac{q_{\min}^{1-\nu}}{z} = \frac{1}{\alpha}\frac{\left(\frac{\theta-1}{\theta}E\left[q\right]\right)^{1-\nu}}{\frac{1}{\alpha}E\left[q\right]^{(1-\nu)}\left(\frac{\theta-1}{\theta}\right)^{(1-\nu)}\left(\frac{\theta}{\theta-(1-\nu)(\rho-1)}\right)^{\frac{1}{\rho-1}}} = \left(\frac{\theta-(1-\nu)\left(\rho-1\right)}{\theta}\right)^{\frac{1}{\rho-1}}$$

and

$$-\left(\frac{1-\nu}{\theta}\right) - \eta = -\left(\frac{1-\nu}{\theta}\right) - \frac{1}{\rho-1} + \frac{1-\nu}{\theta} = -\frac{1}{\rho-1}.$$

Hence,

$$s\left(\overline{q},n\right) = \left(\frac{\theta-(1-\nu)\left(\rho-1\right)}{\theta}\right)\frac{1}{n}.$$

Hence, firms' import behavior is simply governed by the optimality condition

$$\frac{\eta\gamma\left(\sigma-1\right)}{\sigma}S\left(\varphi,n\right)\left(1-s^{D}\left(n\right)\right)s\left(\overline{q},n\right) = wf. \tag{96}$$

To test for the correlation between fixed costs and productivity, we focus on (96) and run regressions of the form

$$ln\left(S_{ist}\left(1-s_{D,ist}\right)s_{iskt}^{m}\right) = \delta_s + \delta_t + \delta_k + x_{ist}'\phi + u_{istk}, \tag{97}$$

where s_{iskt}^{m} is the *smallest* expenditure share of firm i for imports of product k, δ_k is a product fixed effect and x_{ist} is a particular firm attribute. Hence, under (96), ϕ identifies the correlation between characteristic x_{ist} and the exogenous fixed costs of sourcing. The results of this exercise are contained in Table 13 below.