

Skill Bias, Trade, and Wage Dispersion

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Abstract

Wage ratios between different percentiles of the wage distribution have moved in parallel and then diverged in the U.S. in the last 50 years. In this paper, I study the theoretical response of wage ratios to skill-biased technical change and trade integration. I build a simple model of heterogeneous technology and heterogeneous workers that features complementarities between the quality of ideas and abilities. I show that changes to the skill bias of technology and to trade costs can both reproduce the observed pattern since (i) they have similar asymmetric effects on productive vs. unproductive firms, and (ii) positive assortative matching in the labor market transmits this asymmetry across high and low skill workers. Focusing on the different channels through which skill-biased technical change and trade integration operate suggests ways to disentangle the magnitude of each.

Keywords: *intra-industry trade; skill-biased technical change; local change in inequality; intra-firm rent distribution.*

1 Introduction

Wage inequality in the United States has grown since the sixties, both across observable characteristics (e.g. college/high school premium, see Murphy and Welch (1992)) and within them (residual inequality,

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see Juhn, Murphy and Pierce (1993)). This growth has not been uniform throughout the whole wage distribution: a widespread increase in inequality until the late eighties has then evolved into a polarization of the wage distribution, with inequality increasing in the right tail and flattening out, or even decreasing, in the lower tail¹ (see Card and DiNardo (2002), Piketty and Saez (2003) and (2004), and Autor, Katz and Kearney (2005), (2006) and (2008)).

Skill-biased technical change has been considered the most prominent candidate for an explanation (e.g. Bound and Johnson (1992), Autor, Katz and Krueger (1998), Autor, Levy and Murnane (2003)), while international trade integration has been found to contribute to a significant but minor part of this change (Borjas, Freeman, and Katz (1991), Feenstra and Hanson (1996), (1999)). However, most trade-based explanations have been looking for the consequences of exchanges between countries with different skill prices, whereas for a long time international trade has mainly occurred between countries with similar endowments (Baldwin and Martin (1999)). Motivated by this evidence on the behavior of inequality and trade patterns, I ask: can skill-biased technical change and trade integration between identical countries produce the same observed pattern for inequality? I show that because of positive assortative matching in the labor market, the answer is yes. I then argue that focusing on the specific channels of each mechanisms, one can gain further insights into how to disentangle them. My example will concentrate on the intra-firm rent distribution.

My starting point is the well-established finding that this growth in inequality is due to a large increase in the relative demand for skills that has occurred, especially since the late seventies, in the U.S. economy². Wage inequality, as measured for example by the standard deviation of log-wages, or as the ratio between the values at the 90th vs. the 10th percentile in the distribution ($p90/p10$ ratio), has increased sharply until 1987, and increased modestly afterwards³. This deceleration hides a strong increase in the right tail ($p90/p50$) and a constant or decreasing inequality in the lower tail ($p50/p10$) of the distribution⁴, and it emphasizes the necessity to study the evolution of wage inequality in different regions of the skill distribution.

Several studies have documented empirically and justified theoretically how the properties of substitu-

¹The ratio of wages at the 50th vs. 10th percentile, and 90th vs. 50th percentile in the distribution, grew each approximately 10% from 1973 to 1987. After that, the lower tail flattened, while the upper tail continued to grow 10% more through 2004.

²See for example Katz and Murphy (1992) for a discussion of labor demand and supply forces. Juhn, Murphy and Pierce (1993), among other things, also discuss the different timings in the evolution of total and residual wage inequality.

³See Card and DiNardo (2002).

⁴Autor, Katz and Kearney (2005), (2006) and (2008) provide evidence on the polarization of the wage distribution. Piketty and Saez (2003) and (2004) provide evidence on the behavior of wage ratios among top earners.

tion and complementarity of computers with different tasks and abilities can generate these patterns⁵. On the other hand, international trade has been tested as if imports increased the supply of unskilled labor⁶. Some studies have expected trade to reallocate labor force towards sectors with comparative advantage. These reallocations are typically found to be weak and dominated by reallocation of labor force within sectors⁷. Other studies have then considered the consequences of outsourcing of low skill-intensive tasks to unskilled-labor abundant countries, which would look like intra-industry trade, finding larger effects⁸. These approaches are, for that period, at odds with most of the world trade being between developed countries with similar endowments⁹.

In light of this evidence, my contribution focuses the attention squarely on intra-industry trade, and builds a simple model of trade and labor markets capable of incorporating skill-biased technical change. I consider two identical economies with varieties characterized by heterogeneous efficiencies in their technology, in the spirit of Melitz (2003) and Bernard, Jensen, Eaton and Kortum (2003). I extend this framework along the lines of Lucas (1978), by assuming that workers are heterogeneous in their ability to run any firm (if they choose so), while being identical as production workers at the firms' production lines. A firm is then made up by an idea, a manager, and production workers. Complementarities between technology and ability imply positive assortative matching between managers and technological efficiency¹⁰, producing a "superstars" effect as in Rosen (1981). The occupational choice implies that the wage of the manager if she was a production worker plays the role of the fixed cost to access the domestic market, giving rise to increasing returns to scale at the firm level (as in Krugman (1979)), even in a closed economy. A fixed cost of exporting produces the endogenous selection of most productive firms in the foreign market, and trade is assumed to be balanced.¹¹

⁵Bound and Johnson (1992) provides an empirical comprehensive approach. Autor, Katz and Krueger (1998) show evidence of the relation between wage inequality and the timing and diffusion of computers across industries; Autor, Levy and Murnane (2003) investigate the role of computers, skills and tasks in the production function.

⁶For example, Borjas, Freeman and Katz (1991) and Murphy and Welch (1992) convert net imports into labor supply equivalents, first assuming that the impact of imports and exports is the same across skill groups, and then assuming that only imports affect the net supply of unskilled workers.

⁷For example, Bound and Johnson (1992) look for reallocations of workers between industries due to shift in product demand, and actually find that these shifts are slightly *reducing* the demand of college graduates. This finding leads them to look for the consequences of skill-biased technical change on within-industry changes in demand. Weak reallocations across and strong skill-upgrading within detailed sectors in manufacturing industry are also reported in Berman, Bound and Griliches (1994).

⁸Feenstra and Hanson (1996), (1999) calculate a measure of intermediate input outsourcing at sector level. To this end, they use data from input-output matrices to infer the total impact of imports on any given sector.

⁹For example, Baldwin and Martin (1999) document that two-thirds of contemporary world trade occurs among rich countries with similar factor endowments, and three-fourths of this share is two-way trade within narrowly defined industries. See also Helpman (1999), for a discussion.

¹⁰Sattinger (1979) is the first to propose this framework. This paper generalizes his contribution, introducing a fully-fledged general equilibrium model where the outside options are endogenously determined. Sattinger (1993) gives a review and a motivation for using assignment models to study wage distributions.

¹¹The paper is thus consistent with size, skill, wage and productivity premia of exporters (e.g. Bernard and Jensen, (1995),

With these assumptions, I eliminate by design any effect of trade on inequality through deficits or exchanges with countries relatively more endowed with unskilled labor, thus avoiding the most common arguments against trade-based explanations for the evolution of inequality. However, the economy features a wage function that depends on the individual ability, microfounded in a simple model of the labor market: hence, I can study the equilibrium response of wage ratios to trade integration and skill-biased technical change at different points in the skill distribution. I model trade integration as a reduction in the iceberg cost of export, and skill-biased technical change as an increase in the contribution of ideas to firm-level productivity, whereby, because of complementarities, high skill managers gain more than proportionately relative to low skill managers.

To capture the response in any region of the skill distribution, I frame the discussion in terms of wage ratios between two marginally different managers. In the paper, I show that while trade integration and skill-biased technical change operate on local wage ratios through partially different channels, they produce a similar asymmetric effect across firms: low productivity firms face tougher competition, while firms at the high end of the productivity range increase their earnings. Because of positive assortative matching in the labor market, this asymmetry is transmitted across low and high skill managers. I prove that, under some assumptions, there exist unique thresholds abilities above which local inequality increases, and below which local inequality decreases. Whether the wage ratio between two abilities s' and s'' increases or decreases will depend, for both trade and skill-biased technical change, on the position of these abilities s' and s'' with respect to those thresholds. Only studying the evolution of wage dispersion in different regions of the wage distribution is not sufficient to disentangle the source of the pattern.

This observational equivalence may help explain why international trade has been attributed only a limited role in the evolution of inequality observed in the last 50 years. However, I do not aim to propose uni-causal explanations and dismiss the importance of skill-biased technical change or of other channels of trade integration¹²: on the one hand, my model only applies to the manufacturing sector, which has been explicitly studied in this literature¹³, but is certainly not the largest part of the economy; on the other hand, trade with developing countries has grown in importance in recent years¹⁴, and my model addresses - by choice - only intra-industry trade based on love-for-variety motivations. Rather, I intend to emphasize why intra-industry, balanced trade can by itself produce quite articulated behavior

(1997), (1999)) and positive size-wage relation across firms (e.g. Oi and Idson (1999)).

¹²Also, skill-biased technical change and trade integration are by no means the only two explanations put forth. For example, deunionization and declining real minimum wages have also been studied (see for example DiNardo, Fortin and Lemieux (1996) and Lemieux (2006)).

¹³See Berman, Bound and Griliches (1994).

¹⁴See for example Krugman (2008).

on economy-wide wage ratios by proposing a very simple extension of the recently developed theoretical literature on firm-level heterogeneity and trade.

If skill-biased technical change and trade integration can both rationalize the pattern for inequality in the U.S. economy that the literature has documented, what can tell the two causes apart? I propose to exploit the differences in the way skill-biased technical change and trade integration operate. My focus is on the intra-firm rent distribution, where I call "rent" the sum of profits and the manager's wage, less the opportunity cost of ideas and managers in the alternative occupation (zero and the production worker wage, respectively)¹⁵. In the paper, I show that the share of the rent received by the manager is only a function of the relative contribution of managers and ideas to the firm-level productivity. The intra-firm rent distribution is not modified by trade integration because trade costs influence the marginal contribution of managers and ideas in the same way (thus, only competitiveness across firms is affected). Hence, changes in inequality not accompanied by change in the intra-firm rent distribution must be attributed to trade. On the other hand, changes in the intra-firm rent distribution must imply changes in local inequality caused by skill-biased technical change. This result is very dependent on the functional form assumptions, and only provides partial conditions. However, it serves the purpose of illustrating a more general point: progress can be made by explicitly spelling out the different mechanisms through which these two forces operate, and focusing on their different implications at firm level.

My model makes use of assignment concepts used recently in various forms to discuss the impact of information and communication technology on the wage distribution in closed economy (Garicano (2000), Garicano and Rossi-Hansberg (2006)) or the evolution of managerial compensation (Gabaix and Landier (2008), Terviö (2008)). The paper capitalizes on firm-level heterogeneity models (Manasse and Turrini (2001), Melitz (2003), Bernard, Eaton, Jensen and Kortum (2003), Helpman, Melitz and Yeaple (2005), Yeaple (2005), Chaney (2008)) and extends them by enriching the details of the human capital aspects. This contribution is part of a recent literature applying assignment models to trade: see Blanchard and Willmann (2008) for educational and occupational choices, Kremer and Maskin (2006), Antràs, Garicano and Rossi-Hansberg (2006) and Nocke and Yeaple (2008) for applications to offshoring, and Costinot (2009) and Costinot and Vogel (2010) for general theoretical approaches. The relation between trade integration and skill premium has also been intensively studied in alternative frameworks: see Matsuyama (2007) (who assumes a skill-intensive technology to export), Epifani and Gancia (2008) (who

¹⁵For related literature on how wages and profits are distributed within firms, see for example Blanchflower, Oswald, and Sanfey (1996), who infer the existence of sharing rules by merging CPS and the NBER productivity database, and Abowd, Kramarz and Margolis (1999), who propose a statistical decomposition of employer-employee matched dataset.

use two sectors differing in skill intensity and two skills), Verhoogen (2008) (on the implications of skill-upgrading for developing countries), Amity and Davis (2008) and Egger and Kreickemeier (2009) (studying the implication of fair wage concerns on effort), Burstein and Vogel (2010) (who study the effects of trade on between vs. within sector wage inequality), Parro (2010) (who explores the implications of capital-skill complementarity through trade in capital goods), Helpman, Itzhoki and Redding (2010) and Helpman (2010) (for the interplay of trade and labor market frictions on inequality and unemployment)¹⁶. None of these papers studies or compares the response of wage ratios to trade and skill-biased technical change across the ability spectrum.

In the rest of the paper, I will describe the model in closed economy (section 2) and provide a motivation for the theoretical framework used in analyzing wage ratios, applying this to skill-biased technical change (section 3). In section 4, I extend the model to an open economy framework, while in section 5 I show how wage ratios respond to skill-biased technical change and trade integration. Section 6 argues why the intra-firm rent distribution can help in disentangling these two forces. Section 7 provides some concluding remarks.

2 The Closed Economy

In this section, I introduce the framework and explain how the assignment mechanism generates a non-trivial wage function. I then derive the equilibrium in a closed economy.

2.1 Consumers, Managers, and Ideas

The representative consumer maximizes a standard CES utility function where, from an infinite mass of varieties potentially available, a subset J of them is produced and aggregated as

$$Y = \left[\int_{j \in J} y(j)^{(\sigma-1)/\sigma} dj \right]^{\sigma/(\sigma-1)}$$

with $\sigma > 1$. Standard optimization implies that each consumer spends

$$x(j) = \left(\frac{p(j)}{P} \right)^{1-\sigma} X \tag{1}$$

¹⁶A parallel literature, less related, studies the implication of trade on wage inequality in developing countries, where skill premia are also increasing, contrary to the most standard Heckscher-Ohlin predictions. See the review of Goldberg and Pavcnick (2007), and references therein.

on each variety produced, where $P = \left[\int_{j \in J} p(j)^{1-\sigma} dj \right]^{1/(1-\sigma)}$ is the ideal price index of good Y and X is total consumers' expenditure on it. To fix ideas, I think of different j as different varieties, or production lines; however, I will interchangeably use the term "firm", implicitly assuming one product per firm.

Three inputs are necessary for a production line to exist: an idea, a manager, and production workers in proportion to output.

Varieties differ according to the state of the technology available for their production: denoting with $z \in (0, +\infty)$ the quality of an idea, I assume that there is a measure $G(z) = Tz^{-1}$ of ideas at least as good as z . This specification ensures that there are a sufficient number of ideas, however bad, to accommodate any number of managers in equilibrium. Ideas are owned by a mutual fund that maximizes profits and redistributes them equally across agents¹⁷.

The economy is populated by a mass L of agents, which, as in Lucas (1978), can choose to work either as production workers or as managers. Agents are heterogeneous in their managerial ability, while they all have a unit efficiency as production workers. The ability s is also distributed according to a power law: for $s \in [1, +\infty)$, there is a measure $L(s) = Ls^{-1}$ of potential managers with ability of at least s .¹⁸

While in Lucas (1978) potential managers differ by their ability to run larger firms that produce a homogeneous final product, here I assume that there are complementarities between managerial ability and idea efficiency: the total firm's productivity of a pair (z, s) is

$$\varphi(z, s) = z^\kappa s^\alpha$$

with $\kappa > 0$, $\alpha > 0$.¹⁹ The parameter α measures the influence of managers' ability: while $\alpha = 0$ reduces this model to a simple one-sided heterogeneity framework, increasing α lets a firm gain more from a better manager. Moreover, there is a simple mapping between abilities and percentiles in the skill and wage distributions: the ability s is always collocated at the $100(1 - s^{-1})^{th}$ percentile.

Agents who choose to be production workers earn a wage \bar{w} , which is then also the opportunity cost of being a manager. This wage is the numeraire and will be normalized to 1; I leave it here explicitly for clarity. If y units of a variety are to be produced in a firm with productivity φ , y/φ efficiency units of

¹⁷The assumption of equal redistribution is immaterial to the rest of the paper, since I am interested in wage (rather than income) distribution. Juhn, Murphy and Pierce (1993) discuss relative merits of these two alternatives.

¹⁸In Appendix A.1 I show that assuming an exponent of -1 for the quality of ideas and the ability of managers is without loss of generality.

¹⁹This assumption satisfies log-supermodularity as in Costinot (2009).

work from production workers are used. Let

$$x(p) - \frac{\bar{w}x(p)}{p\varphi(z, s)}$$

be the surplus of this firm (the excess of revenues over costs for production workers), if the price p is chosen. When using a manager s , a firm with idea quality z sets a price which solves

$$\max_p x(p) - \frac{\bar{w}x(p)}{p\varphi(z, s)}$$

implying an optimal price of

$$p(z, s) = \frac{\sigma}{\sigma - 1} \frac{\bar{w}}{z^\kappa s^\alpha} . \quad (2)$$

For a given quality z , the optimal price is a function of the ability of the manager s chosen to run the firm. The labor market balances incentives across firms in the choice of managers: this balance is the subject of the next section.

2.2 Assignment

Denoting with $v(z, s)$ the surplus for a firm (z, s) , we can use the revenue function (1) and the optimal price (2) to write $v(z, s)$ as

$$\begin{aligned} v(z, s) &= x(p(z, s)) - \frac{\bar{w}x(p(z, s))}{p(z, s)\varphi(z, s)} = M \left(\frac{z^\kappa s^\alpha}{\bar{w}} \right)^{\sigma-1} \\ M &\equiv \frac{1}{\sigma} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} X P^{\sigma-1} . \end{aligned} \quad (3)$$

The term M measures the size of the market. A larger expenditure level X , or weaker competition through a higher price index P , both make the market bigger and raise the surplus for any firm.

The surplus must cover payments to the manager of ability s , residually determining profits for the idea z . I following Sattinger (1979) and assume that a firm is unable to affect the prevailing labor market conditions: the wage function $w(s)$ is taken as given. The problem of the idea's owner²⁰ is then

$$\pi(z) = \max_{s \in [1, \infty)} \{v(z, s) - w(s)\} .$$

²⁰This is not the only way to characterize the earning functions. Since the problem is symmetric in managers and ideas, we could start with the managers choosing ideas. Alternatively, we could have each side choose the other (as in Sattinger (1979)). In each case, the resulting earning functions would be identical.

The complementarity between managers and ideas creates an incentive for better firms to hire better managers: a marginal increase in s always raises the total surplus, but this increase is larger when the quality of the idea z is higher (i.e., the cross derivative of the surplus $v_{1,2}(z, s)$ is positive). The possibility to choose the manager generates an incentive towards positive assortative matching between managers and ideas.

The optimal ability s is chosen to balance the marginal benefit of a better manager (higher productivity and larger surplus) with her marginal cost (higher wage demanded). In an optimum,

$$v_2(z, s)|_{z=z(s)} = w'(s) \quad (4)$$

gives a condition that can be used to trace out the wage function when the left hand side is evaluated at the idea quality z which chooses s optimally, i.e., at $z = z(s)$.

To build the equilibrium wage function, I proceed under the tentative assumption of positive assortative matching, $z'(s) > 0$, and later show that it holds in equilibrium because of complementarities in production. Matching the measures at the right tail of the distributions, positive assortative matching implies $Tz^{-1} = Ls^{-1}$, or

$$z = ts \Leftrightarrow s = z/t, \text{ with } t \equiv T/L. \quad (5)$$

The parameter t is a measure of the relative size of the technology available in the country. A larger population L increases the availability of managers at all levels of ability, so that any idea z can be matched with a better s ; any potential manager gets hurt by a larger L , though, since the mass of people better than her is also larger²¹.

A simple expression for the marginal rent $w'(s)$ in (4) can be obtained differentiating the surplus (3) with respect to s and plugging $z(s)$ from (5) in it. Integrating this expression over the ability dimension and using the fact that the marginal manager - denote her skill level s_c - must be indifferent between occupations, I obtain the wage function:

$$w(s) = \int_{s_c}^s v_2(z, t)|_{z=z(t)} dt = \frac{\alpha}{\kappa + \alpha} \left(\frac{t^\kappa}{\bar{w}} \right)^{\sigma-1} M \left(s^{(\alpha+\kappa)(\sigma-1)} - s_c^{(\alpha+\kappa)(\sigma-1)} \right) + \bar{w} \quad (6)$$

and with $w(s) = \bar{w}$ below s_c .

The profit function $\pi(z)$ is the difference between the surplus and the wage, and leaves the marginal

²¹To parallel the terminology of Costinot and Vogel (2010), this would be skill-upgrading from the standpoint of the firm, and firm-downgrading (which they call task-downgrading) from the point of view of a manager.

idea z_c indifferent to the alternative of not being used. Using the assignment function (5) in the surplus (3), subtracting the wage (6), and using the fact that $v(z_c, s(z_c)) = \bar{w}$,

$$\pi(z) = v(z, s(z)) - w(s(z)) = \frac{\kappa}{\kappa + \alpha} \left(\frac{t^{-\alpha}}{\bar{w}} \right)^{\sigma-1} M \left(z^{(\kappa+\alpha)(\sigma-1)} - z_c^{(\kappa+\alpha)(\sigma-1)} \right) \quad (7)$$

with $\pi(z) = 0$ below z_c .

The sufficient condition for an optimum will require that $v_{22}(z, s) - w''(s) < 0$ when $z = z(s)$ (i.e., along the optimal assignment), which can be easily shown to be true²².

The equilibrium assignment of managers to firms provides a simple microfoundation for the rent sharing within a firm. The rent to be shared is $v(s, z(s)) - \bar{w}$, the excess of surplus over the sum of managers and ideas' opportunity cost (\bar{w} and 0 respectively); the sharing rule is based on local scarcity of talents vs. ideas' and their contributions to the total productivity of the firm. The share of this rent going to managers' wages is

$$\theta \equiv \frac{\alpha}{\alpha + \kappa} .$$

If managers do not influence the firm's efficiency ($\alpha = 0$), the rent for talent is zero, the equilibrium wage function reduces to the outside option, and we are back to the standard one-sided heterogeneity case similar to Melitz (2003), where workers' contributions are homogeneous and the wage per efficiency unit is flat across ability levels. On the other hand, if $\kappa = 0$ we recover a model similar to Lucas (1978) and Manasse and Turrini (2001), where heterogeneous workers are operating using homogeneous ideas: profits then are zero, and only a non-trivial wage function remains²³.

2.3 Equilibrium

To characterize the equilibrium in a closed economy, it is sufficient to determine the cutoff s_c for managers' ability and the expenditure level X in the economy. These two values must (i) keep in equilibrium the market for production workers and (ii) make the marginal firm indifferent between operating and shutting

²²The second order condition for the optimality of s in the firm problem requires $v_{22}(z, s) - w''(s) < 0$. Differentiating (4) again with respect to s we get $w''(s) = v_{22}(z, s) + v_{12}(z, s) z'(s)$, which implies $w''(z) - v_{22}(z, s) = v_{12}(z, s) z'(s)$. Using (3), we have $v_{12}(z, s) = M \bar{w}^{1-\sigma} \kappa \alpha (\sigma-1)^2 z^{\kappa(\sigma-1)-1} s^{\alpha(\sigma-1)-1} > 0$. Hence, $v_{22}(z, s) - w''(s) < 0 \Leftrightarrow v_{12}(z, s) z'(s) > 0 \Leftrightarrow s'(z) > 0$.

²³A simpler way to reach the same earning functions could be to assume an exogenous firm-level productivity distribution and a fixed rent-sharing rule. For the questions I pose, this choice is not possible. As I will argue below, the proper way to think about skill-biased technical change in this framework is to keep fixed the distribution of managers' ability, while changing the impact of ideas through increases in κ . With this experiment, not only does the share of rents to managers decrease, but the overall firm productivity distribution improves. Studying the effect of a change in an exogenous rent sharing parameter we would miss the second part.

down.

Note first that using the price index definition, the individual firm price (2) and the assignment function (5), and assuming $(\kappa + \alpha)(\sigma - 1) < 1$, the price index has the form

$$P = \frac{\sigma}{\sigma - 1} \bar{w} \left(\frac{\psi}{L} \right)^{1/(\sigma-1)} t^{-\kappa} s_c^{\psi/(\sigma-1)} \quad (8)$$

with

$$\psi \equiv 1 - (\sigma - 1)(\kappa + \alpha) \in (0, 1) . \quad (9)$$

The assumption $(\kappa + \alpha)(\sigma - 1) < 1$ guarantees that there are no firms efficient enough to bring down the price index P to zero. Note that a larger relative measure of technology t reduces the price of the final good aggregate Y . Also, the scale of the economy has welfare consequences: fixing t , the price index is still a decreasing function of the measure of population L . A scale effect typically arises in presence of fixed costs: in this framework, the fixed cost is implicitly given by the opportunity cost that each manager has in terms of her alternative occupation.

With an expression for the price index, we can write down the two general equilibrium conditions.

When s_c is the managers' cutoff, $L\bar{w}(1 - s_c^{-1})$ is earned by production workers. The aggregate expenditure on production workers is always $\frac{\sigma-1}{\sigma}X$.²⁴ Equating these two values, the level of expenditure X compatible with the supply of production workers implied by s_c is

$$X = \frac{\sigma}{\sigma - 1} L\bar{w} (1 - s_c^{-1}) . \quad (10)$$

This curve describes the Labor Market Clearing relation in the market for production workers. When $s_c \rightarrow 1$, total earnings of production workers are zero, and so must be the expenditure X ; as $s_c \rightarrow \infty$, all agents are employed as production workers, and total expenditure on them approaches a finite constant. The expenditure is increasing in the cutoff s_c since as s_c grows, there is a larger supply of production workers, which requires a larger expenditure in equilibrium.

In equilibrium, the idea and the manager in the marginal firm $(s_c, z(s_c))$ are indifferent between production in the firm and their alternative employment, and the surplus function (3) for the indifferent pair of agents is equal to the sum of the outside options; using the assignment relation (5) and the price

²⁴The expenditure on production workers for each firm is $x(\varphi) - v(\varphi)$; substituting in it revenues (1), surplus (3), the assignment function (5) and the price index (8), and integrating over all active firms, we get that the overall expenditure on production workers is $\frac{\sigma-1}{\sigma}X$.

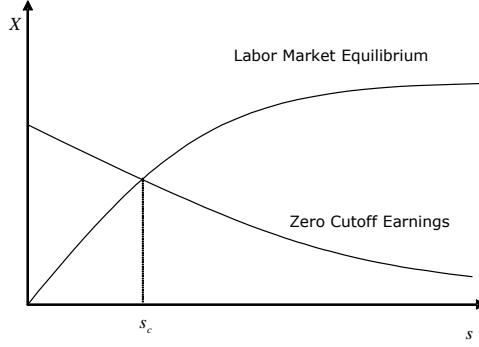


Figure 1: Closed economy equilibrium. This figure shows the equilibrium determination of the cutoff s_c and the expenditure level X in closed economy. The Labor Market Equilibrium represents the locus of pairs (s_c, X) where the expenditure over and the income of production workers are equalized; the Zero Cutoff Earnings is the locus of points where the surplus of the marginal firm $(s_c, z(s_c))$ exactly covers the sum of outside options, so that there is no incentive for entry or exit in the differentiated varieties' sector.

index (8),

$$X = \frac{\sigma}{\psi} L \bar{w} s_c^{-1} . \quad (11)$$

This equation is a Zero Cutoff Earnings condition. As $s_c \rightarrow 1$ the right-hand side becomes a strictly positive and finite number, while as s_c grows toward infinity, this curve goes to zero. The curve is decreasing in s_c : if (s_c, X) is an equilibrium point, and the expenditure becomes smaller, the marginal manager is no longer able to cover her opportunity cost and becomes a production worker.

As shown in Figure 1.1, the equilibrium (s_c, X) is always uniquely determined.

The simple functional forms assumed allows for an explicit solution for s_c in terms of parameters: equating (10) and (11), and using the definition (9) for ψ , I obtain

$$s_c = 1 + \frac{(\sigma - 1)}{1 - (\sigma - 1)(\kappa + \alpha)} \quad (12)$$

$$X = \frac{\sigma}{\psi + \sigma - 1} L . \quad (13)$$

We can also rewrite the earning functions in more explicit terms. Exploiting the equilibrium relation between the indifferent firm and the market size²⁵, we can express wages (6) and profits (7) as:

²⁵Using the assignment function (5) to express the surplus (3) in terms of s , and imposing equality to \bar{w} for the marginal firm, $M = \bar{w}^\sigma t^{-\kappa(\sigma-1)} s_c^{-(\alpha+\kappa)(\sigma-1)} = \bar{w}^\sigma t^{\alpha(\sigma-1)} z_c^{-(\alpha+\kappa)(\sigma-1)}$. I substitute the right hand side in terms of s_c in the expressions for wages (6) to obtain (14), and the right hand side in terms of z_c in (7) to obtain (15).

$$w(s) = \theta \left[\left(\frac{s}{s_c} \right)^{(\alpha+\kappa)(\sigma-1)} - 1 \right] \bar{w} + \bar{w} \quad (14)$$

$$\pi(z) = (1 - \theta) \left[\left(\frac{z}{z_c} \right)^{(\alpha+\kappa)(\sigma-1)} - 1 \right] \bar{w} . \quad (15)$$

Using the expression for s_c in (12) in the price index (8), the profit and wage functions can be written in real terms and only as a function of parameters, as $\pi(z)/P$ and $w(s)/P$ above the cutoffs, and 0 and \bar{w}/P below, respectively.

The equilibrium wage function is determined jointly by the distribution of abilities and technology, through a market mechanism which prices the relative scarcity of each type of factor.

The structure of the real earning functions has some characteristic elements.

The inverse of the price index gives a measure of the opportunity cost of keeping agents employed as production workers: in fact, their real wage is exactly P^{-1} , after normalizing \bar{w} to 1.

The parameter $\theta \equiv \alpha/(\alpha + \kappa)$ is a talent-specific component: total real rents in the firm $\left[(s/s_c)^{1-\psi} - 1 \right] / P$ are split giving a share θ to managers and a share $1 - \theta$ to ideas.

A microeconomic component, s/s_c and z/z_c , determines then earnings differences between different levels of ability within managers, and within ideas.

In terms of the fixed cost, another interpretation is that the manager (or the idea) pays a share θ (or $1 - \theta$) of the fixed cost of being active and gets back her opportunity cost 1 (or 0, respectively). The net fixed payment is $1 - \theta$ for the manager, and $-(1 - \theta)$ for the idea. This interpretation will be important to understand some parts of the open economy response of inequality.

In the next section, I use this framework to evaluate the consequences of skill-biased technical change on the wage ratio at different percentiles in the wage distribution.

3 Skill-Biased Technical Change

This section analyzes the effect of skill-biased technical change on an arbitrary wage ratio $w(s'')/w(s')$ in a closed economy.

Before focusing on the substantive side of the issue, I provide a general motivation for the theoretical framework used.

I define skill-biased technical change as a change in technology which benefits disproportionately

highly skilled managers. I do not attempt to explain the source of this change. Skill-biased technical change is modeled as an exogenous increase in κ . This assumption implies that the percent increase in productivity is biased toward firms which employ better managers: for a given ability s , the elasticity of the productivity of the firm $t^\kappa s^{(\alpha+\kappa)}$ to κ is simply $\kappa \ln ts$, which is increasing in s ²⁶. This is the only way in which, in this framework, skill-biased technical change can be modeled. An increase in α would amount to a change in the distribution of abilities, which instead we want to keep fixed. A proportional increase in the productivity of all the firms is equivalent to an increase in the efficiency of production workers, and hence would not be skill biased.

The elasticity of the wage ratio $w(s'')/w(s')$ with respect to κ is just the difference of the elasticities of the wage function evaluated at each point, $\varepsilon^{(\kappa)}(s'') - \varepsilon^{(\kappa)}(s')$, with $\varepsilon^{(\kappa)}(s) \equiv w_\kappa(s) \kappa / w(s)$. Since the abilities s' and s'' can be chosen arbitrarily, it is convenient to recast the analysis in terms of local changes in wage ratios, i.e., changes in the ratio of wages of two marginally different managers. In the rest of the section, I show why this approach is helpful, and argue that it is a very general way to think about the response of wage ratios to exogenous shocks in different regions of the wage distribution; I will then adopt it in the rest of the paper.

Consider two agents with marginally different levels of ability, s and $s + ds$. The difference in their wage is essentially $w_s(s)$, the marginal price of skills at s , so that the wage ratio is just $1 + w_s(s)/w(s)$. Suppose now that κ increases: if the marginal price of skills is more elastic than the wage, the wage ratio $w(s + ds)/w(s)$, a measure of local inequality, increases. Since the difference in the wage between two arbitrary levels of ability s' and s'' is the sum of all the marginal rents, the response of their ratio $w(s'')/w(s')$ to skill-biased technical change must be related to the integral of the local responses between s' and s'' . The formal argument (in Appendix A.2) shows that for two ability levels s' and s'' , with $s'' > s'$, the elasticity of the wage ratio $w(s'')/w(s')$ to κ is simply

$$\int_{s'}^{s''} \frac{w_s(s)}{w(s)} \eta^{(\kappa)}(s) ds$$

with

$$\eta^{(\kappa)}(s) \equiv \frac{w_{\kappa s}(s) \kappa}{w_s(s)} - \frac{w_\kappa(s) \kappa}{w(s)} \quad (16)$$

being the local change in inequality at s when κ changes. In this notation, $w_\kappa(s) = \partial w(s) / \partial \kappa$ and

²⁶For example, at $t = \kappa = 1$ a 1% increase in κ raises the productivity of a firm employing a top 10% manager by 1.61 percentage points more than the median firm (in fact, $\kappa \ln (ts'')^\kappa - \kappa \ln (ts')^\kappa = \kappa \ln (10/2) = 1.61$).

$w_{\kappa s}(s) = \partial^2 w(s) / (\partial \kappa \partial s)$. The function $\eta^{(\kappa)}(s)$ is the difference between the elasticity of the marginal price of skills and the total price of skills to changes in κ : by construction, $\eta^{(\kappa)}(s) > 0$ if and only if $\varepsilon^{(\kappa)}(s)$ is increasing in s . The (total) change in inequality between s' and s'' is the integral of all the local changes, weighted by $w_s(s)/w(s)$, a positive and unitless measure of the importance of ability differences. When for some s , $\eta^{(\kappa)}(s) > 0$, the local contribution of s is to increase the response of all the wage ratios that contain it, and vice-versa.

Any model about the behavior of wage dispersion is essentially a specification of eq. (16). I now examine in detail how skill-biased technical change affects $\eta^{(\kappa)}(s)$ in this framework.

3.1 Skill-Biased Technical Change and Wage Ratios

In this section, I analyze the local change in inequality implied by skill-biased technical change studying the response of the total and the marginal price of skills in $\eta^{(\kappa)}(s)$ for the wage function (14)²⁷.

An increase in κ affects the wage (the second term in (16)) through (i) rent sharing, (ii) selection, and (iii) assignment. The first term captures a negative "share" effect. For a fixed rent level in the firm, the share of it going to managers decreases, because technology contributes more to differences in firm-level productivities. The second term captures a negative "selection" effect. As κ increases, the productivity of all firms improve, and the marginal firm exits ($\partial s_c / \partial \kappa > 0$ from eq. (12)) because of stiffer competition; since the rent level is just the integral of the marginal rents from the worst firm upwards, the total rent of all the firms also decreases. The third term represents an "assignment" effect, and is always positive: each manager gains from a larger contribution of z to the productivity of the firm.

The elasticity of marginal price of skill $w_s(s)$ to κ (the first term in (16)), has only two components: the slope of the wage tends to decrease because of selection η and to increase through the assignment effect. Since $w_s(s)$ does not contain a share parameter, there is no share effect.

Simple calculations show that $\eta^{(\kappa)}(s)$ is positive if and only if

$$-\underbrace{\frac{(1-\psi)\kappa}{s_c} \frac{\partial s_c}{\partial \kappa}}_{\text{Selection}} + \underbrace{(\sigma-1)\kappa \ln \frac{s}{s_c}}_{\text{Assignment}} > -g_1(s) \underbrace{(1-\theta)}_{\text{Share}} - g_2(s) \underbrace{\frac{(1-\psi)\kappa}{s_c} \frac{\partial s_c}{\partial \kappa}}_{\text{Selection}} + g_2(s) \underbrace{(\sigma-1)\kappa \ln \frac{s}{s_c}}_{\text{Assignment}}$$

where $g_1(s) \equiv \frac{w(s)-1}{w(s)} \in (0, 1)$, $g_2(s) \equiv \frac{w(s)-(1-\theta)}{w(s)} \in (\theta, 1)$, and $g'_i(s) > 0$ for $i = 1, 2$.

For managers close enough to the indifference point s_c , the assignment and share effects are negligible,

²⁷Studying the nominal (as opposed to the real) wage function is without loss of generality since nominal wage ratios and real wage ratios are the same by definition.

and the negative selection effect dominates. Its impact is greater (more negative) on the marginal price (left hand side) than on the total price (right hand side) of skills: part of the selection effect on the level of surplus is borne by profits²⁸. Since the marginal price of skills falls proportionately more than the wage, the wage ratio between two marginally different managers becomes smaller: the local inequality decreases.

For managers skilled enough, the assignment effect becomes dominant: technological change is biased towards better agents. Again, only a fraction $g_2(s)$ of the assignment effect impacts the elasticity of the wage level. The marginal price of skills increases proportionately more than the wage: the ratio of wages between two similar managers becomes higher, and local inequality increases²⁹.

Proposition 1 (proven in Appendix A.3) formally states this result:

Proposition 1. *There exists a unique skill level $s^{(\kappa)} > s_c$ such that the local change in inequality from skill-biased technical change is positive for high abilities and negative for low abilities, i.e., $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$.*

With these results in hand, it is possible to produce dispersion in the tail (as in Piketty and Saez (2004)) by looking at wage ratios between abilities above $s^{(\kappa)}$: since the change in local inequality is always positive, any wage ratio increases with κ . Moreover, it is also possible to replicate divergent patterns of wage ratios in the right vs. the left tail of the distribution (as in Autor, Katz and Kearney (2006)). By picking three abilities such that $s_c < s' < s'' < s^{(\kappa)}$ and $s''' > s^{(\kappa)}$, it is possible to obtain one decreasing and one increasing wage ratio at different percentiles in the distribution.

4 The Open Economy

In this section I show the equilibrium determination in an economy where two identical countries are allowed to trade with each other³⁰. The assumption of identical countries let us focus on the consequences of trade not stemming from differences in factor endowments or technologies.

A firm needs to produce τ units of a good for 1 unit to reach the foreign destination, and f units of production workers are needed to sell in the export market at all. If the price of firm φ is $p(\varphi)$ in the domestic market, it will be $\tau p(\varphi)$ abroad. The surplus from sales on the domestic and export markets

²⁸In fact, if $\theta \rightarrow 0$ also $g_2(s) \rightarrow 0$, and the selection effect cancels.

²⁹As an additional remark, note that if the selection effect is close to zero, the region with local decrease in inequality tends to vanish: selection is necessary for the inequality to decrease among low skill managers.

³⁰All missing algebra details are reported in Appendix A.4.

are given respectively by:

$$v_d(z, s) = M \left(\frac{z^\kappa s^\alpha}{\bar{w}} \right)^{\sigma-1} \quad (17)$$

$$v_x(z, s) = \tau^{1-\sigma} M \left(\frac{z^\kappa s^\alpha}{\bar{w}} \right)^{\sigma-1} - f\bar{w} \quad (18)$$

where $M \equiv \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} X P^{\sigma-1}$.

The earning functions corresponding to equations (6) and (7) are built following steps analogous to the closed economy. The only difference is that the optimal choice of manager (eq. (4)) depends on the export status of the firm. Since this status is not known in advance, I postulate the existence of two cutoffs s_d and s_x (for access on the domestic and export market) and then build separately two sets of first order conditions, for domestic sellers (earning $v_d(z, s)$) and for exporters (earning $v_d(z, s) + v_x(z, s)$). Having obtained two expressions for $w_d(s)$ and $w_x(s)$, I impose two separate indifference conditions, $w_d(s_d) = \bar{w}$ and $w_x(s_x) = w_d(s_x)$, which ensure continuity of the wage function.

The price index in open economy is now a function of the domestic cutoff s_d , and of the export cutoff of the other country, which by symmetry is equal to s_x . Since both these cutoff firms face the same size of the market M , they can be written one as a function of the other:

$$s_x = (\tau^{\sigma-1} f)^{1/(1-\psi)} s_d \quad (19)$$

where I assume that $(\tau^{\sigma-1} f)^{1/(1-\psi)} > 1$ in order to generate the empirically relevant pattern of partitioning in the export behavior of firms, i.e., $s_x > s_d$. The price index can then be written as

$$P = \frac{\sigma}{\sigma-1} \bar{w} \left(\frac{\psi}{L} \right)^{1/(\sigma-1)} t^{-\kappa} \left(1 + \frac{1}{\delta} \right)^{-1/(\sigma-1)} s_d^{\psi/(\sigma-1)} \quad (20)$$

$$\delta \equiv \tau^{1/(\alpha+\kappa)} f^{\psi/(1-\psi)} \quad (21)$$

where δ is an index of distance between the two economies. While the general structure of the price index reflects its shape in closed economy (eq. (8)), the additional term $(1 + 1/\delta)^{-1/(\sigma-1)}$ shows how competition from abroad lowers the price index at home. Note also that heterogeneity in both skill and technology contribute to effectively reduce the distance between the two countries (as $\kappa + \alpha$ grows, δ becomes smaller).

In an open economy, equilibrium will require for each country: (i) equilibrium in the market for

production workers, (ii) indifference for the marginal agent s_d between alternative occupations, and (iii) trade balance.

The total expenditure of firms on production workers is now $\frac{\sigma-1}{\sigma}X + f\bar{w}Ls_x^{-1}$. This expression is found integrating separately labor demand for domestic and export sales, and including the fixed requirement to sell abroad, $f\bar{w}$, in proportion to the mass of exporters Ls_x^{-1} . Using (19), and equating this expenditure to total income of production workers (condition (i)) I obtain

$$X = \frac{\sigma}{\sigma-1}L\bar{w} \left[1 - \left(1 + \frac{1}{\delta} \right) s_d^{-1} \right]. \quad (22)$$

Equation (22) is the parallel in an open economy of eq. (10), the Labor Market Equilibrium condition. It shows how the possibility to sell abroad affects domestic demand of production workers: as economies become more integrated, more workers are demanded to pay the fixed costs of export (δ decreases), and a lower level of overall expenditure X is sufficient to equilibrate demand and supply of production workers.

The indifference of a firm to sell on the domestic market or to shut down (condition (ii)) simply requires the surplus in the domestic market given in (17) to be equal to the sum of the outside options \bar{w} and 0 when evaluated at s_d and $z_d \equiv ts_d$. Substituting in such equality the expression for the price index (20), using (19) and rearranging, we get

$$X = \frac{\sigma}{\psi}L\bar{w} \left(1 + \frac{1}{\delta} \right) s_d^{-1}. \quad (23)$$

This equation is the open economy equivalent of (11), the Zero Cutoff Earnings: it shows how competition from abroad affects occupational choices. Stronger trade integration (lower δ) makes competition stiffer, lowering the price index and increasing the real wage for production workers: at any expenditure level X , the cutoff agent s_d must be better to compete in her own market.

To close the model, we need to make sure that these conditions are compatible in the world economy: if trade balance has to be satisfied (condition (iii)), this entails a relation between the relative wage of production workers in the two economies. When countries are identical, this ratio is simply 1.

Equations (22) and (23) pin down the two endogenous variables of this model, the national income X and the domestic cutoffs s_d . The exporter cutoff s_x can then be found using (19), and the price index using (20). Equating (22) and (23) and solving for s_d , I obtain

$$s_d = \left(1 + \frac{\sigma-1}{1 - (\sigma-1)(\alpha + \kappa)} \right) \left(1 + \frac{1}{\delta} \right). \quad (24)$$

Substituting this value back in (23),

$$X = \frac{\sigma}{\psi + \sigma - 1} L\bar{w} . \quad (25)$$

Figure 1.2 shows graphically how the equilibrium is determined. Larger demand for production workers at any given level of expenditure, combined with stiffer competition on the domestic market for varieties, both imply a stronger selection among domestic firms, i.e., $\partial s_d / \partial \tau < 0$.

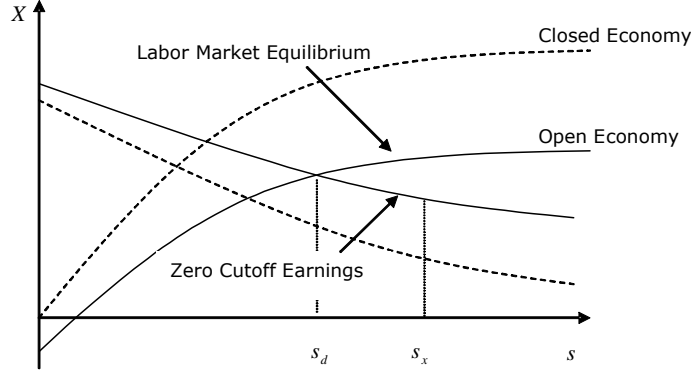


Figure 2: Open economy equilibrium. This figure shows the equilibrium determination of the cutoff s_d and the expenditure level X in an open economy. The possibility to sell abroad implies that a lower level of expenditure (for any supply of production workers) is sufficient for equilibrium: the Labor Market Equilibrium curve shifts down and to the right. On the other hand, competition from abroad implies that the marginal firm must employ a better manager (at any level of domestic expenditure) to stay indifferent: the Zero Cutoff Earnings curve shifts up and to the right. As a result, the cutoff for domestic producers is larger in open economy.

In open economy, the profit and wage functions can be written as

$$\pi(z) = \begin{cases} (1 - \theta) \left[\left(\frac{z}{z_d} \right)^{1-\psi} - 1 \right] & z \in [z_d, z_x) \\ (1 - \theta) \left[(1 + \tau^{1-\sigma}) \left(\frac{z}{z_d} \right)^{1-\psi} - (1 + f) \right] & z \geq z_x \end{cases} \quad (26)$$

and

$$w(s) = \begin{cases} \theta \left[\left(\frac{s}{s_d} \right)^{1-\psi} - 1 \right] + 1 & s \in [s_d, s_x) \\ \theta \left[(1 + \tau^{1-\sigma}) \left(\frac{s}{s_d} \right)^{1-\psi} - (1 + f) \right] + 1 & s \geq s_x \end{cases} . \quad (27)$$

The real earnings can easily be obtained dividing by the price index (20). Below the cutoffs, we still have $\pi(z)/P = 0$ and $w(s)/P = P^{-1}$.

All the components identified in the closed economy case are present, suitably modified, in the open

economy. The existence of an export market now raises the marginal price of skills for managers good enough to access it. An exporting manager pays a share θ of the total fixed cost of the firm $1 + f$, and gets back her opportunity cost 1: her net fixed payment is $1 - \theta(1 + f)$.

In the next section, I use this model to compare the consequences of skill-biased technical change and trade integration on wages ratios in different regions of the income distribution.

5 Wage Dispersion in Open Economy

In this section, I first show that trade integration and skill-biased technical change can both rationalize increasing wage ratios at the top of the distribution and constant or decreasing wage ratios at its bottom. I then illustrate these results numerically with a simple parameterization of the model.

5.1 Observational Equivalence

I start this subsection maintaining the following:

Assumption 1. Fixed costs of exporting are such that $f < \kappa/\alpha$.

This restriction assumes that the fixed cost to access the export market is less than κ/α times the fixed cost to access the domestic market (which is 1). When assumption 1 is satisfied, the net fixed payment to an exporting manager $1 - \theta(1 + f)$ is positive, and the elasticity of the wage function to $i \in \{\tau, \kappa\}$ is, in absolute value, always increasing in s . I will later describe the meaning and the consequences of this assumption, while the numerical simulations will show what happens when it is violated.

I first evaluate the effect of trade integration (a reduction in τ) on the evolution of wage ratios. The local change in inequality is

$$\eta^{(\tau)}(s) = \frac{w_{\tau s}(s)\tau}{w_s(s)} - \frac{w_\tau(s)\tau}{w(s)}$$

where $w_\tau(s) = \partial w(s)/\partial \tau$ and $w_{\tau s}(s) = \partial^2 w(s)/(\partial \tau \partial s)$. Now, $\eta^{(\tau)}(s) < 0$ corresponds to an increase in local inequality following a reduction in trade barriers.

Trade integration affects wages through two channels: (i) a market effect - the reduced marginal cost that exporters face to sell abroad - which increases the value of skills; and (ii) a selection effect - fiercer competition at home - which reduces revenues on the domestic market and select some managers out of the differentiated sector. These two channels have a different importance for non-exporters and exporters.

For non-exporters, the selection effect is the only active channel. Using (27), simple calculations show that $\eta^{(\tau)}(s) > 0$ is always true³¹:

$$-\underbrace{(1-\psi) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau}}_{\text{Selection}} > -g_2^d(s) \underbrace{(1-\psi) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau}}_{\text{Selection}} \forall s \in (s_d, s_x)$$

with $g_2^d(s) \equiv [w(s) - (1 - \theta)] / w(s)$ for $s \in (s_d, s_x)$. As τ falls, both the wage and the marginal price of skill decrease. However, part of the adjustment of the level of surplus is borne by profits, and hence the marginal price of skill falls more: hence, the wage ratio between two marginally different managers also falls.

For exporters, both the market and the selection effects operate. For these agents, $\eta^{(\tau)}(s) < 0$ always holds:

$$-\underbrace{\frac{(\sigma-1)}{\tau^{\sigma-1}+1}}_{\text{Market}} - \underbrace{(1-\psi) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau}}_{\text{Selection}} < -g_2^d(s) \underbrace{\frac{(\sigma-1)}{\tau^{\sigma-1}}}_{\text{Market}} - \underbrace{g_2^x(s) (1-\psi) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau}}_{\text{Selection}} \forall s > s_x$$

where $g_2^x \equiv [w(s) - (1 - \theta(1 + f))] / w(s)$ for $s \geq s_x$. The selection effect still pushes towards a reduction of local inequality. The market effect raises both the total and the marginal price of skills. Again, part of the increase in the surplus level benefits profits, so the marginal price is more sensitive: the local wage ratio tends to increase through this channel. For exporters, the market effect always prevails, and trade integration increases local inequality for all $s > s_x$.

It is then possible to state the following proposition (proven in Appendix A.5):

Proposition 2. *There exists a unique skill level $s^{(\tau)} = s_x$ such that the local change in inequality is positive for high abilities and negative for low abilities, i.e., $\eta^{(\tau)}(s) \leq 0 \Leftrightarrow s \geq s^{(\tau)}$.*

Skill-biased technical change, again under Assumption 1, has the same behavior as in closed economy. In Appendix A.6 I prove the following:

Proposition 3. *There exists a unique skill level $s^{(\kappa)} > s_d$ such that the local change in inequality from skill-biased technical change is positive for high abilities and negative for low abilities, i.e., $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$.*

Trade integration and skill-biased technical change operate through partially different channels. However, they have the same asymmetric effect across firms: the competitive pressure on low productivity

³¹All results are formally proven in Appendix A.5 and A.6.

firms rise, while firms at the high end of the productivity range increase their earnings. Because of positive assortative matching in the labor market, this asymmetry reflects itself across low and high skill managers. Hence, trade integration and skill-biased technical change produce qualitatively similar local changes in inequality and responses of wage ratios: only studying the evolution of wage dispersion in different regions of the wage distribution is not sufficient to disentangle the source of the pattern.

What is the role of Assumption 1? Among domestic sellers, the elasticity of the wage with respect to τ , $\varepsilon^{(\tau)}(s)$, is always increasing (in absolute value) with ability. When Assumption 1 is met, the same is true for exporters. When it is violated, however, the wage of high skilled exporting managers is less sensitive to trade costs reductions than the wage of low skilled exporters: $\varepsilon^{(\tau)}(s)$ is decreasing in s . In this case, a marginal fall in trade costs raises the wages of all exporters, but proportionately more the wage of low skilled ones. The wage ratio between an exporter s and a marginally worse manager become then lower, and the local change in inequality is negative everywhere, not only among domestic sellers. The reason why local inequality decreases is related to the size of the fixed export cost. From the wage function, a share θ of the total fixed cost $(1 + f)$ borne by an exporting firm is paid by the manager; hence, when f is higher, the elasticity of the wage to τ is larger, *ceteris paribus*; however, since f is a fixed cost, its increase does not affect the elasticity of the marginal price of skill. When f is high enough, the total fixed payment to the manager becomes negative (in fact, $f > \kappa/\alpha \iff 1 - \theta(1 + f) < 0$), and the elasticity of the wage becomes larger than the elasticity of the marginal price of skills: hence, $\eta^{(\kappa)}(s) \geq 0$, and local inequality decreases even for exporters.

An analogous argument holds for $\varepsilon^{(\kappa)}(s)$. When $f < \kappa/\alpha$, the local change in inequality goes monotonically from negative to positive values along s (i.e., the elasticity of the wage to κ , $\varepsilon^{(\kappa)}(s)$ is monotonically increasing in s). When $f > \kappa/\alpha$, this is no longer necessarily the case: local inequality always decreases for low-skill domestic sellers, and always increases for high-skill exporters; in between, cases can be constructed where local inequality changes in either direction for the worst exporters³².

In summary, when Assumption 1 is violated, the qualitative predictions of trade and skill-biased technical change no longer coincide. In this case, the behavior of wage ratios is even more articulated. In the next subsection, I provide some numerical examples both when Assumption 1 is satisfied, and when it is not, while in the following section, I suggest a way in which detailed firm-level data may be used to partially disentangle these mechanisms.

I close this section emphasizing that the relation between changes in the wage ratio between two

³²See the discussion in Appendix A.6.

arbitrary abilities s' and s'' and local inequality (Section 3) has to be slightly amended in open economy to accommodate the existence of another destination market. Around the exporters' cutoff, the response of $w(s_x + ds)/w(s_x - ds)$ to changes in $i \in \{\tau, \kappa\}$ is given by $\varepsilon^{(i)}(s_x + ds) - \varepsilon^{(i)}(s_x - ds)$, the difference in the elasticities of the wage. However, while $\varepsilon^{(i)}(s_x - ds)$ only encompasses responses on the domestic market, $\varepsilon^{(i)}(s_x + ds)$ also counts the benefits of the additional export market, and it is not possible to reduce the change in $w(s_x + ds)/w(s_x - ds)$ to a local measure³³. Intuitively, a small difference in ability generates a large change in the skill premium, and hence in the wage ratio. The general change in the wage ratio between s' and s'' in response to a change in $i \in \{\tau, \kappa\}$ can now be written as³⁴

$$\int_{s'}^{s_x} \frac{w_s(s)}{w(s)} \eta^{(i)}(s) ds + I^{(i)}(s', s'') D^{(i)} + \int_{s_x}^{s''} \frac{w_s(s)}{w(s)} \eta^{(i)}(s) ds$$

with

$$I^{(i)}(s', s'') = 1 \text{ if } s_x \in (s', s''), \text{ and } 0 \text{ otherwise}$$

$$D^{(i)} \equiv \lim_{s \rightarrow s_x^+} \varepsilon^{(i)}(s) - \lim_{s \rightarrow s_x^-} \varepsilon^{(i)}(s) .$$

In this notation, $D^{(i)}$ is the adjustment in the response of the wage ratio due to the marginal exporter accessing the foreign market³⁵. This adjustment is positive if and only if the exogenous change in τ or κ is increasing the surplus for the indifferent exporter. Hence, trade integration always increases the response of the wage ratio (i.e., $D^{(\tau)} < 0$), while skill-biased technical change increases it if and only if the marginal increase in κ is raising the total surplus (i.e., $D^{(\kappa)} > 0 \iff \left[\partial (s/s_d)^{1-\psi} / \partial \kappa \right]_{s=s_x} > 0$).³⁶

5.2 A Simple Parameterization

This subsection provides some simple numerical simulations to illustrate the articulated behavior of wage ratios. The exercise consists in showing contour plots of some wage ratios in a reasonable (κ, τ) space. I also show wage ratios as a function of τ , for values of the fixed cost of export f that satisfy and do not satisfy Assumption 1. To parameterize the model, I need three numbers, α , κ , and σ . I adopt the interpretation of the manager as a top executive in the firm because of the availability of data that can help pin down θ in a simple way. Other approaches are certainly possible, and are discussed below. I will

³³Formally, $\varepsilon_i(s)$ is discontinuous at $s = s_x$; hence, $\partial \varepsilon_i(s) / \partial s$ is not defined.

³⁴See Appendix A.7.

³⁵This expression reduces to $\int_{s'}^{s''} \frac{w_s(s)}{w(s)} \eta^{(i)}(s) ds$ if $s_x \notin (s', s'')$.

³⁶See again Appendix A.7.

set $\sigma = 1.1$: such a low value is necessary to satisfy $\psi > 0$ and still leave some room for variation in the values of κ . Luttmer (2007) reports that the slope in the tail of the size distribution of firms is -1.06 ; in my model, this slope imposes $[(\sigma - 1)(\alpha + \kappa)]^{-1} = 1.06$. Bebchuk and Grinstein (2005) report that the average ratio of managerial rents to firms' earnings between 1993 and 2003 has been 0.066; this fact implies $\alpha/\kappa = 0.066$.³⁷ These relations together deliver $\alpha \simeq 0.58$ and $\kappa \simeq 8.85$. In all graphs, I let τ vary between 1 and 4; moreover, I keep α fixed, and let the technology parameter vary between $\kappa/2 = 4.42$ and the maximum κ compatible with $\psi < 1$, which is roughly 9.3. Note that Assumption 1 is satisfied as long as the fixed cost of exporting is less than or equal to $8.85/0.58 \simeq 15.3$ times the fixed cost of accessing the domestic market.

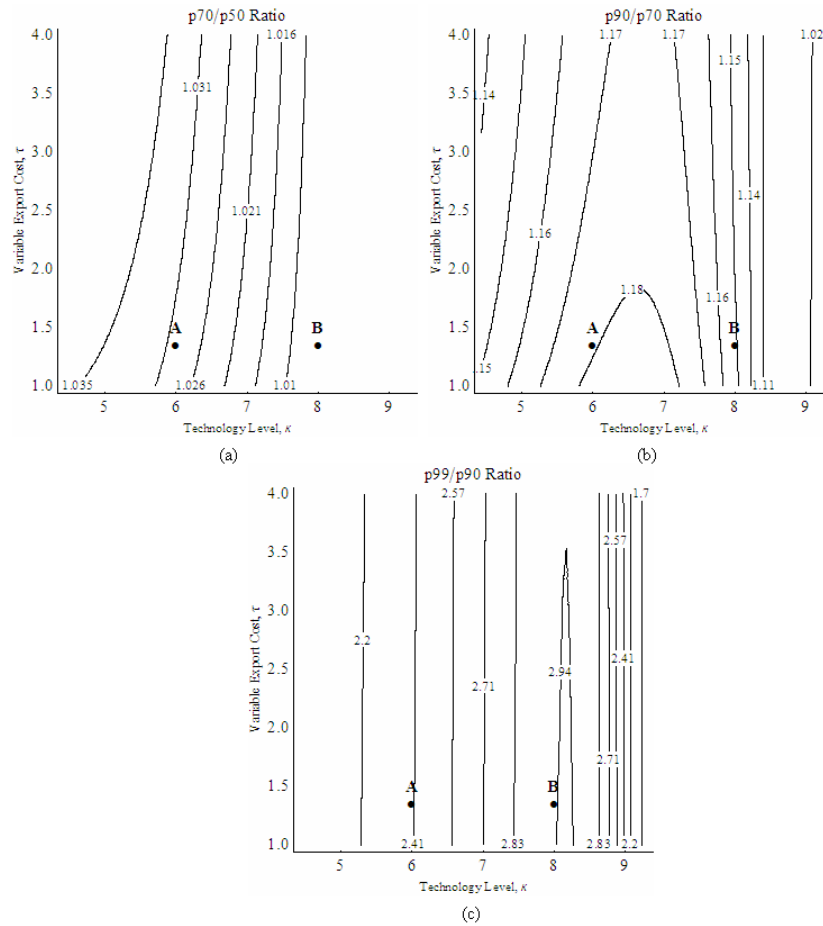


Figure 3: Wage ratios, low fixed costs. This panel shows contour plots for the wage ratio indicated at the top of each figure in the (κ, τ) space. The Figures are drawn setting $\sigma = 1.1$, $\alpha = 0.58$, and $f = 2$. The assumptions $\psi \in (0, 1)$, $f < \kappa/\alpha$, and $\tau^{\sigma-1}f > 1$ hold at all points in the plane.

³⁷In fact, for firms large enough, $w(s)/\pi(z(s)) \simeq \alpha/\kappa$ since the incidence of the opportunity cost of being a production worker is small.

Figure 1.3 focus on the case where $f < \kappa/\alpha$. I choose $f = 2$: to export, a firm must pay twice the fixed cost it pays to sell at home. In any panel, a point in the (κ, τ) space completely characterizes the relevant state in the economy. In particular, each point on (κ, τ) has an associated domestic cutoff and export cutoff (not shown). Points A and B are chosen as an illustration and have the same coordinates in all panels, $A = (6, 1.45)$ and $B = (8, 1.45)$. A movement to the right along the κ axis represents an episode of skill-biased technical change, while a movement down along the τ axis represents an episode of trade integration. Each panel plots the behavior of a different wage ratio: p_{70}/p_{50} , p_{90}/p_{70} , and p_{99}/p_{90} , from left to right. Points A and B are chosen so that p_{70}/p_{50} is the wage ratio between two domestic sellers, p_{90}/p_{70} is the ratio between an exporter and a non-exporter, and p_{99}/p_{90} is the ratio between two exporters.

An economy located at point A has a qualitatively similar behavior in the three wage ratios in response to trade integration and skill-biased technical change. In both cases, the p_{99}/p_{90} and p_{90}/p_{70} ratios increase, and the p_{70}/p_{50} ratio decreases. At point A , an observer only looking at these wage ratios would not be able to disentangle the cause of the change.

At point B , the response to trade is the same as in A . However, technical change is now increasing the wage ratio only in the p_{99}/p_{90} ratio. At such a large initial level of the technology parameter κ , the domestic cutoff is so high that selection and share effect prevail also in the p_{90}/p_{70} region. In this case, the response of the economy is different for the three wage ratios; however, this diversity preserves the same qualitative characteristics as those of point A .

Presumably, τ and κ move at the same time. The simple case of $f < \kappa/\alpha$ is already able to deliver a very articulated path of the wage ratios along an exogenous trajectory of the (κ, τ) space: since there is no monotonicity in the contour plots, inequality can increase and then decrease in the same region over time.

Figures 1.4 shows the behavior of some wage ratios when $f > \kappa/\alpha$. These Figures require a fixed cost of export 20 times higher than the fixed cost to sell at home.

In this example, fixed costs are so high that most of the managers are just domestic sellers: the ratio $p_{99.5}/p_{99}$ is between two exporters, but the $p_{99}/p_{98.5}$ is between an exporter and a domestic seller. Figure 1.4.a shows the ratio $p_{99.5}/p_{99}$ (between two exporters for all values of τ) when $\kappa = 8$ (as in point B). As predicted, this ratio falls with trade integration since the wage of the exporter at $p_{99.5}$ is less sensitive than the one at p_{99} to trade costs reductions. The ratio $p_{99}/p_{98.5}$ (figure 1.4.b) however

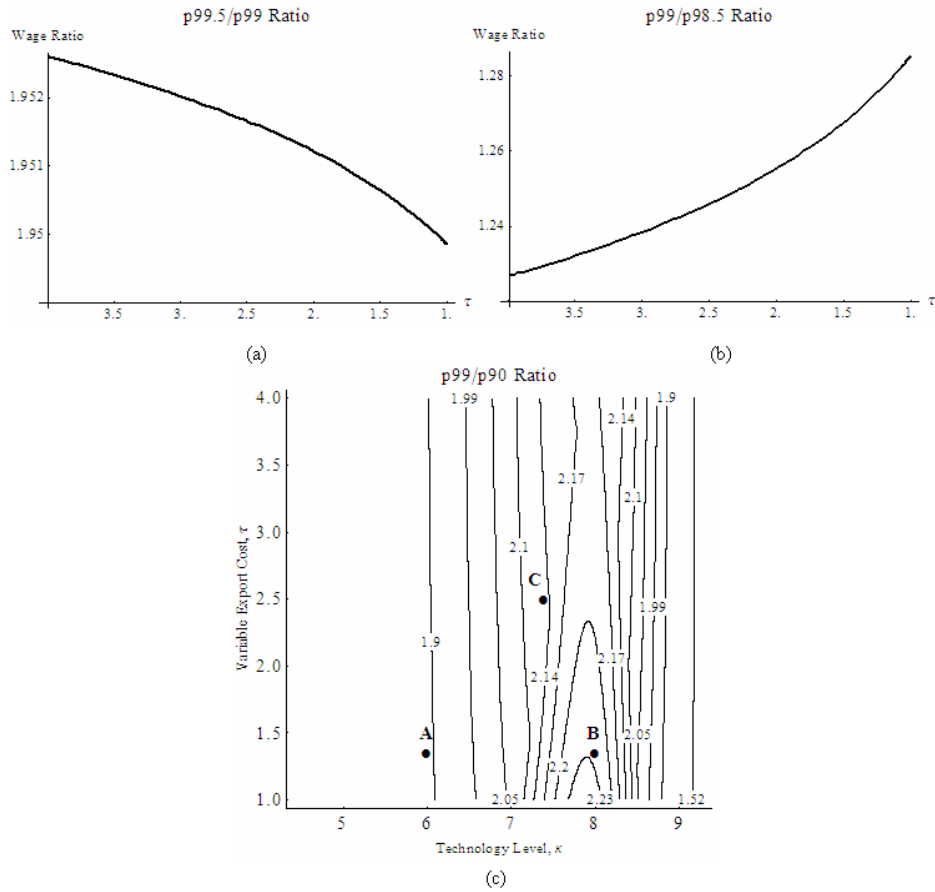


Figure 4: Wage ratios, high fixed costs. Figures 1.4.a and 1.4.b show the reported wage ratios as a function of τ , when $\kappa = 8$. At all points, Figure 1.4.a is the ratio between two exporters, and Figure 1.4.b is the ratio between an exporter and a domestic seller. Figure 1.4.c shows contour plots for the wage ratio $p99/p90$ in the (κ, τ) space. All figures are drawn setting $\sigma = 1.1$, $\alpha = 0.58$, and $f = 20$. The assumptions $\psi \in (0, 1)$, $f > \kappa/\alpha$, and $\tau^{\sigma-1}f > 1$ hold at all points in all Figures.

is always between an exporter and a domestic seller, and it shows that even if local inequality decreases everywhere, some wage ratios may still increase. This behavior is dictated by the adjustment due to the marginal exporter accessing the foreign market, $D^{(\tau)}$, as described in the preceding Subsection. A similar response occurs at point B in figure 1.4.c, where the ratio $p99/p90$ is still between an exporter and a non-exporter. The ratio $p90/p70$ and $p70/p50$ at B , and all three ratios at A (not shown) still decrease in response to trade.

Point C ($\kappa = 7.4, \tau = 2.6$), finally, illustrates a case where a given wage ratio can fall and then increase because of trade. At this point, the exporters' cutoff is slightly above the 99th percentile, and hence the ratio is between two domestic sellers. As τ decreases, the change in local inequality among them is negative, and the $p99/p90$ ratio falls. As τ continue falling, the exporter's cutoff falls below the 99th percentile, the manager at the 99th percentile becomes an exporter, and the ratio starts rising again because of the adjustment $D^{(\tau)}$.

This simple parameterization fails to capture the magnitude of the observed ratios in U.S.: the response of ratios to changes in parameters is quite flat, and levels are also very underestimated. For example, the implied $p90/p50$ ratio at point A in Figures 3.a and 3.b is roughly 1.16, and is not moving much in the (κ, τ) space; by comparison, the same observed wage ratio for males has moved from about 1.6 in 1963 to about 2.3 in 2005 (see Fig. 3 in Autor, Katz and Kearney (2008)).

To summarize, the response of wage ratios to trade and skill-biased technical change can be quite complex, and are qualitatively similar when $f < \kappa/\alpha$, while they may or may not be the same otherwise³⁸. Even if the specific channels are different, the similarity arises because they both act asymmetrically across productive and unproductive firms, and because sorting on the labor market transmits this asymmetry across different skill levels.

In the next section, I argue that focusing the attention on the different channels through which trade integration and skill-biased technical change operate may give further insights on how to disentangle these forces.

6 Intra-Firm Rent Distribution

The response of wage ratios depends on the region of the skill distribution one considers, and on the relative size of domestic and foreign market access costs. To gain further insight, I suggest exploiting the differences in the specific channels through which skill-biased technical change and trade integration

³⁸Also note that as technology becomes more skill biased, Assumption 1 tends to be less and less restrictive.

operate at firm level.

Here I focus on the intra-firm rent distribution. The rent created in a firm by a manager and an idea is given by the sum of profits and manager's wage (i.e., the surplus) less their opportunity cost in the alternative occupation. Noting that the assignment (5) allows us to write $z(s)/z(s_d) = s/s_d$, we can use the earning functions (26) and (27) to express the rent for an exporting and a non-exporting firm as

$$\pi(z(s)) + w(s) - \bar{w} \equiv \begin{cases} \left(\frac{s}{s_d}\right)^{1-\psi} - 1 & s \in [s_d, s_x) \\ (1 + \tau^{1-\sigma}) \left(\frac{s}{s_d}\right)^{1-\psi} - (1 + f) & s \geq s_x \end{cases}.$$

The share of this rent that goes to managers is then

$$\frac{w(s) - \bar{w}}{\pi(z(s)) + w(s) - \bar{w}} = \theta$$

where $\theta \equiv \alpha/(\alpha + \kappa)$; a share $1 - \theta$ is then left for profits. This fact is true for any firm, independently of its export status, and irrespectively of the validity of Assumption 1.

The intra-firm rent distribution is only a function of the relative contribution of types to the overall productivity of the firm. In particular, it is not a function of the level of trade integration, not even in exporting firms. The reason is that in equilibrium, the wage function equates the marginal benefits and costs of a better manager in all markets where the firm chooses to sell; hence, a fraction θ of the additional rent that a higher ability generates in each market is given to the manager: while trade integration affects the level of the rents reaped by a firm, it does not affect the way in which this rent is shared.

These observations suggest that a promising avenue for disentangling the two effects is to look at the intra-firm rent distribution. Firm-level data on employers and employees, properly interpreted, can give a handle on the evolution of θ . Changes in inequality not accompanied by changes in the intra-firm rent distribution must be attributed to trade. Vice-versa, changes in the intra-firm rent distribution must imply changes in local inequality and wage ratios caused by skill-biased technical change³⁹.

A simple way to implement this firm-level analysis is reported in Appendix A.8, and involves having data on the payments to production workers, non-production workers and capital at firm level. Admittedly, this strategy is very dependent on the functional form assumptions, and only provides partial

³⁹Bebchuk and Grinstein (2005) actually show that the ratio of payments to top executives over firms' earnings has grown in the period 1993-2003, i.e., θ has increased. They discuss the role of bargaining between executives and directors when directors have interests aligned with those of the shareholders vs. when they do not. While these topics are certainly interesting, they fall well outside the scope of this model. Moreover, looking to top-executives earning is not the only way to approach the empirical implications of this framework. Appendix A.8 discusses an alternative approach.

conditions: for example, a fall in θ does not exclude a role for trade integration. However, the general point I want to illustrate still remains: progress can be made by explicitly spelling out the different mechanisms through which these two forces operate, and focusing on their different implications at firm level.

7 Conclusion

I have shown that local wage inequality responds in similar ways to both skill-biased technical change and trade integration. Both shocks have asymmetric effects across firms, raising the competitive pressure on low productivity ones, while favoring firms in the right tail of the productivity distribution. Because of positive assortative matching, low and high productivity firms are exactly those who hire low and high skill managers, respectively. As a result, both shocks have asymmetric effects across the ability spectrum: skill-biased technical change and trade integration can - under appropriate parameters' restrictions - both reproduce either parallel or divergent patterns of wage ratios in the lower and the upper tail of the wage distribution, thus being consistent with the evidence on wage inequality in the last 50 years in the United States.

This result suggests the value of modeling the labor market implications of these two mechanisms in order to derive explicitly the dependence of the wage function on economy-wide parameters. I argue that by spelling out their different channels of operation, one may derive restrictions on the behavior of observables that can help disentangle the magnitude of the impact of skill-biased technical change and trade integration.

I acknowledge that this model is still too stylized in many respects to attempt a serious quantification of the importance of these two mechanisms. I emphasize that a reasonable parameterization fails to capture both the level and the magnitude of the change in inequality that has been observed in the U.S. since the sixties. However, it has the virtue of uncovering the link between micro-level behavior and the aggregate evolution of wage ratios, emphasizing new avenues of investigation, such as the intra-firm rent distribution.

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A Appendix

A.1 Invariance of the Productivity Distribution

The assumption of -1 on the exponents of the distribution of ideas and managers is without loss of generality: by suitably redefining the units, we can always recover the same distribution of firm-level productivities and contributions of factor types.

Let $G(z) = Tz^{-\xi_z}$ (with $\xi_z \geq 1$) denote the measure of ideas at least as good as z . The contribution of ideas to the firm-level productivity is z^κ , and its distribution satisfies $T \Pr \{z^\kappa > a\} = Ta^{-\xi_z/\kappa}$. We want to show that there exist a \tilde{z} and $\tilde{\kappa}$ such that (1) the measure of ideas \tilde{z} better than any value a is Ta^{-1} and that (2) the distribution of \tilde{z} still assigns to $\tilde{z}^{\tilde{\kappa}}$ the same distribution that z^κ has. Let $\tilde{z} \equiv z^{\xi_z}$, and $\tilde{\kappa} \equiv \kappa/\xi_z$. Hence, the measure of ideas \tilde{z} better than a is $T \Pr \{\tilde{z} > a\} = T \Pr \{z^{\xi_z} > a\} = Ta^{-1}$: \tilde{z} has a distribution with shape parameter 1. Moreover, $T \Pr \{\tilde{z}^{\tilde{\kappa}} > a\} = T \Pr \{\tilde{z} > a^{\xi_z/\kappa}\} = T (a^{\xi_z/\kappa})^{-1}$ which is then equal to the distribution of z^κ . An analogous argument, with $\tilde{s} \equiv s^{\xi_s}$ and $\tilde{\alpha} \equiv \alpha/\xi_s$, establishes the equivalence for the population of managers. Since z^κ and s^α have the same distribution as $\tilde{z}^{\tilde{\kappa}}$ and $\tilde{s}^{\tilde{\alpha}}$, it must also be true that the product of these two variables, $\tilde{z}^{\tilde{\kappa}}\tilde{s}^{\tilde{\alpha}}$, has the same distribution as $z^\kappa s^\alpha$.

A.2 The Local Change in Inequality

For two ability levels $s'' > s'$, we want to study the direction of the change in $w(s'')/w(s')$ as κ increases. Denote with $w_\kappa(s)$ the derivative of the wage function with respect to κ , and with $\varepsilon^{(\kappa)}(s) \equiv \kappa w_\kappa(s)/w(s)$ the point elasticity of the wage. Then, the elasticity of the wage ratio with respect to κ , call it $\varepsilon(s', s'')$, is simply $\varepsilon(s'') - \varepsilon(s')$. Since the choice of the percentiles (and then of the abilities) is arbitrary, it is convenient to express this elasticity as

$$\varepsilon(s', s'') = \int_{s'}^{s''} \frac{\partial \varepsilon(s)}{\partial s} ds .$$

The elasticity of the wage with respect to κ generally varies with the ability level: the function $\partial \varepsilon(s)/\partial s$ describes this dependence. Moreover, its sign will determine if the local contribution of the ability level s is to increase or decrease *all* the wage ratios that contain it. Calculating $\frac{\partial \varepsilon(s)}{\partial s}$ explicitly,

$$\begin{aligned} \frac{\partial \varepsilon(s)}{\partial s} &= \frac{w_s(s)}{w(s)} \eta^{(\kappa)}(s) \\ \eta^{(\kappa)}(s) &\equiv \frac{w_{\kappa s}(s) \kappa}{w_s(s)} - \frac{w_\kappa(s) \kappa}{w(s)} \end{aligned}$$

where $w_s(s)$ is the marginal wage at s and $w_{\kappa s}(s)$ is the cross-partial derivative of the wage function with respect to κ and s . The sign of $\eta^{(\kappa)}(s)$ is what matters to determine the direction of the local change in inequality.

A.3 Skill-Biased Technical Change in Closed Economy

Recall that eq. (16) defines $\eta^{(\kappa)}(s) \equiv \frac{w_{\kappa s}(s)\kappa}{w_s(s)} - \frac{w_{\kappa}(s)\kappa}{w(s)}$.

Differentiating (14) with respect to κ , multiplying by $\kappa/w(s)$, and normalizing \bar{w} to 1, we have

$$\frac{w_{\kappa}(s)\kappa}{w(s)} = \frac{\theta(\sigma-1)\kappa}{\theta(j(s)-1)+1} \left[-\frac{j(s)-1}{1-\psi} + h(s)j(s) \right] \quad (28)$$

$$\frac{w_{\kappa s}(s)\kappa}{w_s(s)} = (\sigma-1)\kappa h(s) \quad (29)$$

with

$$j(s) \equiv \left(\frac{s}{s_c} \right)^{1-\psi}$$

$$h(s) \equiv \ln \frac{s}{s_c} - (\alpha + \kappa) \frac{1}{s_c} \frac{\partial s_c}{\partial \kappa}$$

and where $\theta \equiv \alpha/(\alpha + \kappa)$ and $\psi \equiv 1 - (\sigma - 1)(\alpha + \kappa)$. The function $j(s)$ is always greater than or equal to 1, $j'(s) > 0$, and is such that $\lim_{s \rightarrow s_c} j(s) = 1$, $\lim_{s \rightarrow \infty} j(s) = +\infty$. The function $h(s)$ is always increasing in s and has the properties $\lim_{s \rightarrow s_c} h(s) = -(\alpha + \kappa) \frac{1}{s_c} \frac{\partial s_c}{\partial \kappa} < 0$, since the cutoff s_c is increasing in κ (this is immediate from eq. (12)), and $\lim_{s \rightarrow \infty} h(s) = +\infty$; hence, $h(s)$ crosses zero only once.

Proposition 1 *There exists a unique skill level $s^{(\kappa)} > s_c$ such that the local change in inequality from skill-biased technical change is positive for high abilities and negative for low abilities, i.e., $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$.*

Proof. For $\eta^{(\kappa)}(s) > 0$ is necessary and sufficient that $\frac{w_{\kappa s}(s)\kappa}{w_s(s)} - \frac{w_{\kappa}(s)\kappa}{w(s)}$ is positive. Using (28) and (29) in this difference and rearranging, we have

$$\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow \frac{w_{\kappa s}(s)\kappa}{w_s(s)} - \frac{w_{\kappa}(s)\kappa}{w(s)} > 0 \Leftrightarrow$$

$$(1 - \theta)h(s) > -\frac{1}{1 - \psi} \theta (j(s) - 1) .$$

The left-hand side starts at $(1 - \theta)h(s_c) < 0$ and always increases with s , crossing zero only once, while

the right-hand side starts in zero and always decreases with s . Hence, there is one and only one $s^{(\kappa)}$ such that $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$. This $s^{(\kappa)}$ is the unique solution of $(1 - \theta)h(s^{(\kappa)}) = -\frac{1}{1-\psi}\theta(j(s^{(\kappa)}) - 1)$. ■

Note that if $h(s_c)$ were zero, the left- and right-hand side would touch for $s = s_c$ and then diverge from each other, so that we would have $s^{(\kappa)} = s_c$ and no region with a negative local change in inequality.

A.4 Equilibrium in an Open Economy

Following the steps indicated in Section 4, simple calculations deliver the earning functions in each country corresponding to (6) and (7) in open economy:

$$\pi(z) = \begin{cases} (1 - \theta) \left(\frac{t^{-\alpha}}{\bar{w}}\right)^{\sigma-1} M \left(z^{1-\psi} - z_d^{1-\psi}\right) & z \in [z_d, z_x) \\ (1 - \theta) \left(\frac{t^{-\alpha}}{\bar{w}}\right)^{\sigma-1} M \left[\left(z^{1-\psi} - z_d^{1-\psi}\right) + \tau^{1-\sigma} \left(z^{1-\psi} - z_x^{1-\psi}\right)\right] & z \geq z_x \end{cases} \quad (30)$$

$$w(s) = \begin{cases} \theta \left(\frac{t^\kappa}{\bar{w}}\right)^{\sigma-1} M \left(s^{1-\psi} - s_d^{1-\psi}\right) + \bar{w} & s \in [s_d, s_x) \\ \theta \left(\frac{t^\kappa}{\bar{w}}\right)^{\sigma-1} M \left[\left(s^{1-\psi} - s_d^{1-\psi}\right) + \tau^{1-\sigma} \left(s^{1-\psi} - s_x^{1-\psi}\right)\right] + \bar{w} & s \geq s_x \end{cases} . \quad (31)$$

To connect the selection of domestic and foreign sellers, I set the surplus in (17) and (18) to \bar{w} and 0, respectively, to characterize s_d and s_x ; substituting the assignment function (5), solving both expressions for M and equating them, I obtain $s_x = (\tau^{\sigma-1}f)^{1/(1-\psi)} s_d$, which is eq. (19).

The price index in open economy is $P = \frac{\sigma}{\sigma-1} \psi^{1/(\sigma-1)} L t^{-\kappa} \bar{w} \left[s_d^{-\psi} + \tau^{1-\sigma} s_x^{-\psi}\right]^{1/(1-\sigma)}$. I use (19) to eliminate $s_x^{-\psi}$ and get the expressions (20) and (21).

To obtain the earning functions (26) and (27) I use the assignment function (5) in the surplus (17), impose equality to \bar{w} , and express the market size M as

$$M = \bar{w}^\sigma t^{-\kappa(\sigma-1)} s_d^{-(\alpha+\kappa)(\sigma-1)} = \bar{w}^\sigma t^{\alpha(\sigma-1)} z_d^{-(\alpha+\kappa)(\sigma-1)}$$

I use this expression to substitute out M and equation (19) to eliminate the exporters' cutoffs in the profit and wage functions (30) and (31): this last step delivers (26) and (27).

A.5 Local Inequality and Trade Integration

Differentiating (27) with respect to τ , and using $j(s) \equiv (s/s_d)^{(\alpha+\kappa)(\sigma-1)}$, the elasticity of wage to τ is a piecewise function of the form:

$$w_\tau(s) \frac{\tau}{w(s)} = \begin{cases} -(1-\psi) \frac{\theta j(s)}{\theta(j(s)-1)+1} \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau} & s \in (s_d, s_x) \\ -(\sigma-1) \left[(\alpha+\kappa) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau} (1+\tau^{1-\sigma}) + \tau^{1-\sigma} \right] \frac{\theta j(s)}{\theta[j(s)(1+\tau^{1-\sigma})-(1+f)]+1} & s > s_x \end{cases} \quad (32)$$

The elasticity of the marginal price of skills is

$$w_{\tau s}(s) \frac{\tau}{w_s(s)} = \begin{cases} -(1-\psi) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau} & s \in (s_d, s_x) \\ -(\sigma-1) \left[(\alpha+\kappa) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau} (1+\tau^{1-\sigma}) + \tau^{1-\sigma} \right] \frac{1}{(1+\tau^{1-\sigma})} & s > s_x \end{cases} \quad (33)$$

Also, note that the square bracket $\left[(\alpha+\kappa) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau} (1+\tau^{1-\sigma}) + \tau^{1-\sigma} \right]$ in the exporter sections of each function is positive, since, differentiating (24) and using $\delta \equiv \tau^{1/(\alpha+\kappa)} f^{\psi/(1-\psi)}$

$$\begin{aligned} (\alpha+\kappa) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau} (1+\tau^{1-\sigma}) + \tau^{1-\sigma} &= -\frac{1}{1+\delta} (1+\tau^{1-\sigma}) + \tau^{1-\sigma} > 0 \Leftrightarrow \\ (1+\delta) \tau^{1-\sigma} &> 1 + \tau^{1-\sigma} \Leftrightarrow (\tau^{\sigma-1} f)^{\psi/(1-\psi)} > 1. \end{aligned}$$

which is always true.

Proposition 2 *There exists a unique skill level $s^{(\tau)} = s_x$ such that the local change in inequality is positive for high abilities and negative for low abilities, i.e., $\eta^{(\tau)}(s) \leq 0 \Leftrightarrow s \geq s^{(\tau)}$.*

Proof. Local inequality increases with trade integration if and only if $\eta^{(\tau)}(s) < 0$.

For domestic sellers, using (32) and (33), this will happen if and only if

$$\begin{aligned} w_{\tau s}(s) \frac{\tau}{w_s(s)} < w_\tau(s) \frac{\tau}{w(s)} &\Leftrightarrow -(1-\psi) \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau} < \frac{\theta(\psi-1)j(s)}{\theta(j(s)-1)+1} \frac{\tau}{s_d} \frac{\partial s_d}{\partial \tau} \Leftrightarrow \\ 1 < \frac{\theta j(s)}{\theta(j(s)-1)+1} &\Leftrightarrow 1-\theta < 0 \end{aligned}$$

which is never true. Hence, local inequality always decreases for domestic sellers.

For exporters, using (32) and (33) local inequality increases if

$$\begin{aligned} w_{\tau s}(s) \frac{\tau}{w_s(s)} < w_\tau(s) \frac{\tau}{w(s)} &\Leftrightarrow \\ \frac{-(\sigma-1) \left[(\alpha + \kappa) \frac{1}{s_d} \frac{\partial s_d}{\partial \tau} (1 + \tau^{1-\sigma}) + \tau^{-\sigma} \right] \tau}{(1 + \tau^{1-\sigma})} < \frac{-\theta j(s) (\sigma-1) \left[(\alpha + \kappa) \frac{1}{s_d} \frac{\partial s_d}{\partial \tau} (1 + \tau^{1-\sigma}) + \tau^{-\sigma} \right] \tau}{\theta [j(s) (1 + \tau^{1-\sigma}) - (1 + f)] + 1} \end{aligned}$$

Since the term in the square bracket is always positive, we can simplify further to obtain

$$\begin{aligned} w_{\tau s}(s) \frac{\tau}{w_s(s)} < w_\tau(s) \frac{\tau}{w(s)} &\Leftrightarrow \frac{1}{(1 + \tau^{1-\sigma})} > \frac{\theta j(s)}{\theta [j(s) (1 + \tau^{1-\sigma}) - (1 + f)] + 1} \\ \Leftrightarrow 1 - \theta(1 + f) > 0 &\Leftrightarrow f < \frac{\kappa}{\alpha} \end{aligned}$$

Hence, local inequality increases for exporters as trade barriers fall if and only if $f < \kappa/\alpha$. ■

A.6 Local Inequality and Skill-Biased Technical Change in Open Economy

The elasticity of wage to skill is now a piecewise function of the form

$$\frac{\kappa}{w(s)} \frac{\partial w(s)}{\partial \kappa} = \begin{cases} -(1 - \theta) g_1(s) + g_2^d(s) (\sigma - 1) \kappa h(s, \tau) & s \in (s_d, s_x) \\ -(1 - \theta) g_1(s) + g_2^x(s) (\sigma - 1) \kappa h(s, \tau) & s > s_x \end{cases} \quad (34)$$

with

$$\begin{aligned} h(s, \tau) &\equiv \left[\ln \frac{s}{s_d} - (\alpha + \kappa) \frac{1}{s_d} \frac{\partial s_d}{\partial \kappa} \right] \\ g_1(s) &\equiv \frac{w(s) - 1}{w(s)}, \\ g_2^d(s) &\equiv \frac{w(s) - (1 - \theta)}{w(s)}, \quad g_2^x(s) \equiv \frac{w(s) - [1 - \theta(1 + f)]}{w(s)} \end{aligned} \quad (35)$$

and the dependence on τ through s_d is always left implicit.

The elasticity of the marginal price of skills to κ is always

$$\frac{\kappa}{w_s(s)} w_{\kappa s}(s) = (\sigma - 1) \kappa h(s) .$$

For a domestic seller, $s \in (s_d, s_x)$, local inequality increases if and only if

$$\begin{aligned} \frac{\kappa}{w_s(s)} w_{\kappa s}(s) &> \frac{\kappa}{w_x(s)} \frac{\partial w_x(s)}{\partial \kappa} \Leftrightarrow \\ h(s, \tau) &> -\frac{\theta}{1-\theta} \frac{1}{1-\psi} (j(s) - 1) \equiv rhs_d(s, \tau) . \end{aligned} \quad (36)$$

In particular, as $s \rightarrow s_d$, the inequality is never satisfied: for the worst managers, local inequality always decreases. For an exporter $s \geq s_x$, local inequality increases if and only if

$$\begin{aligned} \frac{\kappa}{w_s(s)} w_{\kappa s}(s) &> \frac{\kappa}{w_x(s)} \frac{\partial w_x(s)}{\partial \kappa} \Leftrightarrow \\ h(s, \tau) &> -\frac{\theta}{1-\theta(1+f)} \frac{1}{1-\psi} [(1 + \tau^{1-\sigma}) j(s) - (1+f)] \equiv rhs_x(s, \tau) . \end{aligned} \quad (37)$$

I distinguish two cases, based on whether or not Assumption 1 holds.

Case 1: Assumption 1 holds, $f < \kappa/\alpha$

Proposition 3 *There exists a unique skill level $s^{(\kappa)} > s_d$ such that the local change in inequality from skill-biased technical change is positive for high abilities and negative for low abilities, i.e., $\eta^{(\kappa)}(s) \geq 0 \Leftrightarrow s \geq s^{(\kappa)}$.*

Proof. Note that in both (36) and (37), (i) $\partial h(s, \tau) / \partial s > 0$ and $\lim_{s \rightarrow +\infty} h(s, \tau) = +\infty$, while (ii) $\partial rhs_i(s, \tau) / \partial s < 0$ and $\lim_{s \rightarrow +\infty} rhs_i(s, \tau) = -\infty$, for $i \in \{d, x\}$.⁴⁰ Hence, for the values of s where each relation applies: if the inequality is satisfied for an s' , it is also satisfied for all $s > s'$; if it is not satisfied for an s' , it is also not true for all $s < s'$. To prove the existence of this threshold and identify the region it falls in, I check the value of each inequality at $s = s_x$. Then, after some algebra, define:

$$\begin{aligned} lhs(\tau) \equiv h(s_x, \tau) &= \frac{\ln f^{1/(1-\psi)} \tau^{1/(\alpha+\kappa)}}{1 + f^{-\psi/(1-\psi)} \tau^{-1/(\alpha+\kappa)}} - \frac{\alpha + \kappa}{1 + (\sigma - 1)/\psi} \left(\frac{\sigma - 1}{\psi} \right)^2 \\ rhs_d(\tau) \equiv rhs_d(s_x, \tau) &= -\frac{\theta}{1-\theta} \frac{1}{1-\psi} (f\tau^{\sigma-1} - 1) \\ rhs_x(\tau) \equiv rhs_x(s_x, \tau) &= -\frac{\theta}{1-\theta(1+f)} \frac{1}{1-\psi} (f\tau^{\sigma-1} - 1) . \end{aligned}$$

These functions describe the left-hand side and the right-hand side of (36) and (37) when $s = s_x$.

Note that $rhs_d(\tau) > rhs_x(\tau) \forall \tau$. The function $lhs(\tau)$ can be in three positions with respect to this inequality.

⁴⁰The function $rhs_x(s, \tau)$ is decreasing because, by assumption, $f < \kappa/\alpha \Leftrightarrow [1 - \theta(1+f)] > 0$.

(i) Suppose that $rhs_d(\tau) > rhs_x(\tau) > lhs(\tau)$: at $s = s_x$ inequality (37) is not satisfied, and the local inequality for the worst exporters is decreasing; then, it is also decreasing among the all domestic sellers, and there must exists a threshold $s^{(\kappa)} > s_x$ such that local inequality decreases below it and increases above it.

(ii) Suppose that $lhs(\tau) > rhs_d(\tau) > rhs_x(\tau)$: at $s = s_x$ inequality (36) is satisfied, and local inequality for the best domestic sellers is increasing; then, local inequality is also increasing among all exporters, and there must exists a threshold $s^{(\kappa)} \in (s_d, s_x)$ such that local inequality increases above it and decreases below.

(iii) Suppose that $rhs_d(\tau) > lhs(\tau) > rhs_x(\tau)$: at $s = s_x$, (36) is not satisfied while (37) is, and so local inequality increases for all exporters and decreases for all non-exporters. In this case, $s^{(\kappa)} = s_x$. ■

Note that $s^{(\kappa)} = s_x$ (case (iii)) is not a knife edge case. Since $rhs_d(\tau) > rhs_x(\tau)$ holds strictly and $lhs(\tau)$ changes continuously with τ , $s^{(\kappa)} = s_x$ occurs for $\tau \in (\tau_d, \tau_x)$, with $\tau_d : rhs_d(\tau_d) = lhs(\tau_d)$ and $\tau_x : rhs_x(\tau_x) = lhs(\tau_x)$.

To show that each of the cases (i)-(iii) can occur along a path of trade integration, suppose $f = 1$. Then, $\lim_{\tau \rightarrow 1} lhs(\tau) < 0$, $\lim_{\tau \rightarrow 1} rhs_d(\tau) = \lim_{\tau \rightarrow 1} rhs_x(\tau) = 0$. Since, as $\tau \rightarrow \infty$, $lhs(\tau)$ goes monotonically to $+\infty$ while both rhs functions go monotonically to $-\infty$, $lhs(\tau)$ will cross once $rhs_x(\tau)$ and then $rhs_d(\tau)$ from below. In this construction, moving τ from autarky ($+\infty$) to perfect integration ($\tau = 1$) will let the economies visit case (ii), (iii) and finally (i).

Case 2: Assumption 1 does not hold, $f > \kappa/\alpha$.

Assumption 1 does not modify the properties of $h(s, \tau)$ and $rhs_d(s, \tau)$: hence, local inequality always decreases for domestic sellers. When $f > \kappa/a$, however,

$$\frac{\kappa}{w_s(s)} w_{\kappa s}(s) > \frac{\kappa}{w_x(s)} \frac{\partial w_x(s)}{\partial \kappa} \iff h(s, \tau) < \frac{\theta}{\theta(1+f) - 1} \frac{1}{1-\psi} [(1 + \tau^{1-\sigma}) j(s) - (1 + f)]$$

where now $\theta(1+f) - 1 > 0$. Hence, the right-hand side is now positive and increasing in s , and goes to $+\infty$ as $s \rightarrow \infty$. To show that local inequality always increases among the best exporters, note that this inequality is of the form

$$\ln s - A_1 < A_2 s^{1-\psi} - A_3$$

with A_1, A_2 and A_3 positive constants⁴¹. Since $1 - \psi > 0$, for s large enough it is always satisfied, so that local inequality always increases also among the best exporters.

⁴¹In particular, $A_1 \equiv \ln s_d + (\alpha + \kappa) \frac{1}{s_d} \frac{\partial s_d}{\partial \kappa}$, $A_2 \equiv \frac{\theta}{\theta(1+f)-1} \frac{1}{1-\psi} (1 + \tau^{1-\sigma}) s_d^{-(1-\psi)}$ and $A_3 \equiv \frac{\theta}{\theta(1+f)-1} \frac{1}{1-\psi} (1 + f)$.

For intermediate values of s , parameters can be found such that the local inequality for the worst exporters decreases or increases. The accompanying online *Mathematica* notebook plots both sides of the inequality against s for $s \geq s_x$ under two alternative parameter combinations to show it⁴².

A.7 Local Inequality and Wage Ratios in Open Economy

The elasticity of the wage ratio $w(s'')/w(s')$ with respect to $i \in \{\tau, \kappa\}$ is always the difference between the elasticities of the wage evaluated at the two abilities, $\varepsilon^{(i)}(s'') - \varepsilon^{(i)}(s')$. The function $\varepsilon^{(i)}(s')$ is discontinuous at $s = s_x$. I prove this by directly showing $\lim_{s \rightarrow s_x^+} \varepsilon^{(i)}(s) - \lim_{s \rightarrow s_x^-} \varepsilon^{(i)}(s)$.

For $i = \tau$, using (32),

$$D^{(\tau)} \equiv \lim_{s \rightarrow s_x^+} \varepsilon^{(\tau)}(s) - \lim_{s \rightarrow s_x^-} \varepsilon^{(\tau)}(s) = -\theta f \frac{\delta}{\delta + 1} \frac{\sigma - 1}{w(s_x)}$$

which is always negative. For $i = \kappa$, using (34), after some algebra I obtain

$$D^{(\kappa)} \equiv \lim_{s \rightarrow s_x^+} \varepsilon^{(\kappa)}(s) - \lim_{s \rightarrow s_x^-} \varepsilon^{(\kappa)}(s) = \theta f \frac{\kappa(\sigma - 1)}{w(s_x)} h(s_x, \tau)$$

where $h(s_x, \tau)$ is given in (35). It is easy to show that $(\sigma - 1)h(s, \tau) \equiv \partial(s/s_d)^{1-\psi} / \partial \kappa$, so that $h(s_x, \tau)$ is the derivative of $(s/s_d)^{1-\psi}$ for the marginal manager s_x . Note that in principle, $h(s_x, \tau)$ can be positive or negative, and so $D^{(\kappa)} \leq 0 \iff h(s_x, \tau) \leq 0$. The accompanying online *Mathematica* notebook plots $\varepsilon^{(i)}(s)$ under two alternative parameter combinations to show it.

Accounting for the discontinuity, we can rewrite the difference between the two elasticities in terms of local change in inequality as

$$\begin{aligned} \varepsilon^{(i)}(s'') - \varepsilon^{(i)}(s') &= \int_{s'}^{s_x} \frac{\partial \varepsilon^{(i)}(s)}{\partial s} ds + I^{(i)}(s', s'') D^{(i)} + \int_{s_x}^{s''} \frac{\partial \varepsilon^{(i)}(s)}{\partial s} ds = \\ &= \int_{s'}^{s_x} \frac{w_s(s)}{w(s)} \eta^{(i)}(s) ds + I^{(i)}(s', s'') D^{(i)} + \int_{s_x}^{s''} \frac{w_s(s)}{w(s)} \eta^{(i)}(s) ds \end{aligned}$$

with $I^{(i)}(s', s'') = 1$ if $s_x \in (s', s'')$, and 0 otherwise.

⁴²In the notebook, I plot $h(s)$ making use of the fact that, after some algebra, $\frac{\alpha + \kappa}{s_d} \frac{\partial s_d}{\partial \kappa} = \frac{\alpha + \kappa}{1 + (\sigma - 1)/\psi} \left(\frac{\sigma - 1}{\psi} \right)^2 + \frac{1}{\delta + 1} \frac{1}{1 - \psi} (\ln \tau^{\sigma - 1} f)$.

A.8 A Simple Implementation of the Intra-Firm Rent Distribution

Let the total value added of a firm φ be $VA(\varphi)$, and denote with $W_s(\varphi)$ and $W_u(\varphi)$ the total payments to skilled and unskilled workers, respectively. Assume that a firm with $L_s(\varphi)$ skilled workers is actually a set of $L_s(\varphi)$ production lines, i.e., each skilled worker would be a manager. The surplus $v(\varphi)$ in the model is then $\hat{v}(\varphi) \equiv (VA(\varphi) - W_u(\varphi)) / L_s(\varphi)$. To move from the surplus to the rent, we must have an estimate of the opportunity cost of the skilled workers: we can use the wage of an unskilled worker in the same firm, $\hat{w}_u(\varphi) \equiv W_u(\varphi) / L_u(\varphi)$. Hence, the average rent in a firm is $\hat{v}(\varphi) - \hat{w}_u(\varphi)$, and the share of this rent going to the "manager" would be estimated to $\hat{\theta}(\varphi) \equiv (\hat{w}_s(\varphi) - \hat{w}_u(\varphi)) / (\hat{v}(\varphi) - \hat{w}_u(\varphi))$, where $\hat{w}_s(\varphi) \equiv W_s(\varphi) / L_s(\varphi)$ is the average wage for skilled workers. It is easy to show that for $\hat{\theta}(\varphi) \in (0, 1)$, we need that the wage per skilled worker is larger than the wage per unskilled worker and that there are some payments to capital⁴³.

The model would predict that this share is constant across firms, and is not affected by trade integration, so that changes in the average θ over time would measure the intensity of skill-biased technical change.

⁴³Note that, substituting the definitions given in the text, $\hat{\theta}(\varphi) \equiv \frac{W_s(\varphi)/L_s(\varphi) - W_u(\varphi)/L_u(\varphi)}{(VA(\varphi) - W_u(\varphi))/L_s(\varphi) - W_u(\varphi)/L_u(\varphi)}$. Assume that the numerator in $\hat{\theta}(\varphi)$ is positive (i.e., the average wage for skilled workers is larger than the average wage for unskilled workers); then the estimated share will be smaller than 1 as long as

$$(VA(\varphi) - W_u(\varphi)) / L_s(\varphi) - W_s(\varphi) / L_s(\varphi) > 0 \iff VA(\varphi) > W_u(\varphi) + W_s(\varphi)$$

i.e., as long as there are some payment to capital. If this is true, then it is also sufficient for $\hat{\theta}(\varphi)$ to be positive, since the requirement would be $VA(\varphi) / L_s(\varphi) > W_u(\varphi) (1/L_u(\varphi) + 1/L_s(\varphi)) \iff VA(\varphi) > \hat{w}_u(\varphi) (L_s(\varphi) + L_u(\varphi))$.