# Fantastic Beasts and Where to Find Them*

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**Abstract**

Fantastic beasts are magical creatures that cannot be seen unless one looks for them with the eye of the wizard, but that still play a significant role in the world. The fantastic beasts we hunt for and find in the present paper are welfare changes induced by resource shocks that are invisible in quantitative trade models with monopolistic competition and heterogeneous firms if one relies on the pervasive assumption of demand exhibiting constant elasticity of substitution. We argue that, for fantastic beasts to materialize, markups have to vary across firms and firm heterogeneity has to vary across sectors. This is shown both theoretically and empirically exploiting a panel of 76 countries and 17 manufacturing industries for the period 1995-2020.

**KEY WORDS**: Quantitative trade models, Variable Markups, Incomplete Pass-through, Resource Shocks, Immiserizing Growth.

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"[E]conomists should not emancipate themselves from the tyranny of Cobb-Douglas only to enchain themselves in a new Solow CES tyranny."

(Samuelson, 1965; p.346)

"[T]he amusing creatures described hereafter are fictional and cannot hurt you."

(Dumbledore, 2001; p.viii).

# 1 Introduction

Fantastic beasts are magical creatures that cannot be seen unless one looks for them with the eye of the wizard, but still play a significant role in the world, sometimes a fascinating one, other times a dangerous one. The fantastic beasts we are after in the present paper are welfare changes induced by shocks that one fails to see in quantitative trade models with monopolistic competition and heterogeneous firms based on the pervasive assumption of demand exhibiting constant elasticity of substitution (Costinot and Rodriguez-Clare, 2014).

As Samuelson's metaphor implies, constant elasticity of substitution (CES) entails several intertwined restrictions with important implications for welfare analysis. While these have been systematically discussed in closed or fully integrated economy by the recent literature, what one might be missing when evaluating the welfare effects of various shocks in an open economy with trade friction has still to be fully understood.[1] A contribution in this direction can be found in Arkolakis et al. (2019), who show that welfare responses tend to be less pronounced with variable elasticity of substitution (VES).

The shocks we focus on are "resource shocks", which we introduce following a classical interpretation in the spirit of Houthakker (1955), Rybczynski (1955), Solow (1956, 1957) and Jones (1971), according to which, despite that in a Ricardian setup labor is the only explicit input, one may think that there are implicitly other "missing factors", which consist of country- and sector-specific resources that are complementary to labor. These resources are available in fixed endowments, cannot be consumed, produced, nor used for entry, and cannot be traded. Complementarity implies that an exogenous

[1] For a closed economy with a single sector, Dhingra and Morrow (2019) show that, whereas with CES the market equilibrium is constrained efficient, without CES it generally differs from the social optimum. In the same vein, Behrens et al. (2020) characterize the market equilibrium of a multi-sector economy in a non-CES environment and document substantial inefficiency. When the elasticity of substitution differs across sectors, resources are inefficiently allocated across them even with CES with substantial quantitative implications for welfare (see, e.g., Donaldson, 2018).

increase in the endowment of a country's sector-specific resource determines a sector-biased outward shift of its production possibility frontier, which we will also refer to as "growth". This notion of resource shock is also consistent with Costinot et al. (2012). When firms are heterogeneous within sectors, it translates into an increase in the lower bound of the support of their productivity distribution (or, equivalently, in a decrease in the support of the distribution of their unit input requirements).

To highlight the restrictions associated with the CES assumption, let us introduce some definitions (Nocco et al., 2024). Call "absolute markup" the difference between a firms' profit-maximizing price and its marginal cost, and "relative markup" the ratio of the profit-maximizing price to the marginal cost. Then use "absolute pass-through" to refer to the derivative of the profit-maximizing price to the marginal cost, and "relative pass-through" to refer to the corresponding percentage change, that is, the derivative of the logarithm of the profit-maximizing price to the logarithm of the marginal cost. Under CES, the relative markup, the absolute pass-through and the relative pass-through are all constant and common across firms. Only the absolute markup varies and increases with marginal cost, which implies that in equilibrium it is larger for less productive firms as these have higher marginal cost. In addition, both the absolute and the relative pass-throughs are also constant and common across firms. However, while the former is larger than one, the latter is equal to one, which is what the literature refers to as "complete pass-through". These restrictions at the firm level lead to restrictions at the sectoral level. For example, under CES the relative pass-through from the upper bound of the firm productivity distribution to the average price (which we may call relative "growth pass-through") is also complete if firm productivity is Pareto distributed.[2]

We show that a minimal departure from CES allowing for variable relative markup, while retaining constant though incomplete absolute pass-through, is enough to unveil unexpected welfare effects due to incomplete relative "growth pass-through" (IPT) without sacrificing the tractability of quantitive CES models. The minimal departure entails a horizontal translation of the CES demand curve such that it ends up intersecting the vertical axis at a "choke price" above which the quantity demanded is null.

[2]In general, with firm selection, one may think of two types of relative pass-through at the sectoral level (holding input prices constant): from the unconditional average unit input requirement to the conditional average unit input requirement and thereby to the average marginal cost, and from the latter to the average sectoral price. With CES and Pareto, both types are complete, whereas with VES and Pareto only the second type is complete (Huang and Ottaviano, 2024). In contrast, at the firm level, there is only one type of relative pass-through from marginal cost to price, which is complete with CES and incomplete with VES.

The model we rely on features an arbitrary number of sectors that differ in terms of firm heterogeneity and an arbitrary number of countries that differ in terms of their comparative advantage across sectors. It is Ricardian in the sense that labor is the only input and comparative advantage is driven by differences in unit labor requirements. In particular, average requirements vary across sectors, but may still exhibit some statistical overlaps across them due to random dispersion around the mean as in Melitz (2003).

While our model is ready for full-fledged quantitative exercises based on calibration, validation and simulation of all kinds of counterfactual scenarios, in this paper we use its elegant properties in terms of sufficient statistics to construct resource shocks to specific sectors of a country's economy that have no impact on its welfare under CES but sizeable welfare effects under IPT. This can be done as long as sectors differ with regard to the concentration of their firms' unit labor requirements around the sectoral means, which we call "technological concentration" for short.

We target two types of fantastic beasts, which we refer to as "immiserizing growth" and "enriching decline". In the former case, a domestic resource increase that does not change welfare under CES leads to lower welfare under IPT. In the latter case, a domestic resource reduction that does not change welfare under CES leads to higher welfare under IPT. The reason for these divergences is that IPT allows for richer reallocation patterns between firms and sectors than CES does, due to variable markups and incomplete pass-through. The constancy of the pass-through is not essential. However, together with the assumption that firms' labor input requirements are (inverse) Pareto distributed, it generates a simple expression of national welfare as a function of a very limited number of sufficient statistics. The Pareto assumption also buys the opportunity of measuring technological concentration through a single exogenous parameter. This assumption has, nonetheless, its own restrictive implications, which we point out by initially developing the model under general regularity conditions on the distribution of firms' labor input requirements without relying on any specific parametrization.

Using "CES-neutral" to refer to a resource shock that does not change welfare under CES, the main results can be summarized as follows. If an expansionary CES-neutral domestic resource shock hits a sector with low technological concentration, a country may still experience immiserizing growth under IPT, that is, welfare losses. Vice versa, if a contractionary CES-neutral domestic resource shock hits a sector with low technological concentration, the country may still experience enriching decline under IPT, that is, welfare gains. These results are derived both theoretically and empirically for resource shocks of realistic magnitude as proof of concept.

Our results contribute to three main lines of research. Firstly, they contribute to the literature on the gains from trade in quantitative trade models and, in particular, to the ongoing debate about "new gains from trade"in models with imperfect competition and firm heterogeneity (see, e.g., Arkolakis et al., 2012; Melitz and Redding, 2015; Arkolakis et al., 2019). The closest paper to ours is Arkolakis et al. (2019) as our demand system belongs to the class of demand systems they use to quantify the pro-competitive effects of trade. However, to find our fantastic beasts, one has to allow for technological concentration to differ across sectors, which they prevent by assumption to focus on within- rather than between-sector distortions.

From a different angle, our model brings income effects into quasi-linear models with constant absolute pass-through that have been used for trade policy analysis (see, e.g., Melitz and Ottaviano, 2008; Nocco et al., 2019; Nocco et al., 2024). In doing so, it offers a quantifiable open-economy implementation of the setup with additive separable utility, income effects, variable markups and constant absolute pass-through recently put forth by Melitz et al. (2024) for analyzing non-discriminatory industrial policy in closed economy.

Secondly, our findings contribute to the vast literature on the effects of resource shocks, which have been investigated in various setups since early contributions on immiserizing growth (Bhagwati, 1958, Johnson, 1967, Bhagwati, 1968) and the Dutch disease (Corden and Neary, 1982). More generally, it adds to existing studies of the resource curse (see, e.g. Ploeg, 2011, and Ploeg and Poelhekke, 2019, for surveys) and structural change (see, e.g. Kohn et al., 2021). However, while the literature typically identifies quite extreme theoretical conditions for the emergence of immiserizing growth, we show that new trade models with monopolistic competition, firm heterogeneity and variables elasticity of substitution can easily rationalize what has been so far considered a paradoxical result. In this respect, it worthwhile stressing that our definition of "immiserizing growth"differs from the traditional one in that it refers to the welfare impact of a CES-neutral resource shock rather than that of a generic resource shock. It is, therefore, a relative rather than an absolute definition, which is instrumental to show that the analysis of resource shocks with or without CES may produce results that differ not only in magnitude, but also in sign. This is closely related to the motivations of recent studies of structural change, such as Matsuyama (2019) and

Comin et al. (2021), where non-homothetic preferences are shown to play a crucial role.

Thirdly, the paper complements works investigating the implications of assuming a Pareto distribution of firm productivity for predicting trade flows and welfare changes (see, e.g., Head and Mayer, 2014, Bas et al., 2017, Nigai, 2017, Mrázová et al., 2021). These works document the restrictions that the assumption imposes on trade flows (such as constant trade elasticity and adjustment only at the extensive margin), the potential downward bias for trade-induced welfare gains, and the possible misrepresentation of the observed empirical relationship between sales and productivity. By deriving the patterns of trade and welfare predicted by our model for a generic productivity distribution, we can highlight an additional restriction. Specifically, the Pareto assumption conceals the importance of price dispersion (around the demand choke price) as a determinant of the responses of trade and welfare to resource shocks.

The rest of the paper is organized as follows. Section 2 introduces the model with additive-separable VES preferences and a general distribution of technologies. Without making the (inverse) Pareto assumption on the distribution of firms' unit labor requirements, it highlights general and distinctive properties of the setup. Section 3 adds the Pareto assumption and derives the equilibrium of the model, proving existence and uniqueness. It also characterizes the model's "welfare formula" and gravity equation, and its relation to the existing quantitative trade models. Section 4 analyzes the welfare effects of resource shocks and the necessary conditions for the fantastic beasts to materialize. Section 5 hunts for and finds the fantastic beasts in a panel of 76 countries and 17 manufacturing industries in the period 1995-2020. Section 6 concludes.

## 2 A multi-country and multi-sector open economy

There are countably many countries and sectors: indexes $j = 1, ..., J$ indicate a country as a source of supply, indexes $l = 1, ..., J$ indicate a country as a source of demand, and indexes $z = 1, ..., Z$ indicate a sector. Consumption goods are traded across countries. In each country a continuum of varieties of a differentiated consumption good, indexed by $i \in [0, N_l(z)]$, is consumed, where $N_l(z)$ is the measure of varieties of goods in sector $z$ available for consumption in country $l$.

Varieties are supplied by monopolistically competitive firms. Each firm is active in only one country and only one sector, and employs labor to produce one and only one variety under constant returns to scale. Labor is the only input, it is homogeneous, perfectly mobile across sectors but not mobile across countries. Firm entry is unrestricted but costly: producers willing to enter in a country $j$ and sector $z$ pay an exogenous sunk cost in terms of $f_j(z) > 0$ labor units, to develop a new technology in that country and sector pair. After this payment, a firm realizes its idiosyncratic conversion rate of labor per unit of output, as a random draw $c > 0$ from a continuous c.d.f. $G_j(c; z)$ that is specific to country $j$ and sector $z$. After making a successful entry, firms producing in country $j$ might export to any other country $l$ facing a sector-specific iceberg trade costs $\tau_{jl}(z) \geq 1$.

Call $N_j^E(z)$ the measure of entrants in country $j$ sector $z$, then $M_j(z) \leq N_j^E(z)$ is the measure of varieties produced in country $j$ of goods in sector $z$, and only a subset $N_{jl}(z) \leq M_j(z)$ of them is shipped to country $l$. Thus, the measure of available varieties (domestic and imported) in a certain market $l$ is given by $N_l(z) \equiv \sum_{j=1}^{J} N_{jl}(z)$.

## 2.1 Consumers' behavior

In every country $l = 1, ..., J$, preferences are represented by a Cobb-Douglas aggregator across sectors $z = 1, ..., Z$, while consumption bundles of varieties within a sector are ranked according to additive-separable preferences characterized by variable elasticity of substitution:[3]

$$U_l = \prod_{z=1}^{Z} \left[ \sum_{j=1}^{J} \left( \int_0^{N_{jl}(z)} \alpha q_{jl}^c(i;z) - \frac{\gamma}{1-\sigma} q_{jl}^c(i;z)^{1-\sigma} \, di \right) \right]^{\beta(z)} \quad \alpha > 0, \gamma > 0, \sigma < 0, \tag{1}$$

where $q_{jl}^c(i;z)$ is the quantity of good $i$ from sector $z$ produced in country $j$ and consumed in country $l$ while $\beta(z) \in (0,1)$ is a sector-specific share such that $\sum_{z=1}^{Z} \beta(z) = 1$. The sub-utility representing preferences across varieties within a sector is a special case of the class of preferences introduced by Bulow and Pfleiderer (1983), which is defined over the extended parameter space $\alpha \leq 0$ or $\alpha > 0$, $\gamma \neq 0$ and $\sigma < 1$. This class, called "BP" hereafter, implies constant absolute pass-through $1/(1-\sigma)$ from marginal cost to

[3] Two remarks are in order. First, we discuss the case of a Cobb-Douglas aggregator across sectors for the sake of exposition, but the analysis goes through for every homothetic aggregator. Second, $\alpha$ is the marginal utility of any variety when its consumption is null. For $\alpha > 0$, one may simply think of $\alpha \equiv 1$ without loss of generality: as preferences are not heterogeneous across varieties nor countries, only a parameter between $\alpha$ and $\gamma$ is sufficient to represent the taste for differentiation relative to the absolute willingness to pay (which is captured by $\gamma/\alpha$). With extended notation, variety, sector and country-and-sector-specific parameters $\alpha_l(i;z) > 0$ and $\gamma_l(i;z) > 0$ could be introduced to keep track of within-sector patterns of vertical and horizontal differentiation respectively. We abstract from the corresponding sources of differentiation by willingness to pay (*quality*) and by country of origin (*Armington*).

profit-maximizing price. It includes the quadratic case for $\alpha > 0$, $\gamma > 0$ and $\sigma = -1$, as well as the CES case for $\alpha = 0$, $\gamma < 0$ and demand elasticity to price equal to $1/\sigma > 1$. With $\alpha > 0$, $\gamma > 0$ and $\sigma < 0$, a variety's marginal utility at zero consumption to be positive and finite, and thus for its demand curve to feature a choke price, which will be a key feature of our setup in the wake of Arkolakis et al. (2019).[4]

Individual consumers in country $l$ earning a wage $w_l > 0$ take the set of available varieties and prices as given and maximize (1) subject to the budget constraint

$$\sum_{z=1}^{Z} \sum_{j=1}^{J} \int_0^{N_{jl}(z)} p_{jl}\left(i;z\right) q_{jl}^{c}\left(i;z\right) di = w_l \ , \tag{2}$$

where $p_{jl}\left(i;z\right)$ is the price (at destination) of a variety $i$ of goods from sector $z$ produced in country $j$ and sold to country $l$.

Between sectors, the marginal utility is unbounded. Therefore, every consumer of every country $l$ will demand varieties from every sector $z$. Within sector, the marginal utility from consumption of a certain variety is finite ($\alpha > 0$ is necessary for this result). This implies that there is a choke price at which the optimal consumption of a variety is null. Let $\hat{p}_l(z) > 0$ be the price that implies zero demand in country $l$ for a variety in sector $z$. The Marshallian individual demand function in country $l$ for a variety of sector $z$ sold in country $l$ at a price $p$, regardless where production occurs, is given by:

$$q_l^*\left(p;z\right) = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} \left(1 - \frac{p}{\hat{p}_l(z)}\right)^{\frac{1}{-\sigma}} \ , \quad \forall j. \tag{3}$$

The firm-level elasticity of demand to price is fully described by the relative price with respect to the choke price $\varepsilon_l(p;z) = \frac{1}{-\sigma}\frac{p/\hat{p}_l(z)}{1-p/\hat{p}_l(z)}$ in absolute value. Restricting the analysis to $\varepsilon_l(p;z) > 1$, the ratio $\varepsilon_l(p;z)/(\varepsilon_l(p;z) - 1)$ defines the markup rate:

$$mkp_{jl}^*(p;z) = \frac{\frac{1}{-\sigma}}{(1 + \frac{1}{-\sigma}) - \hat{p}_l(z)/p} \ , \quad \forall j, \tag{4}$$

[4] The positive and normative properties of the general BP class with monopolistic competition and firm heterogeneity are studied by Melitz et al. (2024), who prove that BP preferences are necessary and sufficient for constant absolute pass-through when utility is additive separable. See Mrazova and Neary (2017) for a discussion of the relation between absolute pass-through and the log-convexity of the demand curve. In particular, our $\sigma$ is related to their curvature parameter $\rho$, such that $\sigma = \rho - 1$. The quadratic case corresponds to $\sigma = -1$, in which $\rho = 0$. Instead $\sigma \in (-1, 0)$ implies $\rho = 1 + \sigma \in (0, 1)$ whereas $\sigma < -1$ implies $\rho < 0$. In the terminology of their manifold, BP preferences belong to the sub-pass-through family characterized by incomplete constant absolute pass-through.

Hence, the markup rate in a given destination depends on the local price relative to local choke price. Specifically, the markup rate is a decreasing function of the relative price $p/\hat{p}_l(z)$, which implies that within a sector varieties for which consumers have higher demand are those sold at higher markup.

## 2.2 Firms' behavior

A firm located in country $j$, competing in sector $z$, endowed with a technological coefficient $c$, hires labor in the same country at a competitive wage $w_j$ as input in a linear production function:

$$q_j(c;z) = \frac{\ell_j(c;z)}{c} \tag{5}$$

where $\ell_j(c;z)$ is the firm's employment. The marginal cost of production in country $j$ is $w_j c$. For goods produced in country $j$ and shipped to country $l$, the marginal cost of production and delivery is $\tau_{jl}(z) w_j c$.

Since consumers in a given country have the same income, the aggregate demand function in destination $l$ is equal to individual demand (3) times market size $L_l$. Thus, the marginal revenue of a variety sold at price $p_{jl}(c;z)$ is given by $p_{jl}(c;z)/mkp^*_{jl}(p_{jl}(c;z);z)$. Then, equating marginal revenue to marginal cost under the assumption that national markets are segmented yields the profit-maximizing price:

$$p_{jl}(c;z) = \frac{-\sigma\hat{p}_l(z) + \tau_{jl}(z) w_j c}{1-\sigma} \,. \tag{6}$$

with constant absolute pass-through from delivered marginal cost to price equal to $1/(1-\sigma)$. Substituting (7) in the Marshallian demand (6) shows that the technological coefficient that implies a zero demand in country $l$ for a good of sector $z$ produced in country $j$ is the export cutoff:

$$c^*_{jl}(z) = \frac{\hat{p}_l(z)}{\tau_{jl}(z) w_j} \,. \tag{7}$$

Therefore, the marginal cost for profitably producing in country $j$ and shipping to country $l$ is bounded above by the choke price in the destination country $l$. While firms with $c \leq c^*_{jl}(z)$ sell in country $l$, firms with $c > c^*_{jl}(z)$ optimally choose not to serve it.

The measure of firms producing in country $j$ and serving market $l$ consists of the fraction of entrants in the sector, $N^E_j(z)$, whose unit labor requirements do not exceed the export cutoff:

$$N_{jl}(z) = G_j(c^*_{jl}(z);z)N^E_j(z)\ . \tag{8}$$

A firm's price $p_{jl}(c;z)$, markup rate $mkp_{jl}(p;z)$, output $q_{jl}(c;z)$, employment $\ell_{jl}(c;z)$, revenue $r_{jl}(c;z)$ and profit $\pi_{jl}(c;z)$ can all be expressed in terms of the choke price $\hat{p}_l(z)$ and the cutoff $c^*_{jl}(z)$:

$$\begin{aligned}
p_{jl}(c;z) &= \frac{\hat{p}_l(z)}{1-\sigma}\left(-\sigma + \frac{c}{c^*_{jl}(z)}\right) \\
mkp_{jl}(c;z) &= \frac{1}{1-\sigma}\left(1-\sigma\frac{c^*_{jl}(z)}{c}\right), \\
q_{jl}\,(c;z) &= L_l\left(\frac{\alpha}{(1-\sigma)\gamma}\right)^{\frac{1}{-\sigma}}\left(1-\frac{c}{c^*_{jl}(z)}\right)^{\frac{1}{-\sigma}}, \\
\ell_{jl}(c;z) &= \frac{\hat{p}_l(z)L_l}{w_j}\left(\frac{\alpha}{(1-\sigma)\gamma}\right)^{\frac{1}{-\sigma}}\left(1-\frac{c}{c^*_{jl}(z)}\right)^{\frac{1}{-\sigma}}\frac{c}{c^*_{jl}(z)} \\
r_{jl}\,(c;z) &= \frac{\hat{p}_l(z)L_l}{1-\sigma}\left(\frac{\alpha}{(1-\sigma)\gamma}\right)^{\frac{1}{-\sigma}}\left(1-\frac{c}{c^*_{jl}(z)}\right)^{\frac{1}{-\sigma}}\left(-\sigma+\frac{c}{c^*_{jl}(z)}\right), \\
\pi_{jl}\,(c;z) &= \hat{p}_l(z)L_l\left(\frac{\alpha}{(1-\sigma)\gamma}\right)^{\frac{1}{-\sigma}}\frac{-\sigma}{1-\sigma}\left(1-\frac{c}{c^*_{jl}(z)}\right)^{1+\frac{1}{-\sigma}}.
\end{aligned} \tag{9}$$

## 2.3 Sectoral expenditure share and price concentration

A distinctive feature of this setup is that consumers' expenditure and indirect utility are characterized by moments of the distribution of prices relative to the choke price. Specifically, define the following two moments of the distribution of prices relative to the choke price in country $l$ sector $z$ among goods shipped from country $j$:

$$\begin{aligned}
\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z) &\equiv N_{jl}(z)^{-1}\int_0^{N_{jl}(z)}\frac{p(i)}{\hat{p}_l(z)}\left(1-\frac{p(i)}{\hat{p}_l(z)}\right)^{\frac{1}{-\sigma}}di, \\
\bar{\bar{p}}_{jl}(z) &\equiv 1 - N_{jl}(z)^{-1}\int_0^{N_{jl}(z)}\left(-\sigma+\frac{p(i)}{\hat{p}_l(z)}\right)\left(1-\frac{p(i)}{\hat{p}_l(z)}\right)^{\frac{1}{-\sigma}}di\ .
\end{aligned}$$

The difference between the two moments has an intuitive interpretation. High concentration of prices around the choke price (i.e., $p(i) \approx \hat{p}_l(z)$ for all $i \in [0, N_{jl}(z)]$) implies $\bar{p}_{jl}(z) \approx \bar{\bar{p}}_{jl}(z) \approx 1$; otherwise $0 < \bar{p}_{jl}(z) < \bar{\bar{p}}_{jl}(z) < 1$ holds. Hence, the larger the difference $\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)$, the wider the dispersion of prices away from the choke price.

Then, country $l$'s expenditure and sub-utility associated with the consumption of sector $z$'s varieties sourced from country $j$ can be respectively expressed as:

$$\begin{aligned} e_{jl}(z) &\equiv \int_0^{N_{jl}(z)} p(i) q_l^* \left(p(i); z\right) di = \hat{p}_l(z) \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} \left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z), \\ u_{jl}(z) &\equiv \int_0^{N_{jl}(z)} \alpha q_l^* \left(p(i); z\right) - \frac{\gamma}{1-\sigma} q_j^* \left(p(i); z\right)^{1-\sigma} di = \frac{\gamma}{1-\sigma} \left(\frac{\alpha}{\gamma}\right)^{\frac{1-\sigma}{-\sigma}} \left(1 - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z). \end{aligned}$$

In the spirit of Dixit and Stiglitz (1977), total expenditure $\sum_{j=1}^{J} e_{jl}(z)$ on sector $z$'s varieties can be decomposed into a quantity index $\mathbb{Q}_l(z) \equiv (1/\alpha) \sum_{j=1}^{J} u_{jl}(z)$ and a price index $\mathbb{P}_l(z) \equiv \sum_{j=1}^{J} e_{jl}(z) / \mathbb{Q}_l(z)$ such that $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta_l(z) w_l$, where the sector-specific expenditure share is given by:

$$\theta_l(z) \equiv \frac{\beta(z)\eta_l(z)}{\sum_{s=1}^{Z} \beta(s)\eta_l(s)} \in (0,1) \,. \tag{10}$$

This deviates from the exogenous Cobb-Douglas expenditure shares due to the presence of the endogenous coefficient:

$$\eta_l(z) \equiv \frac{\mathbb{P}_l(z)}{\hat{p}_l(z)} = \frac{(1-\sigma) \sum_{j=1}^{J} \left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z)}{\sum_{j=1}^{J} \left(1 - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z)} \in (0,1) \,, \tag{11}$$

which is a country-and-sector-specific index of price concentration (as summarized by the sectoral price index) relative to the choke price.

Therefore, the fact that BP preference are non-homothetic among varieties within sectors implies that sectoral expenditure shares are endogenous and depend on the concentration in the within-sector price distributions. Importantly, the exogenous Cobb-Douglas shares ($\beta(z)$) coincide with the endogenous expenditure shares ($\theta_l(z)$) if and only if all sectors exhibit the same concentration in their price distributions, as in this case $\eta_l(z) = \eta_l$ holds for all $z$.

## 2.4 Equilibrium

With CES preferences a variety's marginal utility tends to infinity as its consumption goes to zero. As all varieties always contribute positive marginal utility, they are all demanded in all countries wherever they are produced. In contrast, with BP preferences a variety's marginal utility at zero consumption is positive and finite, which implies that in a country some varieties may not be demanded or some sectors may not be producing. To compare with the typical case discussed in the quantitative trade literature, based on CES preferences, we focus here on an equilibrium with diversification, such that in all countries all sectors produce.

An equilibrium with diversification is characterized by a strictly positive measure of entrants in every country and sector pair, such that $N_j^E(z) > 0$ for all $j = 1, ..., J$ and $z = 1, ..., Z$. This implies that in every destination market there is a measure of domestic incumbent firms in each sector, such that $c_{ll}^*(z) > 0$ for all $l = 1, ..., J$ and $z = 1, ..., Z$, hence, the domestic cutoff $c_{ll}^*(z) \equiv c_l^*(z)$ determines the choke price $\hat{p}_l(z) = w_l c_l^*(z)$.

For this reason we will first define an open economy equilibrium with diversification in the context of this model and then discuss existence, uniqueness and properties of the equilibrium in the next section.

In every country and sector pair $(j, z)$ with a strictly positive measure of entrants, free entry implies that the expected value of a new entry unconditional on being successful matches the entry cost:

$$\text{FEC}: \quad \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} \pi_{jl}(c; z) dG_j(c; z) = w_j f_j \qquad \forall (j, z). \qquad . \tag{12}$$

Output market clearing requires that sales made by all firms in a sector $z$ that serve a destination country $l$ add up to the expenditure of that country in that sector:

$$\text{OMC}: \quad \sum_{j=1}^{J} N_{jl}(z) \int_0^{c_{jl}^*(z)} r_{jl}(c; z) \frac{dG_j(c; z)}{G_j(c_{jl}^*(z); z)} = \theta_l(z) w_l L_l \qquad \forall (l, z). \tag{13}$$

Labor market clearing requires that the total sales of all firms producing in a country $j$ are equal to the country's aggregate labor income (from production and entry):

$$\text{LMC}: \quad \sum_{z=1}^{Z} \sum_{l=1}^{J} N_{jl}(z) \int_0^{c_{jl}^*(z)} r_{jl}(c; z) \frac{dG_j(c; z)}{G_j(c_{jl}^*(z); z)} = w_j L_j \quad \forall j. \tag{14}$$

Given a set of preference parameters $\{\alpha, \gamma, \{\beta(z)\}_{z=1}^{Z}\}$, market sizes $\{L_j\}_{j=1}^{J}$, entry costs $\{f_j\}_{j=1}^{J}$, a distribution of technological coefficients $\{G_j(c; z)\}_{j=1, z=1}^{J, Z}$ and a set of bilateral sector specific trade costs $\{\tau_{jl}(z)\}_{j=1, l=1, z=1}^{J, J, Z}$, the diversified equilibrium consists of:

a) a vector of wages $w_l > 0$ for every country $l = 1, 2, ...J$

b) a vector of choke prices $\hat{p}_l(z) = c_l^*(z) w_l > 0$ for every country $l = 1, 2, ...J$ and sector $z = 1, 2, ..., Z$

c) a vector of measures of entrants $N_j^E(z) > 0$ for every origin country $j = 1, 2, ...J$ and sector $z = 1, 2, ..., Z$

that satisfy

*i*) the system of $J \times Z$ free entry conditions (12),

*ii*) the system of $J \times Z$ sectoral output market clearing conditions (13),

*iii*) the system of $J$ aggregate labor market clearing conditions (14),

once the export cutoff (7), the measure of exporters (8), the definitions of firm-level profit and revenue in (18), sectoral expenditure share (10) and sectoral price concentration (11) are understood. Without loss of generality, labor in one of the countries is taken as numeraire, such that the corresponding wage is 1 before and after any change in the fundamentals of the economy.

## 2.5 Welfare

Given the price vector $\boldsymbol{p}_l$ for available varieties in destination market $l$ and local wage $w_l$, the indirect utility enjoyed by the representative consumer is a Cobb-Douglas aggregation of the sectoral utility-based quantity indexes $V(\boldsymbol{p}_l, w_l) = \prod_{z=1}^{Z} \mathbb{Q}_l(z)^{\beta(z)}$. The budget constraint $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta_l(z) w_l$ then implies:

$$\mathbb{Q}_l(z) = \frac{\theta_l(z)}{\mathbb{P}_l(z)} w_l = \frac{\theta_l(z)}{\eta_l(z)} \frac{w_l}{\hat{p}_l(z)} = \frac{\beta(z)}{\bar{\eta}_l} \frac{w_l}{\hat{p}_l(z)},$$

where $\bar{\eta}_l \equiv \sum_{z=1}^{Z} \beta(z)\eta_l(z) \in (0,1)$ is the weighted average of the sectoral price concentration indexes in country $l$, with weights given by the sectoral shares in consumer's preferences. Measuring welfare as indirect utility then yields:

$$V(\boldsymbol{p}_l, w_l) = \bar{\eta}_l^{-1} \prod_{z=1}^{Z} \left( \frac{\beta(z)}{\hat{p}_l(z)} \right)^{\beta(z)} w_l = \bar{\eta}_l^{-1} \prod_{z=1}^{Z} \left( \frac{\beta(z)}{c_l^*(z)} \right)^{\beta(z)}, \tag{15}$$

where the last equality follows from $\hat{p}_l(z) = c_l^*(z) w_l$ as implied by equation (7).

Expression (15) offers two insights. First, welfare in country $l$ is a geometric average across sectors of the country's sectoral productivity cutoffs $1/c_l^*(z)$. Second, for given choke prices and wage, through $\bar{\eta}_l$ welfare is higher when, on average, prices are more dispersed away from the sectoral choke price.

## 2.6 Gravity equation

The equilibrium with diversification predicts a structural gravity representation of trade flows. To obtain this, start from the definition of expenditure, call $X_{jl}(z) = e_{jl}(z)L_l$ the value of imports of country $l$ from country $j$ in sector $z$ and let $X_l(z) \equiv \sum_{j=1}^{J} X_{jl}(z)$ be the aggregate expenditure of country $l$ in goods of sector $z$ sourced from anywhere. Substituting for the measure of exporters $N_{jl}(z) = G_j(c_{jl}^*(z); z)N_j^E(z)$ as per expression (8) yields a gravity equation in terms of the measure of entrants in each country and sector:

$$X_{jl}(z) = \frac{\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) G_j(c_{jl}^*(z); z)N_j^E(z)}{\sum_{m=1}^{J} \left(\bar{p}_{ml}(z) - \bar{\bar{p}}_{ml}(z)\right) G_m(c_{ml}^*(z); z)N_m^E(z)} X_l(z) \ .$$

Free entry (12) implies that the cost of entry in a country $j$ sector $z$, that is $N_j^E(z)w_j f_j$, equals total profit in that sector and country. Let $\delta_j(z) = \Pi_j(z)/R_j(z) \in (0,1)$ be the fraction of aggregate profit $\Pi_j(z) \equiv N_j^E(z) \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} \pi_{jl}(c; z) dG_j(c; z)$ over aggregate revenue $R_j(z) \equiv N_j^E(z) \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} r_{jl}(c; z) dG_j(c; z)$ in country $j$ sector $z$, such that $N_j^E(z)w_j f_j = \delta_j(z)R_j(z)$. Output market clearing (13) implies $X_l(z) = \theta_l(z)Y_l$, where $Y_l \equiv w_l L_l$ denotes income in country $l$.

Under free entry, total revenue coincides with total labor income (associated with both production and entry). Define $\rho_j(z) \in (0,1)$ as the share of employment in sector $z$ of country $j$. Then, labor market clearing (14) at the sectoral level yields $R_j(z) = \rho_j(z)w_j L_j$, that allows to substitute for $N_j^E(z) = \delta_j(z)\rho_j(z)L_j/f_j$. This completes the characterization of the gravity equation:

$$X_{jl}(z) = \left( \frac{\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) G_j(c_{jl}^*(z); z)\delta_j(z)\rho_j(z)L_j/f_j}{\sum_{m=1}^{J} \left(\bar{p}_{ml}(z) - \bar{\bar{p}}_{ml}(z)\right) G_m(c_{ml}^*(z); z)\delta_m(z)\rho_m(z)L_m/f_m} \right) \theta_l(z)Y_l \ , \tag{16}$$

where the expression in brackets is the fraction of expenditure in goods of sector $z$ that country $l$ sources from country $j$.

With respect to the family of structural gravity equations that are popular in the quantitative trade literature, an important difference emerges: the role played by price dispersion at origin relative to the choke price at destination. All the rest given, country $l$ sources relatively less from an origin $j$ if this is characterized by more concentrated sellers at the choke price ($\bar{\bar{p}}_{jl}(z) \approx 1$, $\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z) \approx 0$) than by less concentrated sellers ($\bar{\bar{p}}_{jl}(z) < \bar{p}_{jl}(z) < 1$). A finite choke price is necessary for this channel to operate.

It is, however, not sufficient. In particular, we will show that assuming an Inverse Pareto distribution of technological coefficients makes such channel immaterial.

## 2.7 Distribution of technologies

Closing the model to make it amenable to quantitative analysis requires to take a stand on the distributions of unit labor requirements across sectors and countries. The existing literature has mostly focused on Pareto distributions, Log-normal distributions or combinations of the two types, as proxies of the actual empirical distributions. We argue that a much more flexible Beta distribution can be assumed instead, without affecting much the complexity of symbolic and numerical analysis.

In particular, we can state the following:

**Proposition**. *Let the distribution of unit labor requirements across potential entrants in country j and sector z be a 3-parameter Beta, with shape parameters* $\kappa_1 > 0$ *and* $\kappa_2 > 0$ *and location parameter* $c_j^{max}(z) > 0$*, such that its p.d.f. is given by*

$$g_j(c;z) = \frac{c^{\kappa_1 - 1}(c_j^{max}(z) - c)^{\kappa_2 - 1}}{\frac{\Gamma(\kappa_1)\Gamma(\kappa_2)}{\Gamma(\kappa_1+\kappa_2)} c_j^{max}(z)^{\kappa_1+\kappa_2-1}}, \quad c \in [0, c_j^{max}(z)].$$

*Consider aggregate variables (i.e. expenditure, utility, average revenue and average profit) in a market segment* $(j, l, z)$*. The following results hold:*

**(i)** *If there is no selection (* $c_{jl}^*(z) = c_j^{max}(z)$*), then aggregate variables are determined in closed form.*

**(ii)** *If there is selection (* $c_{jl}^*(z) < c_j^{max}(z)$*), then determining aggregate variables only requires integration of a continuous and smooth (i.e. with continuous derivatives) function on the given compact support* $[(-\sigma)/(1-\sigma), 1]$*, for which the limit of integration is bounded and thus standard numerical integration applies.*

**(iii)** *If there is selection (* $c_{jl}^*(z) < c_j^{max}(z)$*) and* $\kappa_2$ *is a natural number, then the expected profit at entry that a firm producing in country j sector z earns from sales in country l can be computed in closed form:*

$$\bar{\pi}_{jl}(z) = \frac{w_l L_l \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} (-\sigma)}{\omega_{jl}(z)^{\kappa_1+\kappa_2-1}} c_l^*(z)^{\kappa_1+\kappa_2} \sum_{k=0}^{\kappa_2-1} a(\kappa_1, \kappa_2, \sigma; k) \left( \frac{\omega_{jl}(z)}{c_l^*(z)} - \frac{1}{1-\sigma} \right)^{\kappa_2-1-k}, \quad (17)$$

*where* $\omega_{jl}(z) \equiv [w_j c_j^{max}(z)\tau_{jl}(z)]/[w_l(1-\sigma)]$ *and the coefficients* $a(\kappa_1, \kappa_2, \sigma; k)$ *are positive real numbers determined by exogenous parameters only.*

**(iv)** *For a given vector of wages* $\{w_j\}_{j=1}^J$*, the sector-z specific non-linear system of J free entry conditions in J domestic cutoffs* $\{c_j^*(z)\}_{j=1}^J$ *has a solution in* $\Re^J$ *and it is unique.*

**Proof**. See Appendix D.

This result shows that a setup with BP preferences remains highly tractable for quantitative analysis even when complemented by a more flexible distribution of technology than those typically used in the literature.[5]

# 3 Implications of a Pareto distribution of technologies

Distinctive features of our setup are variable markups, incomplete constant absolute pass-through, and cross-sector variation in the concentration of firms' technological coefficients. However, to understand their implications, the flexibility of the distributional assumptions on $G_j(c; z)$ and the exact value of the constant absolute pass-through $1/(1-\sigma)$ are not of first-order importance. Therefore, while the entire analysis would go through also in the general case discussed so far, we now prefer to make two simplifying assumptions that, by making the model's parametrization more parsimonious, better serve the purpose of comparing our results with those in the literature. First, we impose $\sigma = -1$, which makes a variety's sub-utility in definition (1) quadratic and its demand linear in own price. Second, we assume that the distribution of unit labor requirements is Inverse Pareto. Nonetheless, we will highlight what these parametric restrictions imply.

After substituting for $\sigma = -1$, the firm performance measures listed in (9) can be

[5]A 3-parameter Beta nests the (Inverse) Pareto distribution and the uniform distribution as special cases, and approximates well distributions of the exponential family (including Normal and Lognormal). Furthermore, the parametric restriction on $k_2$ being a natural number (not necessary, but convenient for analytical tractability) comes at little loss of generality since the shape of the p.d.f. is essentially determined by the difference $\kappa_1 - \kappa_2$ which is unrestricted and can be any real number.

rewritten as:

$$
\begin{aligned}
p_{jl}(c;z) &= \frac{\hat{p}_l(z)}{2}\left(1+\frac{c}{c^*_{jl}(z)}\right), \\
mkp_{jl}(c;z) &= \frac{1}{2}\left(1+\frac{c^*_{jl}(z)}{c}\right), \\
q_{jl}(c;z) &= \frac{L_l}{2\gamma}\frac{\alpha}{c^*_{jl}(z)}\left(c^*_{jl}(z)-c\right), \\
\ell_{jl}(c;z) &= \frac{L_l}{2\gamma}\frac{\alpha\tau_{jl}(z)}{c^*_{jl}(z)}\left(c^*_{jl}(z)\,c-c^2\right), \\
r_{jl}(c;z) &= \frac{w_jL_l}{4\gamma}\frac{\alpha\tau_{jl}(z)}{c^*_{jl}(z)}\left(c^*_{jl}(z)^2-c^2\right), \\
\pi_{jl}(c;z) &= \frac{w_jL_l}{4\gamma}\frac{\alpha\tau_{jl}(z)}{c^*_{jl}(z)}\left(c^*_{jl}(z)-c\right)^2,
\end{aligned}
\tag{18}
$$

where we have used the relation $\hat{p}_l(z) = w_j\tau_{jl}(z)c^*_{jl}(z)$ between the choke price, the wage and the cutoff. Optimal consumer expenditure and indirect sub-utility simplify to:

$$
\begin{aligned}
&e_{jl}(z) = \hat{p}_l(z)\frac{\alpha}{\gamma}\left(\bar{p}_{jl}(z)-\bar{\bar{p}}_{jl}(z)\right)N_{jl}(z)\,, \\
&u_{jl}(z) = \frac{\alpha^2}{2\gamma}\left(1-\bar{\bar{p}}_{jl}(z)\right)N_{jl}(z)\,,
\end{aligned}
$$

where $\bar{p}_{jl}(z)$ and $\bar{\bar{p}}_{jl}(z)$ are the first and second moments of the distribution of prices relative to the choke price in country $l$ and sector $z$ across varieties sold from country $j$:

$$
\bar{p}_{jl}(z) \equiv N_{jl}(z)^{-1}\int_0^{N_{jl}(z)}\frac{p(i)}{\hat{p}_l(z)}di \quad \text{and} \quad \bar{\bar{p}}_{jl}(z) \equiv N_{jl}(z)^{-1}\int_0^{N_{jl}(z)}\left(\frac{p(i)}{\hat{p}_l(z)}\right)^2 di\,.
$$

Introduce now the assumption that the distribution of unit labor requirements is Inverse Pareto on the support $[0, c^{max}_j(z)]$, with a country- and sector-specific location parameter $c^{max}_j(z) > 0$ and a sector-specific shape parameter $k(z) > 1$ such that $G_j(c;z) = (c/c^{max}_j(z))^{k(z)}$. The mean of the distribution is $\frac{k(z)}{k(z)+1}c^{max}_j(z)$. For $k(z) \to 1$ the distribution becomes uniform (maximum dispersion), whereas for $k(z) \to \infty$ the distribution degenerates to a unit mass point at $c^{max}_j(z)$, describing maximum concentration at the upper bound of the support. Henceforth, we will refer to the parameter $k(z)$ as the "technological concentration" of sector $z$.

The Inverse Pareto assumption imposes a discipline on the distribution of prices relative to the choke price within a country-sector. The choke price in sector $z$ country $l$ is such that the firm-level demand is null, which corresponds to $c = c^*_{jl}(z)$ for a firm producing in any country $j$. The relative price is given by $\frac{p_{jl}(c;z)}{\hat{p}_l(z)} = \frac{1}{2}(1 + c/c^*_{jl}(z))$ and it is distributed over the support $c \in [0, c^*_{jl}(z)]$ according to the truncated Inverse Pareto $G^*_{jl}(c;z) = (c/c^*_{jl}(z))^{k(z)}$. As a result, the first and second moments of the relative price distribution

$$\bar{p}_{jl}(z) = \frac{2k(z)+1}{2(k(z)+1)} \equiv \mu_1(z) \tag{19}$$

$$\bar{\bar{p}}_{jl}(z) = \frac{2k(z)^2+4k(z)+1}{2(k(z)+2)(k(z)+1)} \equiv \mu_2(z) \tag{20}$$

are no longer endogenous, as they are only determined by the parameter of technological concentration $k(z)$ with no role for origin or destination country characteristics. The consequences for welfare, equilibrium allocations and trade flows are not innocuous, as we discuss in the following sections.

## 3.1 Implications for welfare

Given that the first and second moments of the within-sector relative price distribution are the same across countries, the index (11) of concentration of prices relative to the choke price is the same in all destinations independently of available varieties. In particular, we have $\eta_l(z) \equiv \eta(z)$ with:

$$\eta(z) = \frac{2(\mu_1(z) - \mu_2(z))}{1 - \mu_2(z)} = \frac{2k(z)+2}{2k(z)+3},$$

which is increasing in technological concentration as measured by $k(z)$. As a result, sectoral expenditure shares (10) are the same across countries ($\theta_l(z) \equiv \theta(z)$) so that, in sectors with a relatively more (less) concentrated distribution of technological coefficients, the equilibrium expenditure shares are everywhere larger (smaller) than their corresponding Cobb-Douglas shares: $\theta(z) > (<) \beta(z)$.

With $\eta_l(z) \equiv \eta(z)$, welfare (15) is unaffected by changes in price dispersion and becomes a geometric average of sectoral productivity cutoffs:

$$W_l = \bar{\eta} \prod_{z=1}^{Z} \left( \frac{\beta(z)}{c^*_l(z)} \right)^{\beta(z)}, \tag{21}$$

where $\bar{\eta}$ can be set to 1 without loss of generality. The fact that the Inverse Pareto neutralizes the role of changes in relative price dispersion confirms the conclusion by Melitz and Redding (2015) that with firm selection the moments of the micro structure matter for welfare.

## 3.2 Implications for equilibrium allocations and trade flows

Evaluating the free entry condition (12) under the Inverse Pareto assumption yields:

$$\text{FEC*}: \quad \sum_{l=1}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left( \frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} = 1 \ \ \forall (j,z)\,, \tag{22}$$

where

$$c_j^{aut}(z) \equiv \left( \frac{c_j^{max}(z)^{k(z)}}{\zeta_\Pi(z) L_j/f_j} \right)^{\frac{1}{1+k(z)}}$$

is the cutoff cost in sector $z$ of country $j$ in autarky (i.e $\tau_{jl}(z) \to \infty$ for every $j \neq l$) and $\zeta_\Pi(z) > 0$ is a decreasing transformation of technological concentration $k(z)$ (see appendix B.1 for a detailed derivation).

Turning to output market clearing, condition (13) evaluated under Inverse Pareto distribution yields:

$$\text{OMC*}: \quad \sum_{j=1}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left( \frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} w_j f_j N_j^E(z) = w_l f_l N_l^{E\ aut}(z) \quad \forall (l,z) \tag{23}$$

where

$$N_l^{E\ aut}(z) = \theta(z)\delta(z)\frac{L_l}{f_l}$$

is the measure of entrants in sector $z$ of country $l$ in autarky and $\delta(z) \in (0,1)$ is the profit-to-revenue ratio, which is decreasing in $k(z)$ and constant across countries (see appendix B.1 for a detailed derivation).

The labor market clearing condition (14) becomes:

$$\text{LMC*}: \quad \sum_{z=1}^{Z} \left( \frac{f_j N_j^E(z)}{\delta(z)} \sum_{l=1}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left( \frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} \right) = L_j \quad \forall j \tag{24}$$

where the left hand side is the aggregate labor demand in country $j$.

Under Inverse Pareto the moments of the relative price distribution ($\bar{p}_{jl}(z)$ and $\bar{\bar{p}}_{jl}(z)$), the expenditure shares ($\theta_l(z)$), and the profit-to-revenue ratios ($\delta_j(z)$) do not depend on country specific characteristics. The structural gravity equation simplifies to:

$$X_{jl}(z) = \left( \frac{\left(\tau_{jl}(z) w_j c_j^{max}(z)\right)^{-k(z)} \rho_j(z) L_j / f_j}{\sum_{m=1}^{J} \left(\tau_{ml}(z) w_m c_m^{max}(z)\right)^{-k(z)} \rho_m(z) L_m / f_m} \right) \theta(z) Y_l \,, \tag{25}$$

which is a sectoral structural gravity equation (Head and Mayer, 2014) with trade elasticity equal to $-k(z)$, despite variable markups and sectoral choke prices (Arkolakis et al., 2019).

## 3.3 Uniqueness and existence of the equilibrium

For a compact characterization of the equilibrium, it is useful to simplify the notation by defining two sets of changes in variables and three sets of bundling parameters.

In the case of variables, we exploit the fact that, as we have seen, the autarkic cutoffs and the measures of entrants can be expressed in closed form as functions of exogenous parameters. Specifically, define $x_l(z)$ as the trade-induced change in the cutoff of sector $z$ in country $l$, and $y_j(z)$ as the trade-induced change in the measure of entrants in sector $z$ country $j$:

$$x_l(z) \equiv \left( \frac{c_l^*(z)}{c_l^{aut}(z)} \right)^{1+k(z)} \quad \text{and} \quad y_j(z) \equiv \frac{N_j^E(z)}{N_j^{E\ aut}(z)}.$$

As for the bundling parameters, we introduce the following definitions:

$$T_{jl}(z) \equiv \frac{\tau_{jl}(z)^{k(z)}}{L_l / L_j} \,, \quad K_{jl}(z) \equiv \left( \frac{c_l^{aut}(z)}{c_j^{aut}(z)} \right)^{1+k(z)} \,, \quad E_{jl}(z) \equiv \frac{f_j N_j^{E\ aut}(z)}{f_l N_l^{E\ aut}(z)} = \frac{L_j}{L_l} \,,$$

which allow us to collect in $T_{jl}(z)$ the trade costs from country $j$ to country $l$ weighted by relative market size, in $K_{jl}(z)$ the autarkic productivity cutoff of country $j$ relative to country $l$, and in $E_{jl}(z)$ the autarkic patterns of entry in country $j$ relative to country $l$.

With this simplified notation the structure of the equilibrium conditions can be written in compact form as:

$$\text{FEC}^{**} \quad : \quad \sum_{l=1}^{J} \frac{K_{jl}(z)}{T_{jl}(z)} \left(\frac{w_l}{w_j}\right)^{1+k(z)} x_l(z) = 1 \qquad \forall (j,z)$$

$$\text{OMC}^{**} \quad : \quad \sum_{j=1}^{J} \frac{K_{jl}(z)E_{jl}(z)}{T_{jl}(z)} \left(\frac{w_l}{w_j}\right)^{k(z)} x_l(z)y_j(z) = 1 \qquad \forall (l,z)$$

$$\text{LMC}^{**} \quad : \quad \sum_{z=1}^{Z} \theta(z) y_j(z) = 1 \qquad \forall j.$$

Accordingly, the equilibrium of the model consists of the solution of a system of $J + 2 \cdot J \cdot Z$ non-linear coupled equations in as many unknowns $\{w_j, x_j(z), y_j(z)\}$ for $j = 1, ..., J$ and $z = 1, ..., Z$. For a given vector of relative wages, FEC** is a linear system of $J \cdot Z$ equations in as many unknowns $x_j(z)$, which, once substituted in OMC**, yields a linear system of $J \cdot Z$ equations in as many unknowns $y_j(z)$. Therefore, the system of FEC** and OMC** determines a unique matrix of relative cutoff costs $x_j(z)$ and relative firm entry $y_j(z)$ for a given vector of relative wages.

*Uniqueness.* Taking the wage in country 1 as numéraire without loss of generality and rearranging the system of FEC** and OMC** within sector $z$ shows that the trade-induced change in country $m$'s cutoffs $x_m(z)$ is increasing in its relative wage $w_m/w_1$, whereas the trade-induced change in the measure of entrants $y_m(z)$ is decreasing in its relative wage $w_m/w_1$ for every sector:[6]

$$x_m(z) = 1 - \left(\frac{w_1}{w_m}\right)^{1+k(z)} \sum_{l \neq m}^{J} \frac{K_{ml}(z)}{T_{ml}(z)} \left(\frac{w_l}{w_1}\right)^{1+k(z)} x_l(z) \qquad \forall (m,z), \tag{26}$$

$$y_m(z) = \frac{1}{x_m(z)} - \left(\frac{w_m}{w_1}\right)^{k(z)} \sum_{j \neq m}^{J} \frac{K_{jm}(z)E_{jm}(z)}{T_{jm}(z)} \left(\frac{w_1}{w_j}\right)^{k(z)} y_j(z) \qquad \forall (m,z). \tag{27}$$

The comparative statics in (26) and (27) corroborate the interpretation of the left hand side of LMC** as the country's aggregate labor demand function in open economy relative to autarky. Specifically, after substituting for $y_j(z)$ for $z = 1, ..., Z$ as implied by the system of FEC** and OMC**, the weighted sum of sectoral trade-induced changes in the numbers of entrants $\sum_{z=1}^{Z} \theta(z) y_j(z)$ is decreasing in the the country's own rela-

[6] These properties hold not only locally (i.e. holding $\{x_l, y_j : l, j \neq m\}$ constant), but also globally: every $x_k$ for $k \neq m$ is increasing in $w_k/w_1$ and decreasing in $w_m/w_1$ and every $y_n$ for $n \neq m$ is decreasing in $w_n/w_1$ and increasing in $w_m/w_1$. A detailed derivation of the argument for uniqueness is discussed in Appendix B.2.

tive wage $w_m/w_1$ and increasing in the relative wage of other countries. It follows that, if an open economy equilibrium with diversification exists, then the monotonicity of the relative labor demand function in every country guarantees that the equilibrium is unique.[7]

*Existence*. For arbitrary parameter configurations for preferences, technologies, market sizes and trade costs, an open economy equilibrium with diversification is not granted. But, the equilibrium conditions FEC** and OMC** can be used to define a subset within the space of relative wage vectors $(1, w_2/w_1, ..., w_J/w_1) \in \Re_+^{J-1}$ that hosts (if nonempty) the equilibrium. An open economy equilibrium with diversification exists only if:

$$x_m(z) > 0 \iff \frac{w_m}{w_1} > [1 - x_m(z)]^{\frac{1}{1+k(z)}} > 0 \qquad \forall(m,z), \tag{28}$$

$$y_m(z) > 0 \iff \frac{w_m}{w_1} < [1 - x_m(z) y_m(z)]^{-\frac{1}{k(z)}} \qquad \forall(m,z). \tag{29}$$

After substitution of $\{x_k(z), y_k(z) : k = 1, ..., J\}$ as implied by FEC** and OMC**, conditions (28) and (29) define a finite number of interior sets $int\{\Omega_m(z)\}$ of compact sets $\Omega_m(z) \in \Re_+^{J-1}$, one for each sector $z = 1, ..., Z$. An equilibrium exists only if the intersection set $\Omega_m = \Omega_m(1) \cap ... \cap \Omega_m(Z)$ is nonempty for every country $m = 1, ..., J$. Thus, as FEC** and OMC** yield $(x_m(z), y_m(z))$ for all $(m,z)$ as closed form expressions of model parameters and the vector of wages, the necessary condition for existence of an open economy equilibrium with diversification is:

$$0 < x_m(z) < 1 \text{ and } 0 < y_m(z) < 1/x_m(z) \qquad \forall(m,z). \tag{30}$$

The system of $J-1$ labor market clearing conditions LMC** is a continuous vector-valued and real-valued function defined on the compact set $\Omega = \Omega_1 \cap ... \cap \Omega_J$ mapping to the $(J-1)$-dimensional unit vector $\mathbf{1}$, i.e. $f : \Omega \to \Re_+^{J-1}$ such that $f(\omega) = 1$ for all $\omega \in \Omega \subset \Re_+^{J-1}$. If $\mathbf{1} \in \Omega$ held, then Brouwer Fixed-Point Theorem for $f : \Omega \to \Omega$ would imply that a solution to LMC** exists in $\Omega$. Given that $w_m/w_1 = 1$ for all $m = 1, ..., J$ satisfies (28) and (29) for every country $m$ and sector $z$, $\mathbf{1} \in \Omega$ actually holds. Hence, Brouwer Fixed-Point Theorem implies that (30) is a necessary and sufficient condition

[7]This argument is constructive: a numerical solution can be obtained by starting with a guess for the vector of relative wages and then augmenting the relative wage for countries with too much entry (i.e. those with $\sum_{z=1}^{Z} \theta(z) y_j(z) > 1$) and decreasing it for countries with not enough entry (i.e. those with $\sum_{z=1}^{Z} \theta(z) y_j(z) < 1$) until convergence is reached.

for existence of an open economy equilibrium with diversification.

*In practice*. An inspection of lower bounds defined in (28) and upper bounds defined in (29) suggests that $\Omega$ is not empty if trade costs are sufficiently high: everything else being the same, higher trade costs, through $T_{jm}(z)$ and $T_{ml}(z)$, decrease the lower bound and increase the upper bound of the feasible support for a relative wage in every country and sector.

Furthermore, an immediate test for existence of the equilibrium (sufficient but not necessary) consists of solving the FEC** and the OMC** given the same wage across countries $w_m/w_1 = 1$ for all $m = 1, ..., J$ and check if the solution satisfies, sequentially, first (28) and then (29). Once this test is passed then we know that an open equilibrium with diversification exists (at least the one with wage equalization across countries) and the LMC** condition can be solved by iteration starting with the guess $w_m/w_1 = 1$ for all $m = 1, ..., J$ and updating the relative wage up for countries with too much entry and down otherwise.[8]

## 3.4 Discussion

Under the Pareto assumption, our model belongs to the class of general equilibrium trade theories with monopolistic competition under free entry and additive-separable preferences discussed in Arkolakis et al. (2019), who extend previous work in Arkolakis et al. (2012) by relaxing the assumption of CES preferences.[9]

In this section we briefly discuss some related salient implications of trade in an open economy equilibrium with diversification. We start with gains from trade and then we look at trade-induced reallocation across sectors.

*Gains from trade*. The model predicts that in an equilibrium with diversification (i.e. such that $c_j^*(z) > 0$ and $N_j^E(z) > 0$ hold for all $j$'s and all $z$'s), there are gains from trade for every country generated in every sector. This can be seen by rewriting FEC* as:

$$\left(\frac{c_j^*(z)}{c_j^{aut}(z)}\right)^{1+k(z)} = 1 - \sum_{l \neq j}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left(\frac{w_l}{w_j}\frac{c_l^*(z)}{c_j^{aut}(z)}\right)^{1+k(z)} < 1 \quad \forall (j,z)\,, \tag{31}$$

[8]Both procedures can be readily illustrated in the special case of symmetric countries, for which closed form solutions of the necessary and sufficient conditions are obtained. See Appendix B.2.

[9]To see this, with reference to the group of firms producing in country $j$ sector $z$ and selling to a destination $l$, call $v \equiv \hat{p}_l(z)/[\tau_{jl}(z)w_j c] = c_{jl}^*(z)/c \geq 1$ the measure of efficiency of a firm endowed with productivity $1/c$ relative to the other firms in the group. This change of variable makes our analysis in sections 2 and 3 isomorphic to the one in Arkolakis et al. (2019). Appendix C compares these setups in detail.

which implies a lower cost cutoff in open economy $c_j^*(z) < c_j^{aut}(z)$ for every country $j$ and sector $z$. Since welfare (21) is a geometric average of sectoral cutoff productivities $1/c_j^*(z)$, it is necessarily higher under trade than autarky. Furthermore, despite tougher selection, in every sector the measure of varieties available (sourced from anywhere) rises as the cutoff falls:

$$N_l(z) = \frac{\gamma}{\alpha} \frac{\theta(z)}{(\mu_1(z) - \mu_2(z))\, c_l^*(z)}\,, \tag{32}$$

which is obtained from the system of output market clearing conditions $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta(z)w_l$ and choke price relations $\hat{p}_l(z) = w_l c_l^*(z)$ evaluated under Inverse Pareto.[10]

Gains from trade are a classical result, that is customary in frameworks that feature a constrained-efficient equilibrium (such as with CES preferences). In our setup, however, the equilibrium is not constrained efficient. In particular, as discussed in Dhingra and Morrow (2019) and Melitz et al. (2024) for a closed (or fully integrated) economy, the way markups vary under our assumption on preferences implies that better-performing firms are smaller and worse-performing firms are larger than in the social optimum. Yet, trade enhances efficiency even without starting from a constrained-efficient allocation.

*Trade-induced reallocations.* Rewrite the sectoral version of labor market clearing LMC* as an *export equation*

$$N_j^E(z) \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} r_{jl}(c;z) dG_j(c;z) = \rho_j(z) w_j L_j\,,$$

and rewrite the output market clearing OMC* as an *import equation*

$$\sum_{m=1}^{J} N_m^E(z) \int_0^{c_{mj}^*(z)} r_{mj}(c;z) dG_m(c;z) = \theta(z) w_j L_j\,.$$

Accordingly, the ratio of total sales of country $j$ in sector $z$ (to itself and to the rest of the world) divided by total purchase of country $j$ in sector $z$ (from the world including the country itself) is given by $\rho_j(z)/\theta(z)$. It follows that a positive sectoral trade balance is characterized by a sectoral income share that is larger than the corresponding expenditure share: $\rho_j(z) > \theta(z)$.

The system of a sector's labor market clearing condition LMC* and free entry condi-

[10] See Appendix A for a derivation of this result and other aggregate outcomes.

tion FEC* yields the equilibrium relationship between the sector's employment share, $\rho_j(z)$, and the measure of entrants, $N_j^E(z)$, in open economy and in autarky. These are respectively determined as:

$$\frac{N_j^E(z)}{\delta(z)} = \rho_j(z)\frac{L_j}{f_j} \text{ and } \frac{N_j^{E\ aut}(z)}{\delta(z)} = \theta(z)\frac{L_j}{f_j}. \tag{33}$$

The sectoral profitability rate $\delta(z)$ (which is fixed by technological concentration $k(z)$ under Pareto) and market size relative to entry cost $L_j/f_j$ are country and sector specific characteristics that do not vary by trade regime. Therefore, a positive sectoral trade balance in a given sector is associated with relatively more firm entry in open economy than in autarky:

$$\frac{N_j^E(z)}{N_j^{E\ aut}(z)} = \frac{\rho_j(z)}{\theta(z)}. \tag{34}$$

Average employment per entrant $\rho_j(z)L_j/N_j^E(z) = \theta(z)L_j/N_j^{E\ aut}(z)$ is not affected by trade, and this is also true for average labor cost, revenue and profit per entrant. Sectors in which the country is specialized (i.e. those with a positive trade balance) grow unambiguously in terms of employment, sales, profit and measure of firms relative to import-competing sectors (i.e. with a negative trade balance).[11]

As long as this outcome comes with tougher selection, average employment, average revenue and average profit among incumbent firms increase. Furthermore, the measure of incumbent domestic firms unambiguously shrinks in import-competing sectors, both due to a lower measure of potential entrants and tougher selection.[12]

At the country level, the entry of firms is bounded by market size and technological characteristics only, independently on the degree of trade openness. This can be seen by substituting FEC* in the aggregate LMC*, which yields an upper bound to the measure of entrants at the country level:

$$\sum_{z=1}^{Z} \frac{N_l^E(z)}{\delta(z)} = \sum_{z=1}^{Z} \frac{N_l^{E\ aut}(z)}{\delta(z)} = \frac{L_l}{f_l} \quad \forall l. \tag{35}$$

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[11]This result also holds when comparing different levels of trade openness, with respect to sectors whose income shares expand (or shrink) in response to a trade shock.

[12]These effects are present in every sector as cutoffs fall in all sectors when moving from trade to autarky. However, as we will discuss in the next section, if a shock other than a change in trade barriers hits an economy that is already open, then the changes in the cost cutoffs depend on the general equilibrium adjustment of wages.

Therefore, if changes in trade openness lead to more entry in one sector, this must be compensated by less entry in other sectors.

All these considerations about trade-induced reallocation suggest that the ratio of sectoral income share $\rho_j(z)$ (which is endogenous) to the sectoral expenditure share $\theta(z)$ (which is fixed under Pareto) can be considered as a model-based index of "revealed comparative advantage".

# 4 Growth and welfare

How shall we think of growth in the model? Labor is the only factor of production. However, following a classical interpretation (in the spirit of Houthakker, 1955, Rybczynski, 1955, Solow, 1956, 1957 and Jones, 1971), there are also "missing factors", defined as country- and sector-specific resources complementary to labor, which are provided in fixed endowments, cannot be consumed, produced nor to cover entry requirements, and cannot be traded.

Complementarity implies that an exogenous increase in the endowment of a country's sector-specific resource determines a corresponding sector-biased outward shift of its production possibility frontier, which we will refer to as "growth".

## 4.1 Growth as a resource shock

To translate growth in terms of the fundamentals of the model, recall that the exogenous technological coefficient $c_j^{max}(z)$ corresponds to the maximum units of labor per unit of output that a firm producing in sector $z$ of country $j$ operates with. The vector of these technological coefficients characterizes the smallest production possibility frontier of country $j$ for a given endowment of labor $L_j$, defined as the locus of points $\{L_j/c_j^{max}(z) : z = 1, ..., Z\}$ were only the highest possible unit labor requirements to be used in all sectors. For this reason, in the wake of Eaton and Kortum (2002), we call $c_j^{max}(z)$ the "state of technology" of sector $z$ in country $j$. We then define an exogenous growth shock in sector $s \in \{1, ..., Z\}$ of ('home') country $h \in \{1, ..., J\}$ as a sudden and permanent reduction in the exogenous labor requirement $c_h^{max}(s)$, while keeping all other exogenous characteristics of the economy unchanged. We will refer to such shock as an improvement in the state of technology of sector $s$ in country $h$.

To understand the consequences of the shock, let '0' and '1' label the equilibrium allocations before and after the shock respectively. Given the vector of relative wages before the shock, FEC** for sector $s$ in country $h$ can be rearranged as follows:

$$\sum_{l=1}^{J} a_{hl}^{0}(s)\left(\frac{x_l^1(s)}{x_l^0(s)}\right) = \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)}\right)^{k(s)} < 1,$$

$$\text{where } a_{hl}^{0}(s) \equiv \frac{K_{hl}^{0}(s)}{T_{hl}^{0}(s)}\left(\frac{w_l^0}{w_h^0}\right)^{1+k(s)} x_l^0(s) \text{ and } \sum_{l=1}^{J} a_{hl}^{0}(s) = 1.$$

Inverting the implied linear system to solve for the changes in the cutoff costs yields:[13]

$$\begin{aligned}\left(\frac{c_h^{*1}(s)}{c_h^{*0}(s)}\right) &= \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)}\right)^{2k(s)} \\ &< \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)}\frac{c_l^{max\ 1}(s)}{c_l^{max\ 0}(s)}\right)^{k(s)} = \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)}\right)^{k(s)} = \left(\frac{c_l^{*1}(s)}{c_l^{*0}(s)}\right) < 1. \end{aligned} \tag{36}$$

Therefore, for given pre-shock relative wages, sector $s$'s post-shock equilibrium cutoff costs are lower in all countries, with the most pronounced fall in country $h$. Furthermore, when relative wages are held at their pre-shock values, the free entry conditions of all other sectors are not affected by the shock and no changes thus occur in their cutoff costs in any country.

Now consider OMC** for sector $s$ in country $h$. Given the vector of relative wages before the resource shock hits:

$$\sum_{j=1}^{J} b_{jh}^{0}(s)\frac{y_j^1(s)}{y_j^0(s)} = \frac{x_h^0(s)}{x_h^1(s)}\left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{k(s)} = \left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{2k(s)} > 1$$

with

$$b_{jh}^{0}(s) \equiv \frac{K_{jh}^{0}(s)E_{jh}^{0}(s)}{T_{jh}^{0}(s)}\left(\frac{w_h^0}{w_j^0}\right)^{k(s)} x_h^0(s)y_j^0(s) \text{ and } \sum_{j=1}^{J} b_{jh}^{0}(s) = 1,$$

where the second equality is implied by the FEC**. Inverting the associated linear system to solve for the changes in the measures of firms yields:

$$\frac{N_j^{E1}(s)}{N_j^{E0}(s)} = \frac{y_j^1(s)}{y_j^0(s)} = \left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{2k(s)} > 1\ \forall j. \tag{37}$$

[13] Appendix B.3 reports detailed derivations for this paragraph.

Therefore, in all countries, more firms are willing to enter the sector hit by the shock. Also in this case, when wages are held constant at their pre-shock values, output market clearing conditions in other sectors are not affected and thus their measures of entrants do not change.

At pre-shock wages, sector $s$'s cutoff costs fall and its measures of entrants rise in all countries, though to a greater extent in country $h$, whereas they do not change in all other sectors. Then, $y_j^1(s) > y_j^0(s)$ and $y_j^1(z) = y_j^0(z)$ for all $z \neq s$ and all $j =\in \{1, ..., J\}$ imply that labor demand exceeds labor supply in all countries, but to a greater extent in country $h$. Hence, as all countries' labor supplies are exogenously fixed, to restore market clearing wages have to increase everywhere, but to a greater extent in country $h$. Higher wages increase the cutoff costs and decrease the measure of entrants. For sector $s$, they thus dampen the fall in the cutoff cost and the rise in the measure of entrants with respect to their pre-shock values. For all other sectors, they lead to larger cutoffs and smaller measures of entrants with respect to the pre-shock equilibrium. The more a country's wage increases relative to the other countries, the more its cutoffs rise and the measures of its firms fall in the sectors that are not hit by the shock.

## 4.2 Extended "welfare formula"

The previous section has clarified how a resource shock propagates in the general equilibrium of the multi-country multi-sector economy: at least in some sector in every country higher relative wages lead to higher cutoff costs and thus higher choke prices. This raises concerns about the possibility of net welfare losses, which can be readily addressed by noticing that the model generates a handy "welfare formula" in the wake of Arkolakis et al. (2012) and Arkolakis et al. (2019). The formula provides a convenient way to summarize the prediction of the model for welfare in response to trade shocks as in those paper, but also to other shocks, such as a resource shock.

To see this, consider value added $w_j L_j$ in country $j$ and recall that $\theta(z) w_j L_j$ is the expenditure of country $j$ on goods of sector $z$. A fraction of this expenditure is allocated to domestic production, and to characterize this fraction we define country $j$'s domestic trade share in sector $z$ as $\lambda_{jj}(z) \equiv X_{jj}(z)/[\theta(z) w_j L_j]$, where domestic sales in the domestic market are given by

$$X_{jj}(z) = \underbrace{N_j^E(z)[w_j c_j^{max}(z)]^{-k(z)} \hat{p}_j(z)^{k(z)}}_{\text{measure of firms}} \underbrace{\zeta_X(z) \hat{p}_j(z) L_j}_{\text{average firm sales}} .$$

Substituting for the choke price $\hat{p}_j(z) = w_j c_j^*(z)$, and for the measure of entrants $N_j^E(z)$ as implied by the system of free entry and sectoral labor market clearing conditions $\delta(z)\rho_j(z)w_j L_j = f_j w_j N_j^E(z)$ yields the following expression for the cutoff cost in terms of the sectoral domestic trade share $\lambda_{jj}(z)$ and sectoral employment share $\rho_j(z)$:

$$c_j^*(z)^{1+k(z)} = \frac{\theta(z) c_j^{max}(z)^{k(z)}}{\zeta_X(z)\delta(z)} \frac{f_j}{L_j} \frac{\lambda_{jj}(z)}{\rho_j(z)}.$$

Further substitution for the cutoff cost in (21) allows us to evaluate welfare through a formula that recalls the one based on CES preferences in Arkolakis et al. (2012):

$$W_j = \prod_{z=1}^{Z} B(z) \left( \frac{c_j^{max}(z)^{k(z)}}{L_j/f_j} \frac{\lambda_{jj}(z)}{\rho_j(z)} \right)^{-\frac{\beta(z)}{1+k(z)}} \tag{38}$$

where $B(z) \equiv \beta(z)^{\beta(z)} \left(\zeta_X(z)\delta(z)/\theta(z)\right)^{\frac{\beta(z)}{1+k(z)}}$ is a constant bundle of sector-specific taste parameters and technological concentration. It follows that the only endogenous outcomes needed to evaluate the sectoral cutoff costs and thus welfare are the sectoral domestic trade shares $\lambda_{jj}(z)$ and the sectoral employment shares $\rho_j(z)$. Arkolakis et al. (2019) point out that this should be the case for a class of models ours belongs to, so this conclusion confirms their result. However, differently from them, we allow technological concentration to vary across sectors, which is crucial not only for understanding the welfare effect of the resource shock we study, but also for gaining deeper insights into the welfare effects of the very same trade shocks they focus on.[14]

## 4.3 Incomplete "growth pass-through"

Expression (38) is the analogue in our framework of equation (25) in Melitz and Redding (2013), who characterize the equilibrium sectoral productivity cutoffs in the context of multi-country, multi-sector trade models with monopolistic competition for CES preferences across varieties.[15]

[14] Appendix C.3 shows that the impact of a foreign trade shock is smaller for lower cost pass-through on average (which is the point of Arkolakis et al., 2019), but also in sectors characterized by lower technological concentration $k(z)$ with smaller expenditure share $\theta(z)$. Clearly, this differential effect vanishes if differences in sectoral concentration are not considered so that $\theta(z) \equiv \beta(z)$ holds.

[15] With respect to their notation, we have: the equilibrium cost cutoff equal to the inverse of the equilibrium productivity cutoff $c_j^*(z)^{1+k(z)} \equiv 1/(\varphi_{jjz}^*)^{k(z)}$, the upper bound of the cost support equal to the

Comparison with Melitz and Redding (2013) sheds light on a key difference from Arkolakis et al. (2012) not considered in Arkolakis et al. (2019). While with CES preferences a change in the upper bound of the support for the cost distribution that holds the ratio of sectoral domestic trade shares and employment shares constant is fully passed on to the equilibrium cutoff costs, this is not the case in the presence of a choke price:

$$\left.\frac{\partial \log c_j^*(z)}{\partial \log c_j^{max}(z)}\right|_{\frac{\lambda_{jj}(z)}{\rho_j(z)}} = \begin{cases} \frac{k(z)}{1+k(z)} \in (0,1) & \text{with a choke price} \\ 1 & \text{with CES preferences} \end{cases}. \tag{39}$$

The reason for this different behavior, which we may call incomplete relative "growth pass-through", is that, while with CES preferences without a choke price, the cutoff cost in the destination market does not matter for average firm sales, with a choke price in the destination market average firm sales are proportional to the local cutoff cost.

This means that, in the presence of a choke price, if the average cost in sector $z$ of country $j$ decreases through an exogenous reduction in the upper bound $c_j^{max}(z)$ and we do not observe any change in the ratio of domestic trade share $\lambda_{jj}(z)$ over employment share $\rho_j(z)$, then average firm sales fall less than proportionately and the remaining adjustment takes place through more firm entry. Furthermore, the entry of firms is more pronounced in sectors with lower technological concentration. In contrast, with CES preferences the model would not predict any adjustment in either the intensive or the extensive margins and, therefore, also asymmetries across sectors in technological concentration would not play a role.

## 4.4 Welfare response to a resource shock

The incomplete relative growth pass-through documented in (39) has two implications. First, observed changes in domestic trade shares and employment shares are translated into welfare changes at a discount rate $-[1+k(z)]$ that is larger in absolute value than the trade elasticity $-k(z)$. Second, sectors with lower technological concentration contribute proportionally less to welfare changes, as implied by the fact that the

lower bound of productivity support augmented by fixed market access cost $c_j^{max}(z)^{k(z)} \equiv 1/[f_{jjz}\varphi_{min}^{k(z)}]$, a fixed cost of entry $f_j \equiv f_{Ejz}$ (which could be made sector specific also in our setup with no loss of tractability), sectoral consumption shares $\theta(z) \equiv \beta_z$, a profitability index $\zeta_X(z)\delta(z) \equiv \frac{k(z)-(\sigma_z-1)}{(\sigma_z-1)}$, endogenous sectoral employment shares defined as $\rho_j(z) \equiv L_{jz}/\bar{L}_j$, and endogenous domestic trade share defined as $\lambda_{jj}(z) \equiv \lambda_{jjz}$.

proportional change from $k(z)$ to $1+k(z)$ declines as $k(z)$ grows. Hence, a sector's contribution to welfare change is attenuated the more its pass-through is incomplete.

Rewriting welfare (38) to emphasize the relative growth pass-through $k(z)/[1+k(z)]$ yields

$$W_j = \prod_{z=1}^{Z} B(z) \left( \frac{f_j c_j^{max}(z)^{k(z)}}{L_j} \frac{\lambda_{jj}(z)}{\rho_j(z)} \right)^{-\frac{\beta(z)}{k(z)} \frac{k(z)}{1+k(z)}} \tag{40}$$

which converges to the formula in Arkolakis et al. (2012) as the pass-through becomes increasingly complete (i.e. $k(z)/[1+k(z)] \to 1$). Otherwise, expression (40) offers a parametrization of the results discussed (on average) in Arkolakis et al. (2019) that remains highly tractable despite allowing for sectoral heterogeneity in the degree of technological concentration.

In this respect, it is instructive to further contrast our framework characterized by incomplete pass-through, in the sense of (39), with its analogue under CES. The goal here is not to assess the impact of any specific resource shock, but rather to highlight what (40) and its CES analogue imply for our understanding of welfare changes when they are calibrated on the same observed outcomes (i.e. on the same historical data on sectoral domestic trade shares and employment shares) and as on the same exogenous variation.

Let $h$ indicate again the home country. Given welfare (40) and computing percentage changes in log-difference before and after the shock as $\Delta \ln x = \ln x^1 - \ln x^0$ yields

$$\begin{aligned}
&\Delta \ln W_h = \\
&= -\sum_{z=1}^{Z} \beta(z) \Delta \ln(c_h^*(z)) \\
&= -\sum_{z=1}^{Z} \frac{k(z)}{1+k(z)} \frac{\beta(z)}{k(z)} \left[ \Delta \ln f_h - \Delta \ln L_h + \Delta \ln \lambda_{hh}(z) - \Delta \ln \rho_h(z) + k(z) \Delta \ln c_h^{max}(z) \right].
\end{aligned}$$

Consider a shock that hits sector $s$ only in country $h$ (i.e. $\Delta \ln c_h^{max}(s) < 0$ and $\Delta \ln c_h^{max}(z) = 0$ for every $z \neq s$) while keeping all other exogenous parameters (endowments, entry costs, trade costs, technological concentration and preferences) unchanged everywhere.

The impact of this shock is given by:

$$\Delta \ln W_h^{IPT} = \tag{41}$$
$$= \underbrace{\sum_{z=1}^{Z} \frac{k(z)}{1+k(z)} \frac{\beta(z)}{k(z)} \left[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)\right]}_{\text{GE trade effect}} - \underbrace{\frac{k(s)}{1+k(s)} \beta(s) \Delta \ln c_h^{max}(s)}_{\text{direct resource effect}},$$

where we emphasize (in red) that the formula accounts for incomplete pass-through (IPT). Crucially, the shock is designed to work in a controlled environment, with the same interpretation of the "ex-post" result in Arkolakis et al. (2012). Hence, changes in observed outcomes (sectoral domestic trade shares and employment shares) are attributed, by design, to the resource shock. Only under these circumstances, the general equilibrium (GE) effect of the resource shock can be identified and disentangled from the direct effect.

On these premises, if a model equivalent to ours, but with CES preferences across varieties, were calibrated on the same shock and observed outcomes, it would compute a change in welfare equal to:[16]

$$\Delta \ln W_h^{CES} = \sum_{z=1}^{Z} \frac{\beta(z)}{k(z)} \left[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)\right] - \beta(s) \Delta \ln c_h^{max}(s). \tag{42}$$

A comparison between expression (41) and (42) shows that, once calibrated on the same data, the two models' assessments of the welfare changes differ only because of the incomplete pass-through featuring in the IPT expression, which scales down both the GE trade effect and the direct resource effect relative to the CES expression.

## 4.5 Hunting for fantastic beasts

In the Introduction we have defined a "fantastic beast" as a magical creature that cannot be seen unless a wizard searches for it, but that plays a significant role in the real world.

Following this metaphor, we now want to search for welfare changes induced by a resource shock that we fail to see when we look at the data through a CES lens. A shock

[16] We do not report the derivations, since we refer to the canonical multi-country, multi-sector, quantitative trade model with heterogeneous firms, free entry and monopolistic competition, discussed in Arkolakis et al. (2012) and in the handbook chapter by Melitz and Redding (2013).

like this is defined by inverting (42) for $\Delta \ln W_h^{CES} = 0$ to obtain

$$\beta(s)\Delta \ln c_h^{max}(s) = \sum_{z=1}^{Z} \frac{\beta(z)}{k(z)} \left[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)\right] \tag{43}$$

which we can substitute in the wizard's world that is visible through the IPT lens:

$$\Delta \ln W_h^{IPT} = \sum_{z=1}^{Z} \left( \frac{k(z)}{1+k(z)} - \frac{k(s)}{1+k(s)} \right) \frac{\beta(z)}{k(z)} \left[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)\right]. \tag{44}$$

The necessary conditions to find fantastic beasts, that is, to see welfare changes that are invisible through a CES lens, are:

1. **Multi-sector economy**: the contribution of the sector hit by a resource shock in explaining why welfare responses under IPT deviate from those under CES is null; therefore, in a one-sector economy $\Delta \ln W_h^{CES} = 0$ implies $\Delta \ln W_h^{IPT} = 0$.
2. **Open economy**: in closed economy the domestic trade share equals one by construction, hence $\Delta \ln \lambda_{hh}(z) = 0$ holds, and the employment share is fixed such that $\rho_h(z) = \theta(z)$, hence $\Delta \ln \rho_h(z) = 0$ holds; therefore, the GE effect of a resource shock does not operate, only the direct resource effect is at work and it implies proportional welfare changes $\Delta \ln W_h^{IPT} = \frac{k(s)}{1+k(s)} W_h^{CES}$.
3. **Heterogeneity in technological concentration across sectors**: if sectors have the same pass-through (i.e., the same technological concentration $k(s) = k(z) = k$), then welfare responses under IPT do not deviate from those under CES; therefore, if there is no heterogeneity in technological concentration across sectors, then $\Delta \ln W_h^{CES} = 0$ implies $\Delta \ln W_h^{IPT} = 0$ and, more generally, $\Delta \ln W_h^{IPT} = \frac{k}{1+k} W_h^{CES}$. This directly speaks to the "elusive pro-competitive effect" result in Arkolakis et al. (2019) by highlighting that variable elasticity of demand and incomplete pass-through lead to smaller welfare changes than CES and complete pass-through.

It follows that, only by modelling a multi-sector open economy with heterogeneous technological concentration across sectors, one can hope to see some fantastic beasts.

In addition, a sufficient condition to find fantastic beasts can be established as follows. Let $\boldsymbol{z}^{\boldsymbol{h}}_{(-)}$ refer to country $h$'s sectors where the ratio $\rho_h(z)/\lambda_{hh}(z)$ decreases after the shock, such that $[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)] < 0$; these are "bad" sectors with a negative contribution to the country's welfare. Analogously, let $\boldsymbol{z}^{\boldsymbol{h}}_{(+)}$ refer to country $h$'s sectors where the ratio $\rho_h(z)/\lambda_{hh}(z)$ increases after the shock, such that $[\Delta \ln \rho_h(z) - \Delta \ln \lambda_{hh}(z)] >$

0; these are “good” sectors with a positive contribution to the country’s welfare. This grouping is country-specific and has to be interacted with heterogeneity in sector-specific technological concentration to see whether fantastic beasts exist: welfare losses in response to a positive resource shock (“immiserizing growth”), or welfare gains in response to a negative resource shock (“enriching decline”) despite no welfare changes under CES.

Consider a positive CES-neutral resource shock in sector $s$ of country $h$, such that (43) holds for $\Delta \ln c_h^{max}(s) < 0$. Then, a sufficient condition for the country to experience "immiserizing growth" is that the following requirements are both met with at least one of them met strictly:

(i) Technology is less concentrated in the sector hit by the resource shock than in the country’s “bad” sectors, i.e. $k(s) \leq \min_{z \in z^h_{(-)}} \{k(z)\}$;

(ii) Technology is more concentrated in the sector hit by the resource shock than in the country’s “good” sectors, i.e. $k(s) \geq \max_{z \in z^h_{(+)}} \{k(z)\}$.

If these requirements were met with opposite signs, and at least one were met strictly, then we would have a sufficient (though not necessary) condition for "enriching decline".

# 5 Theory with numbers

As proof of concept, we now set out to see whether any fantastic beasts can be identified in real-world data, that is, whether in an IPT setup there is any scope for welfare-reducing positive resource shocks (“immiserizing growth”) or welfare-improving negative resource shocks (“enriching decline”) that would be welfare-neutral in a CES one.

Consider again a sudden and permanent increase in resources specific to the production of sector $s$ in home country $h$, while keeping all other exogenous parameters (i.e. domestic and foreign economic fundamentals) constant. As previously discussed, such shock takes the form of an improvement in the country- and sector-specific state of technology. Formally, let $\boldsymbol{L}$, $\boldsymbol{f}$, $\boldsymbol{\tau}$ and $\boldsymbol{c}$ be the vectors of parameters respectively capturing market sizes, fixed costs, trade costs and the upper bound of the support of the technological coefficients for all countries and sectors. Then we have:

**Definition**. *A 'CES-neutral' domestic resource shock in country h sector s is a change from* $c_h^{max}(s)$ *to* $c_h^{max}(s)' \neq c_h^{max}(s)$ *such that* $\boldsymbol{L}' = \boldsymbol{L}$, $\boldsymbol{f}' = \boldsymbol{f}$, $\boldsymbol{\tau}' = \boldsymbol{\tau}$, $c_j^{max}(z)' = c_j^{max}(z)$

*for all countries* $j \neq h$ *sectors* $z = 1, ..., Z$, $c_h^{max}(z)' = c_h^{max}(z)$ *for all sectors* $z \neq s$ *and* $\Delta \ln c_h^{max}(s) = c_h^{max}(s)' - c_h^{max}(s)$ *satisfies condition* (43).

According to this condition, computing a CES-neutral domestic resource shock in sector $s$ of country $h$ requires trade and production data on the sectoral domestic trade shares $\lambda_{hh}(z)$ and employment shares $\rho_h(z)$, in addition to estimates of the sectoral Cobb-Douglas consumption shares $\beta(z)$ and trade elasticities $k(z)$.[17]

Before looking at the data, we pause to remark that, although a resource shock is different from a trade shock, a CES-neutral domestic resource shock can only exist in open economy. In this respect, the consequences of a CES-neutral resource shock belong to the welfare responses that are channeled through trade.[18]

## 5.1 Data and sources

The main data source we use is the Trade in Value-Added database (TiVA 2023) by the OECD. It provides information on production, consumption, international trade and global economic integration based on Inter-Country Input-Output (ICIO) tables. The data is available annually for the period 1995-2020, for 76 countries (including all OECD countries and the rest of the world) and 45 industries classified by economic activity. Among these, we consider only manufacturing, which corresponds to 17 sectors in the TiVA industry classification.

We use data on consumption in value-added (*CONS_VA*) by source country and sector, and value added (*VALU*) by origin, destination and sector to compute: $\beta(z)$ as the cross-country average expenditure shares at the sector level; $\lambda_{jj}(z)$ as the domestic

[17]We proceed along same lines of the sufficient statistic approach by Arkolakis et al. (2012). This is not immune to criticism. In particular, as pointed out by Melitz and Redding (2015), when using endogenous outcomes as sufficient statistics, one needs to assume that no change occurs in the structural parameters of the model across countries and over time. This is less of a concern in our case as the goal of our proof of concept is not to predict welfare changes, but rather to compare two different model-specific computations of welfare changes, $\Delta \ln W_h^{IPT}$ and $\Delta \ln W_h^{CES}$, for identical values of endogenous outcomes and structural parameters.

[18]The peculiarity of a resource shock is that it directly affects a country and a sector in isolation from other countries and sectors (see, e.g., Pelzl and Poelhekke, 2021, or Caliendo et al., 2018 within the quantitative trade literature). This 'local' feature makes the resource shock fundamentally different from a perturbation to other parameters shaping the equilibrium of the model. For example, changes in trade costs $\tau_{jl}(z)$ affect sector-specific but bilateral parameters, with more than one country involved; changes in market sizes $L_j$ and entry costs $f_j$ concern country-specific parameters that affect all sectors; changes in technological concentration $k(z)$, which impact on expenditure shares $\theta(z)$ and profitability $\delta(z)$, or change in tastes that influence $\beta(z)$, are all about sector-specific parameters, but hit all countries simultaneously.

consumption share in value-added at country-sector level; and the sectoral employment share $\rho_j(z)$ as the share of value-added at country-sector level.

The key parameter of our analysis is the concentration of the distribution of technologies by sector $k(z)$. The structural interpretation of the gravity equation we have provided implies that $k(z)$ corresponds to the trade elasticity. We can, therefore, use existing estimates of the trade elasticity at TiVA industry-level that are available in the literature, relying specifically on those provided by Fontagné et al. (2022).[19]

## 5.2 Preliminary evidence

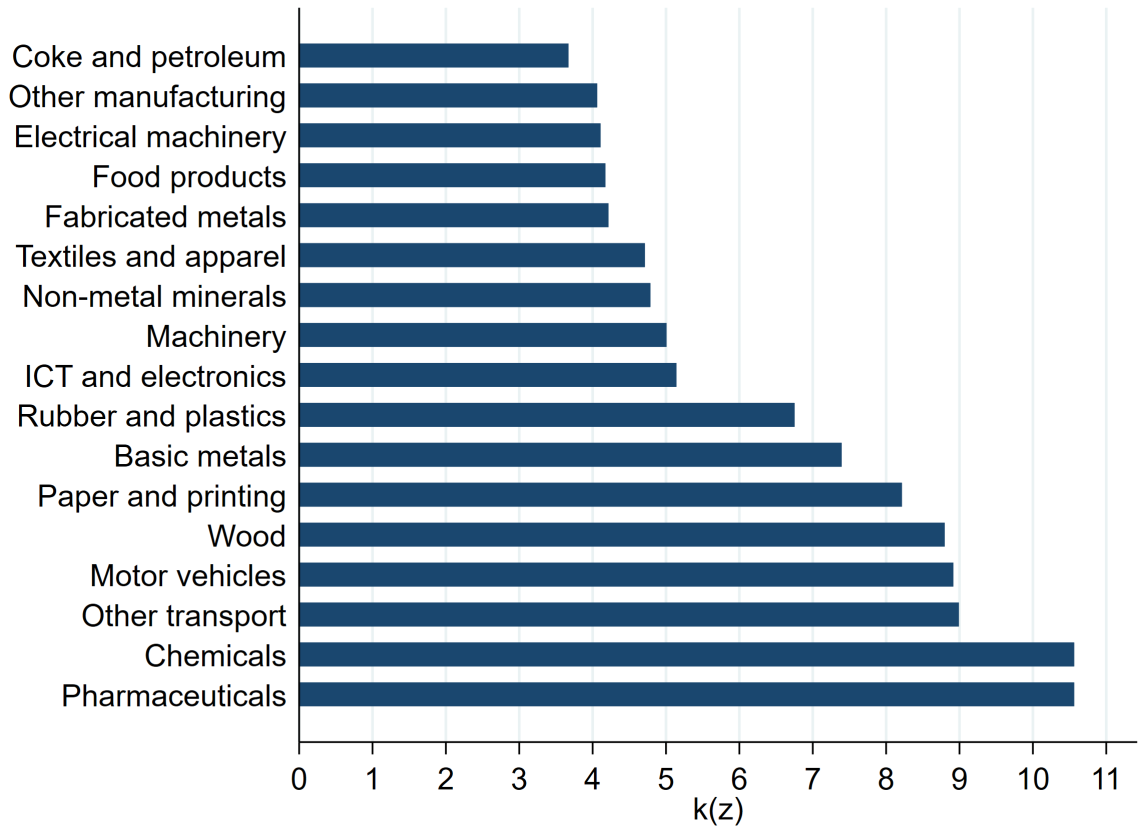
In light of the discussion in Section 5.5, to find fantastic beasts in a real-world multi-sector open economy it is necessary that there is variation in the technological concentration across sectors as well as in the sectoral domestic trade shares, employment shares and their ratios within countries over time.

We start by looking at the variation of the technological concentration parameter. Figure 1 reports the estimated $k(z)$ across manufacturing industries. Sector C-19 *Coke and refined petroleum product* is the one with the lowest concentration, $k(z) = 3.67$; sectors C-21 *Pharmaceuticals products* and C-20 *Chemical products* exhibit the highest concentration, $k(z) = 10.56$.[20] Accordingly, based on their concentration parameters, a resource shock in the petroleum industry is associated with a relative growth pass-through of $78, 59\%$, while a resource shock in the pharmaceutical or chemical industries is associated with a relative growth pass-through of $91, 35\%$.

We turn next to the variation over time in the sectoral domestic trade shares, the employment shares and their ratios at the country level. Using data from all 76 countries, all 17 manufacturing industries and all years from 2000 to 2020, we compute 1-year, 3-year and 5-year changes in log difference of country-specific sectoral domestic trade shares, sectoral employment shares and their balance. The three panels in Figure 2 report the results for the three statistics referred to the 5-year change; the 1-year and 3-year changes exhibit similar patterns. In both the domestic trade shares and the em-

[19] We have chosen this source because the estimates of the trade elasticity are computed at the level of TiVA sectors and are thus consistent with the rest of our analysis. For a comparison with other estimates in the literature and an assessment of robustness, we refer the interested reader to Fontagné et al. (2022).

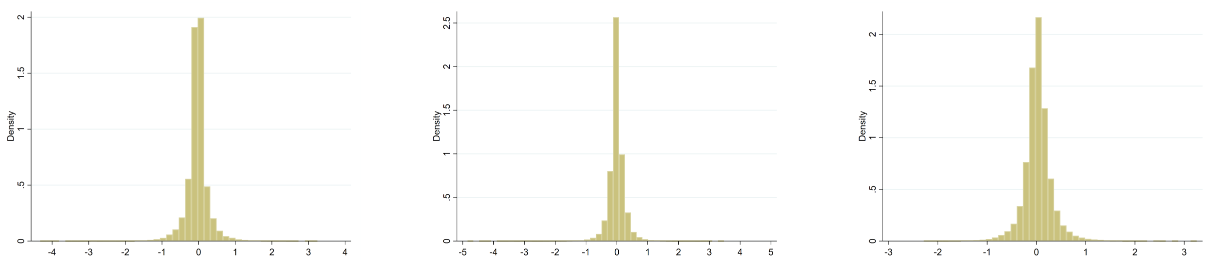
[20] Recall that higher concentration implies that upon entry a firm is more likely to draw a unit labor requirement closer to the upper bound of the support, and it is therefore more likely to subsequently leave the market without producing. Higher concentration also means that, among firms that actually produce, there is a larger share of small firms that have much higher labor unit input requirements than those of the much fewer more efficient and larger firms in the sector.




Note: sectors are the manufacturing industries in TiVA classification; estimates of the parameter $k(z)$ are from Fontagné et al. (2022).

Figure 1: Estimates of the parameter of technological concentration by sector

ployment shares there is substantial variation around zero, which is suggestive of a stationary data generating process in log differences. Furthermore, while some sectors expand while other sectors shrink, the employment shares do not change proportionally with the domestic trade shares.



$\Delta \ln \lambda_{jj}(z)$ $\Delta \ln \rho_j(z)$ $\Delta \ln \rho_j(z) - \Delta \ln \lambda_{jj}(z)$

Figure 2: Change in domestic trade shares and employment shares

Based on these premises, it is now possible to apply equation (43) to compute the

CES-neutral resource shock in sector $s$ of country $h$ and then evaluate the corresponding IPT welfare change through expression (44).
[21]

Table 1 reports the summary statistics for the computed CES-neutral resource shock $\Delta \ln c_h^{max}(s)$. The mean of the distribution is 2.3% over a 1-year interval, 7.4% over a 3-year interval and 12.6% over a 5-year interval. While the mean is positive, there is enough variation to allow for both shocks that improve the state of technology ($\Delta \ln c_h^{max}(s) < 0$) and shocks that worsen it ($\Delta \ln c_h^{max}(s) > 0$). This feature comes from the the data rather than by construction and makes it feasible to look for immiserizing growth as well as enriching decline.

Table 1: Summary statistics on CES-neutral resource shock

| | mean | std. dev. | p10 | p50 | p90 |
|---|---|---|---|---|---|
| 1-year % change | 2.310 | 36.347 | -25.620 | 2.027 | 31.675 |
| 3-year % change | 7.416 | 56.648 | -39.029 | 5.513 | 57.068 |
| 5-year % change | 12.581 | 70.299 | -44.904 | 8.932 | 74.473 |

Before doing that, however, it would be reassuring to know that the computed CES-neutral resource shocks are in the same ballpark as actual resource shocks documented in the literature. As a recent example falling in our period of observation, Caliendo et al. (2018) document a shale oil boom of 9% TFP growth in North Dakota over the period from 2007 to 2012 and a 14.6% TFP growth in the industry of Computers and Electronics in California during the same period. Hence, our CES-neutral resource shocks are broadly in line with actual technological shocks reported in other studies.

## 5.3 Finding fantastic beasts

Expressions (43) and (44) makes clear that the exact values of CES-neutral $\Delta \ln c_h^{max}(s)$ and its impact on $\Delta \ln W_h^{IPT}$ depend on which sector $s$ and which country $h$ those expressions are applied to. Moreover, while the sign of the CES-neutral shock is not sector specific, the sign of a country's welfare response depends on the technological concentration of the sector hit by the shock relative to all other sectors.

[21] It should be emphasized that we are not saying that the only shock affecting the data generating process was a resource shock. Several shocks (e.g. foreign shocks as in Arkolakis et al., 2012) might well have hit the economy, and thus driven the observed changes of the domestic trade shares and employment shares. However, this immaterial for the computation of the counterfactual CES-neutral resource shock that would have had the same observed effects as the combined actual shocks.

To highlight the importance of the sectoral choice, we first consider the US case. While we focus on 5-year percentage changes for parsimony, the conclusions are all confirmed also using 1-year and 3-year changes. In the 21 years from 2000 to 2020, the CES-neutral domestic resource shock over 5 years is found to be expansionary only four times: in year 2013 with $\beta(s)\Delta \ln c_h^{max}(s) = -0.05\%$, year 2016 with $\beta(s)\Delta \ln c_h^{max}(s) = -0.005\%$, year 2019 with $\beta(s)\Delta \ln c_h^{max}(s) = -0.18\%$, and year 2020 with $\beta(s)\Delta \ln c_h^{max}(s) = -0.7\%$.

For the sake of argument, we choose the sector $s$ to be shocked based on observed resource shocks. In particular, we refer to Caliendo et al. (2018) who document resource shocks for two sectors: *Coke and petroleum* (henceforth, simply 'oil'), a sector with the lowest concentration $k(oil) = 3.67$ and a consumption share $\beta(oil) = 5.90\%$; and *ICT and electronics* (henceforth, simply 'ICT'), a sector with about median concentration $k(ict) = 5.16$ and a consumption share $\beta(ict) = 6.75\%$. Plugging these numbers in expression (43), we calculate a positive CES-neutral resource shock for the US oil sector during 2009-2013 equal to $\Delta \ln c_{US}^{max}(oil) = -0.05\%/\beta(oil) = -0.85\%$. By expression (44), the corresponding change in US welfare amounts to $\Delta \ln W_{USA}^{IPT} = +0.015\%$. Hence, in the time period under consideration, the elasticity of US welfare to the domestic CES-neutral domestic resource shock in the oil sector is 0.020. Analogously, we can calculate a positive CES-neutral resource shock for the US ICT sector for the same period, which evaluates to $\Delta \ln c_{USA}^{max}(ict) = -0.05\%/\beta(ict) = -0.74\%$. The corresponding welfare change is $\Delta \ln W_{USA}^{IPT} = +0.018\%$, with the elasticity of welfare to the shock equal to 0.025. In both cases, the IPT setup captures welfare gains that do not arise under CES.

Having highlighted the importance of the sectoral dimension, we can extend the analysis to all 76 countries, 17 sectors and years in the dataset. We again focus on 5-year time intervals, with 1-year and 3-year intervals generating similar results.

Pooling all countries, Figure 3 depicts how welfare changes in response to an expansionary CES-neutral domestic resource shock ($\Delta \ln c_{US}^{max} < 0$). Each dot refers to a sector. Its vertical coordinate corresponds to the mean point estimate of the impact on country's welfare given that the shock hits a given sector whose technological concentration is reported on the horizontal axis in ascending order, as in Figure (1). Around the mean, the figure also reports the 95% confidence intervals, based on variation across countries and years. The figure shows that immiserizing growth materializes when an expansionary CES-neutral resource shock hits a sector characterized by relatively low

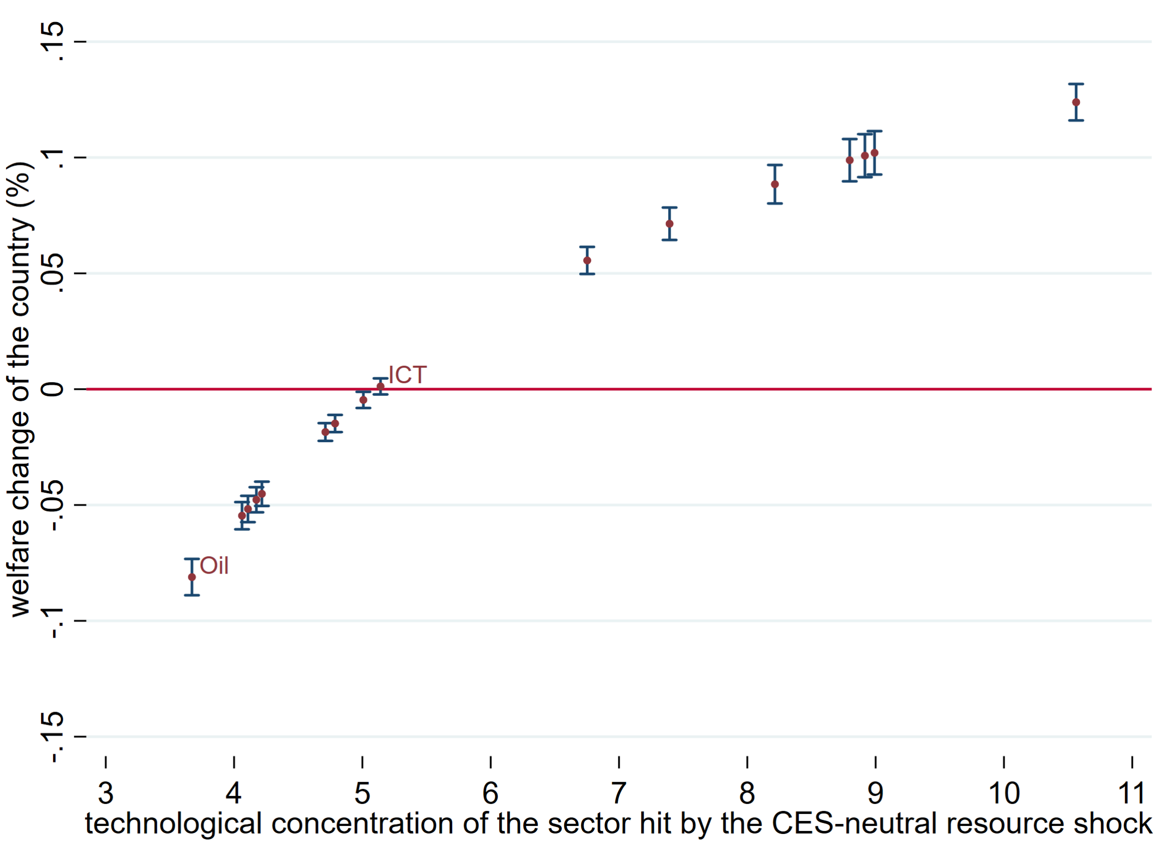



Figure 3: Welfare response to expansionary resource shocks

technological concentration. In contrast, when the shock hits a sector with high technological concentration, the IPT welfare-change is positive.

Symmetric results are portrayed in Figure 4 for contractionary CES-neutral domestic resource shocks. Mean point estimates are again precisely estimated, and reveal enriching decline when the shock hits sectors characterized by low concentration.

In both figures we keep track of the two sectors discussed in the US case: Oil and ICT. The first sector is the typical sector considered in the literature on the “Dutch disease" as the cause of immiserizing growth. As this is the sector with the lowest technological concentration and thus the most incomplete pass-through, our model provides new insights on the classical result based on monopolistic competition, firm heterogeneity and markup distortions. The second sector is important for recent growth episodes, as suggested by Caliendo et al. (2018). Interestingly, in the two figures ICT appears to be the sector associated with the concentration threshold below which immizerizing growth or enriching decline materialize. In other words, a CES-neutral resource shock in ICT is indeed neutral also from welfare viewpoint.

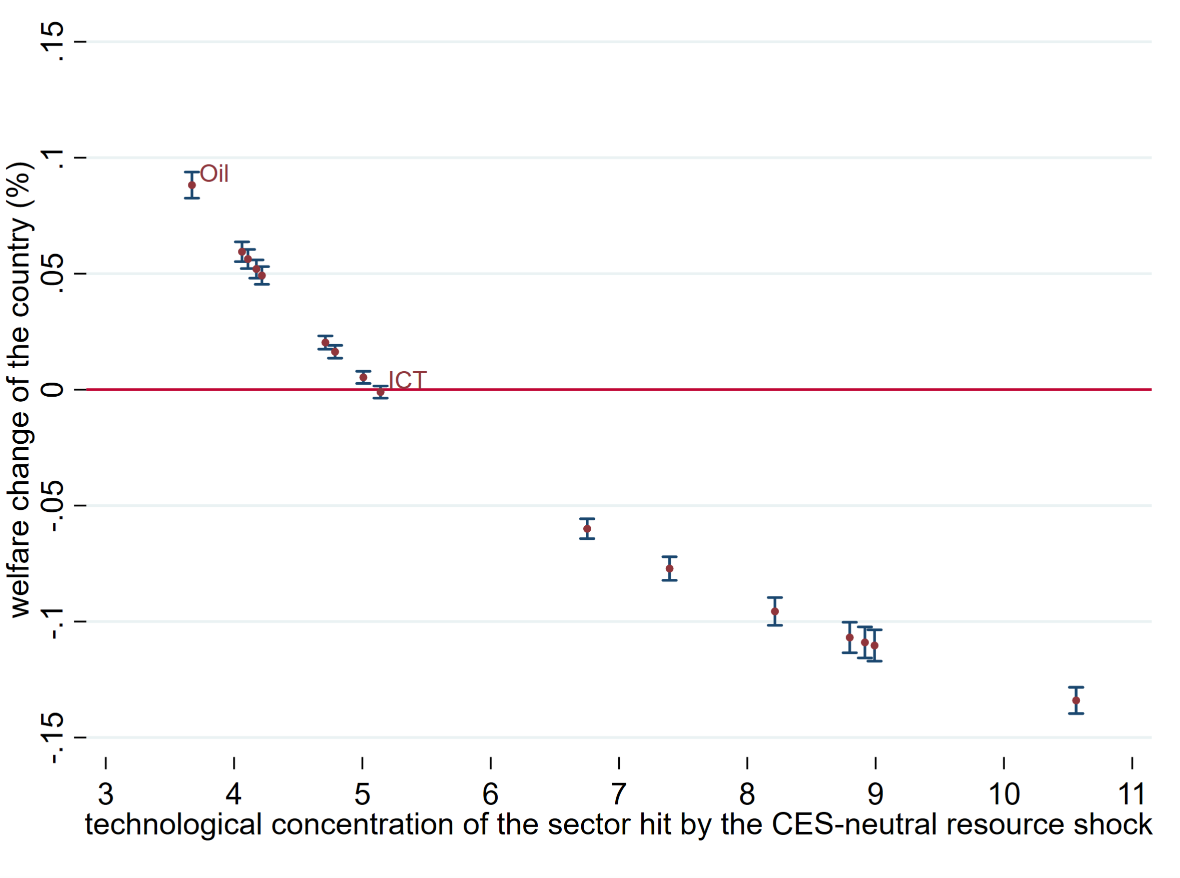



Figure 4: Welfare response to contractionary resource shocks

Figures 5 and 6 repeat the exercise across 15 selected countries. For each country, Figure 5 depicts the welfare change in response to an expansionary CES-neutral resource shock against technological concentration, with 95% confidence intervals relying here on time variation only. The conclusion that we have reached on the pooled analysis finds confirmation in the overall association of lower concentration with smaller positive or more negative welfare changes. There is, however, a lot of cross-country variation due to different sectoral specialization. While the evidence of immiserizing growth is not statistically significant everywhere, in all significant cases it materializes when the shock hits sectors with lower technological concentration, in particular the oil sector.

When interpreting the magnitude of the effects, one should consider that, on average, each sector weighs less than 6% in consumers' expenditure. At the median magnitude of a CES-neutral shock (i.e. 8.9%), even if the shock were entirely transmitted to welfare in proportion with the consumption share (i.e., there were no general equilibrium feedback effects and pass-through were complete), the response would have be

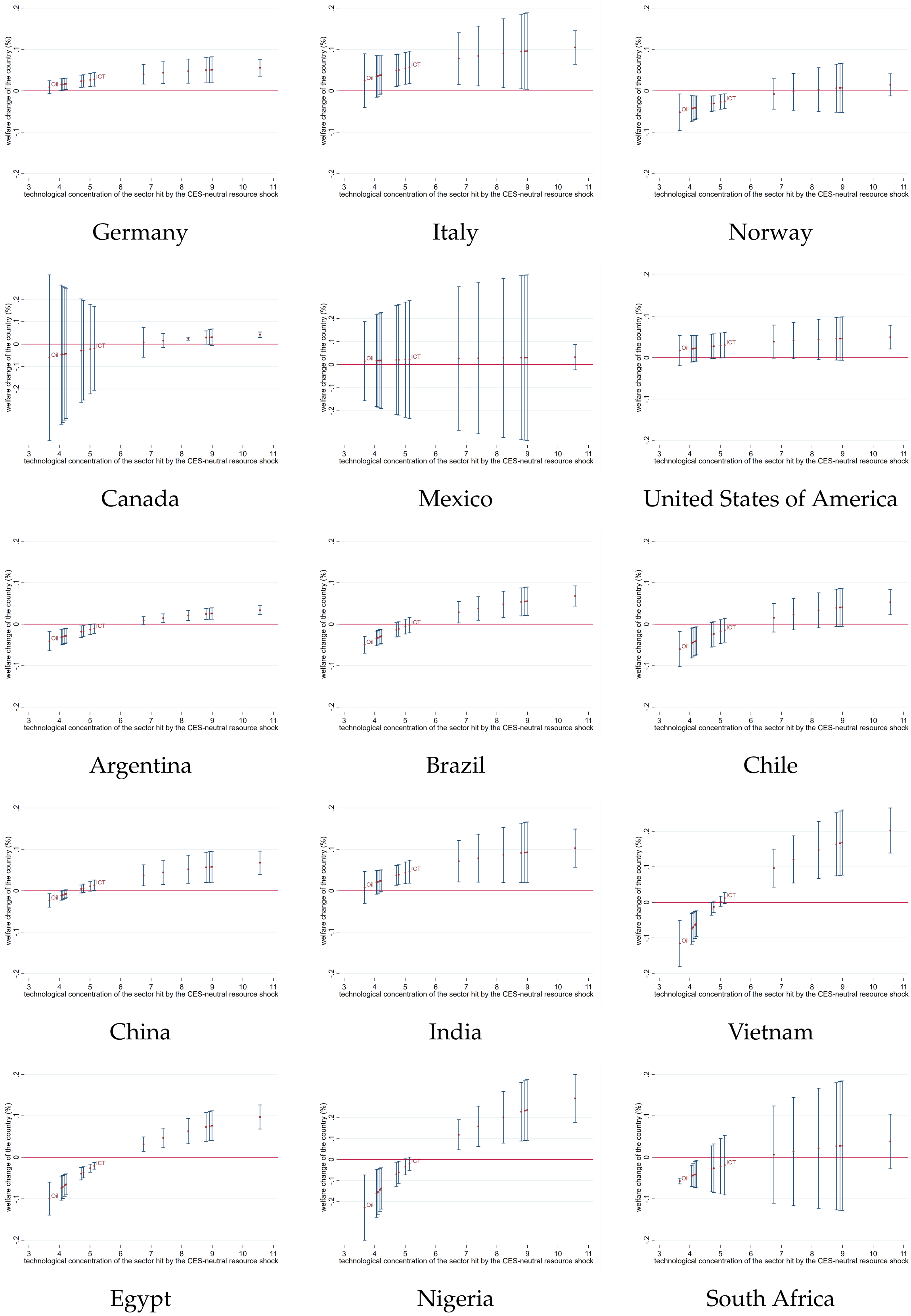



Figure 5: Welfare responses to expansionary resource shocks, in selected economies

smaller than 0.5%. Thus, to help comparison across countries, the scale of the vertical axis is set in the range $\pm 0.2\%$ everywhere.

Although the pattern of an increasing average relationship is common, the range of sectors conducive to immizerizing growth if hit by a CES-neutral expansionary shock varies substantially. For example, a shock hitting ICT implies statistically significant welfare losses in three countries (Norway, Argentina and Egypt) and statistically significant welfare gains in five countries (Germany, Italy, USA, China and India).

Figure 6 considers a contractionary CES-neutral resource shock. There is a robust negative relationship between technological concentration and welfare changes everywhere, with statistically significant evidence of enriching decline when the shock hits sectors with lower technological concentration.

# 6 Conclusion

We have developed a quantitative trade model with incomplete constant absolute pass-through (IPT) that can predict both “immiserizing growth”and “enriching decline”when standard models featuring CES demand and thus complete pass-through predict neither. In the former case, a domestic resource increase that does not change welfare under CES leads to lower welfare under IPT. In the latter case, a domestic resource reduction that does not change welfare under CES leads to higher welfare under IPT.

We have shown that the reason for these divergences is that IPT allows for richer reallocation patterns between firms and sectors than CES does. We have argued that constant absolute pass-through is not essential. Nevertheless, together with the assumption that firms’ labor unit input requirements are (inverse) Pareto distributed, it leads to a simple expression of national welfare as a function of a very limited number of sufficient statistics. The Pareto assumption also allows for measuring the concentration of such technological coefficients across firms through a single exogenous parameter.

Using "CES-neutral" to refer to a resource shock that does not change welfare under CES, we can summarize our results as follows. If an expansionary CES-neutral domestic resource shock hits a sector with low technological concentration, a country may still experience immiserizing growth, that is, a welfare loss under IPT. Vice versa, if a contractionary CES-neutral domestic resource shock hits a sector with low technological concentration, the country may still experience enriching decline, that is, a

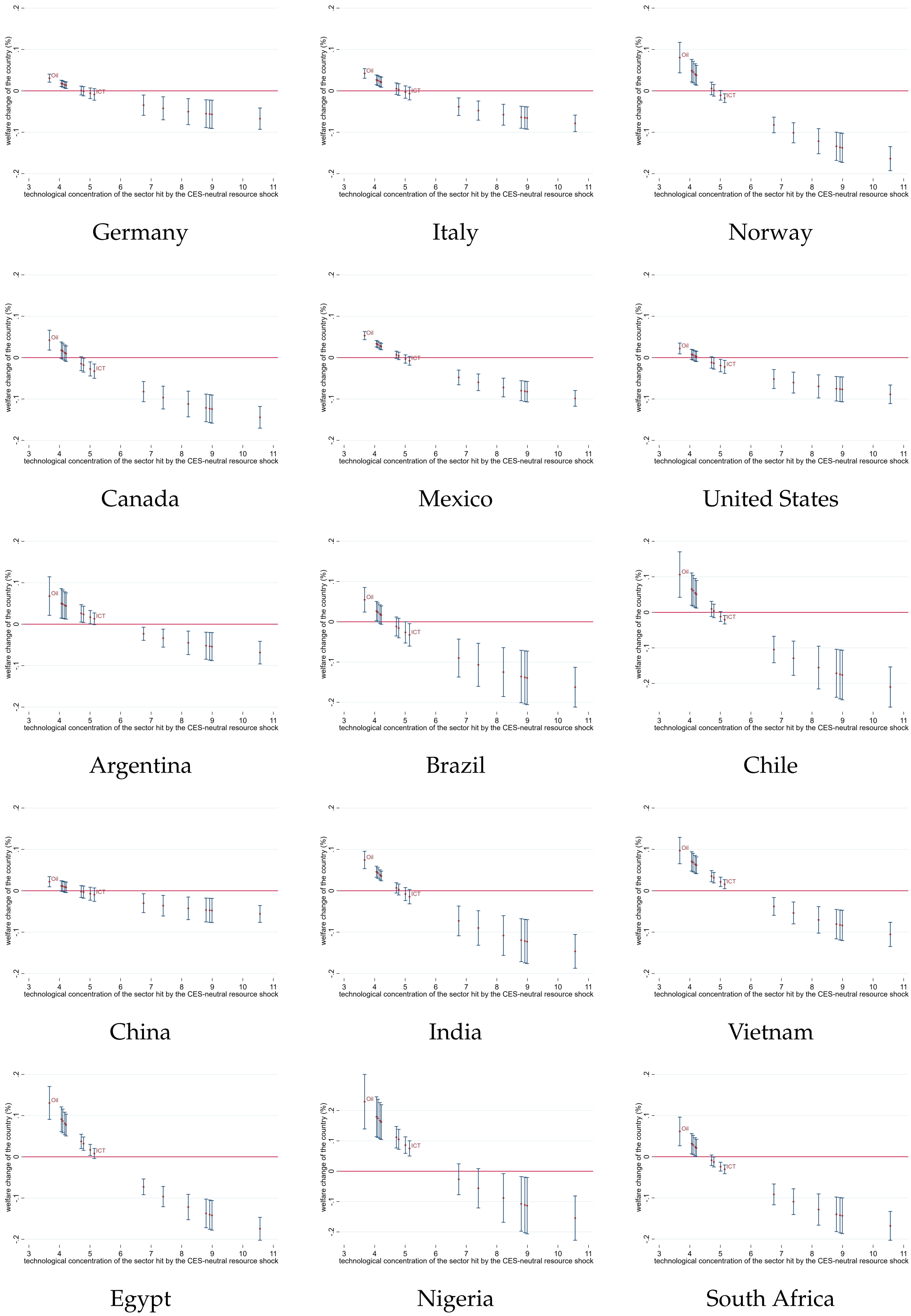



Figure 6: Welfare responses to restrictive resource shocks, in selected economies

welfare gain under IPT. These results are derived both theoretically and empirically for resource shocks of realistic magnitude as proof of concept.

Despite these novel insights, in this paper we have not exploited the full potential of our model, which is ready for full-fledged positive and normative quantitative exercises based on calibration, validation and simulation of all kinds of counterfactual scenarios, without much additional complexity with respect to the commonly used CES-based quantitative trade models. We leave these exercises to future exploration.

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# Appendices

## A Detailed derivations of results in the general framework

In this section we describe in detail Bulow and Pfleiderer (1983) preferences and the derivations of the expressions for expenditure shares and welfare, discussed in the main text.

### A.1 Properties of BP preferences

Consumption of a variety $i$ of a good of sector $z$ produced in country $j$ provides utility to a consumer located in country $l$ given by

$$u_{jl}(z): \quad \alpha q^c_{jl}(i;z) - \frac{\gamma}{1-\sigma} q^c_{jl}(i;z)^{1-\sigma}, \ \{\alpha > 0, \gamma > 0, \sigma < 0\} \text{ or } \{\alpha \leq 0, \gamma < 0, \sigma \in (0,1)\}.$$

This implies first, second and third order derivatives:

$$\begin{aligned} u'_{jl}(z): &\quad \alpha - \gamma q^c_{jl}(i;z)^{-\sigma}, \\ u''_{jl}(z): &\quad \sigma\gamma q^c_{jl}(i;z)^{-\sigma-1}, \\ u'''_{jl}(z): &\quad -(1+\sigma)\sigma\gamma q^c_{jl}(i;z)^{-\sigma-2}. \end{aligned}$$

If $\sigma < 0$ then $\gamma > 0$ is a necessary condition for decreasing marginal utility $u''_{jl}(z) < 0$, thus, there exists a finite "choke price" such that the marginal utility is null. Under this scenario, there is a maximum consumption below which marginal utility is positive $u'_{jl}(z) \geq 0$, provided that $\alpha > 0$. If $\sigma \in (-1, 0]$ the marginal utility is decreasing and convex, i.e. $u'''_{jl}(z) \geq 0$, otherwise, for $\sigma < -1$ the marginal utility is decreasing and concave, i.e. $u'''_{jl}(z) \leq 0$. Quadratic preferences are the case for $\sigma = -1$ implying linear marginal utility.

If $\sigma \in (0, 1)$ then $\gamma < 0$ is a necessary condition for a decreasing marginal utility, i.e. $u'' < 0$, and it is concave, i.e. $u'''_{jl}(z) \leq 0$. Under these circumstances, $\alpha \leq 0$ is a necessary condition for the marginal utility going to zero as quantity goes to infinity. However, the marginal utility at zero consumption goes to infinity, thus, there is no choke price. CES preferences are the special case for $\alpha = 0$ and the elasticity of substitution across varieties is equal to $1/\sigma$.

Both specifications, either $\{\alpha > 0, \gamma > 0, \sigma < 0\}$ or $\{\alpha \leq 0, \gamma < 0, \sigma \in (0,1)\}$, imply a constant absolute pass-through of marginal cost on price, equal to $1/(1-\sigma)$. However, the attractive feature of the specification with $\{\alpha > 0, \gamma > 0, \sigma < 0\}$ is that it prescribes a finite choke price. When preferences are defined in a multi-country and multi-sector framework, such as in (1), the optimal demand for a consumer with marginal utility of income $\lambda_l > 0$ is null at the choke price

$$\hat{p}_l(z) \equiv \frac{\alpha}{\lambda_{jl}(z)}, \quad \lambda_{jl}(z) \equiv \frac{\sum_{j=1}^{J} u_{jl}(z)}{\beta(z) U_l} \lambda_l.$$

## A.2 Firm-level variables

This setup, has relevant implications for several equations characterizing the model. Starting with utility (1), the corresponding expression under Bulow-Pfleiderer preferences can be obtained by replacing 2 by $1-\sigma$. Thus, focusing on the case in which $\alpha > 0, \gamma > 0$ and $\sigma < 0$ hold, the Marshallian demand function is given by

$$q_l^*(p;z) = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} \left(1 - \frac{p}{\hat{p}_l(z)}\right)^{\frac{1}{-\sigma}},$$

with price elasticity

$$\varepsilon_l(p;z) = \frac{1}{-\sigma} \frac{p/\hat{p}_l(z)}{1 - p/\hat{p}_l(z)}.$$

This expression implies that markup is given by

$$mkp_{jl}^*(p;z) = \frac{\varepsilon_l(p;z)}{\varepsilon_l(p;z) - 1} = \frac{\frac{1}{-\sigma}}{(1 + \frac{1}{-\sigma}) - \hat{p}_l(z)/p},$$

and for a linear technology the profit-maximizing price is given by

$$p_{jl}(c;z) = mkp_{jl}^*(p;z)\tau_{jl}(z)w_j c = \frac{-\sigma\hat{p}_l(z) + \tau_{jl}(z)w_j c}{1-\sigma}$$

while the relation between the choke price and the cutoff unit labor requirement is $\hat{p}_l(z) = w_j\tau_{jl}(z)c_{jl}^*(z)$. The firm performance measures are:

- Price

$$p_{jl}(c;z) = \frac{\hat{p}_l(z)}{1-\sigma}\left(-\sigma + \frac{c}{c_{jl}^*(z)}\right),$$

- Markup

$$mkp_{jl}^*(c;z) = \frac{1}{1-\sigma}\left(1 - \sigma\frac{c_{jl}^*(z)}{c}\right),$$

- Quantity

$$q_{jl}(c;z) = L_l \left(\frac{\alpha}{(1-\sigma)\gamma}\right)^{\frac{1}{-\sigma}} \left(1 - \frac{c}{c_{jl}^*(z)}\right)^{\frac{1}{-\sigma}},$$

- Employment

$$\ell_{jl}(c;z) = \tau_{jl}(z)c_{jl}^*(z)L_l \left(\frac{\alpha}{(1-\sigma)\gamma}\right)^{\frac{1}{-\sigma}} \left(1 - \frac{c}{c_{jl}^*(z)}\right)^{\frac{1}{-\sigma}} \frac{c}{c_{jl}^*(z)},$$

- Revenue

$$r_{jl}\left(c;z\right)=\frac{w_j\tau_{jl}(z)c^*_{jl}(z)L_l}{1-\sigma}\left(\frac{\alpha}{(1-\sigma)\gamma}\right)^{\frac{1}{-\sigma}}\left(1-\frac{c}{c^*_{jl}(z)}\right)^{\frac{1}{-\sigma}}\left(-\sigma+\frac{c}{c^*_{jl}(z)}\right),$$

- Profit

$$\pi_{jl}\left(c;z\right)=w_j\tau_{jl}(z)c^*_{jl}(z)L_l\left(\frac{\alpha}{(1-\sigma)\gamma}\right)^{\frac{1}{-\sigma}}\frac{-\sigma}{1-\sigma}\left(1-\frac{c}{c^*_{jl}(z)}\right)^{1+\frac{1}{-\sigma}}.$$

Therefore, for a given wage $w_j$ and cost cutoff $c^*_{jl}(z)$, all firm level variables are univariate continuous functions of the relative cost $c/c^*_{jl}(z)\in(0,1)$.

### A.3 Sectoral expenditure share

Given utility (1) and budget constraint (2), a consumer of country $l$ allocates expenditure over a quantity $q^c_{jl}\left(i;z\right)\geq 0$ based on the following first order condition for an interior solution

$$\frac{\beta(z)U_l}{\sum_{j=1}^J u_{jl}(z)}\left(\alpha-\gamma q^c_{jl}\left(i;z\right)^{-\sigma}\right)=\lambda_l p_{jl}\left(i;z\right)\quad\forall(j,z),$$

and the budget constraint

$$\sum_{z=1}^Z\sum_{j=1}^J\int_0^{N_{jl}(z)}p_{jl}\left(i;z\right)q^c_{jl}\left(i;z\right)di=w_l\,.$$

Given a binding budget constraint, let the marginal utility of income be $\lambda_l>0$. Introducing the definitions of sectoral quantity index $\mathbb{Q}_l(z)$ and sectoral price index $\mathbb{P}_l(z)$ yields:

$$\frac{\beta(z)U_l}{\mathbb{Q}_l(z)}\left(1-\frac{\gamma}{\alpha}q^c_{jl}\left(i;z\right)^{-\sigma}\right)=\lambda_l p_{jl}\left(i;z\right)\quad\forall(j,z),$$
$$\sum_{z=1}^Z\mathbb{P}_l(z)\mathbb{Q}_l(z)=w_l\,.$$

The choke price $\hat{p}_l(z)>0$ is defined as the minimum (finite) price at which the consumer of country $l$ optimally allocates zero consumption on a variety of sector $z$. Thus, evaluating the first order condition for consumption at the choke price for any pair of sectors, $z'$ and $z''$, and then taking the ratio yields:

$$\frac{\hat{p}_l\left(z''\right)}{\hat{p}_l\left(z'\right)}\frac{\mathbb{Q}_l(z'')}{\mathbb{Q}_l(z')}=\frac{\beta(z'')}{\beta(z')}.$$

The definition $\eta_l(z)\equiv\mathbb{P}_l(z)/\hat{p}_l\left(z\right)$ implies:

$$\frac{\mathbb{P}_l(z'')\mathbb{Q}_l(z'')}{\mathbb{P}_l(z')\mathbb{Q}_l(z')}=\frac{\beta(z'')\eta(z'')}{\beta(z')\eta(z')}.$$

Substituting in the budget constraint yields:

$$\sum_{z''=1}^{Z} \mathbb{P}_l(z'')\mathbb{Q}_l(z'') = \frac{\mathbb{P}_l(z')\mathbb{Q}_l(z')}{\beta(z')\eta(z')} \sum_{z''=1}^{Z} \beta(z'')\eta(z'') = w_l \,,$$
$$\mathbb{P}_l(z')\mathbb{Q}_l(z') = \frac{\beta(z')\eta(z')}{\sum_{z''=1}^{Z} \beta(z'')\eta(z'')} w_l.$$

Therefore, the expenditure share in goods from any sector $z'$ is given by $\theta(z') = \frac{\beta(z')\eta(z')}{\sum_{z''=1}^{Z} \beta(z'')\eta(z'')}$.

# B Results with quadratic preferences and Inverse Pareto

In this section we outline the solution of the model with quadratic preferences and a distribution of technological coefficients given by $G_j(c;z) = (c/c_j^{max}(z))^{k_j}$. Quadratic preferences correspond to the special case in (1) for $\sigma = -1$. All derivations of firm-level variables and aggregate variables go through after this parametrization, thus, we do not repeat them.

## B.1 Aggregate variables given an Inverse Pareto distribution

Expenditure and utility due to individual consumption in country $l$ on goods from sector $z$ sourced from country $j$ are:

$$\begin{aligned} e_{jl}(z) &= \frac{\alpha}{\gamma}\left(\mu_1(z) - \mu_2(z)\right)\hat{p}_l(z) N_{jl}(z) \\ u_{jl}(z) &= \frac{\alpha^2}{2\gamma}\left(1 - \mu_2(z)\right) N_{jl}(z). \end{aligned}$$

Quantity index and price index in a certain country $l$ and sector $z$ are

$$\begin{aligned} \mathbb{Q}_l(z) &\equiv (1/\alpha)\sum_{j=1}^{J} u_{jl}(z) = \frac{\alpha}{2\gamma}\left(1 - \mu_2(z)\right) N_l(z), \\ \mathbb{P}_l(z) &\equiv \sum_{j=1}^{J} e_{jl}(z)/\mathbb{Q}_l(z) = \left(\frac{2(\mu_1(z) - \mu_2(z))}{1 - \mu_2(z)}\right)\hat{p}_l(z). \end{aligned}$$

The system of output market clearing $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta(z) w_l$ and choke price $\hat{p}_l(z) = w_l c_l^*(z)$ yields the measure of varieties of sector $z$ available in country $l$ sourced from anywhere:

$$N_l(z) = \frac{\gamma}{\alpha}\frac{\theta(z)}{\left(\mu_1(z) - \mu_2(z)\right) c_l^*(z)} \,.$$

Aggregate revenue made by firms producing in country $j$ sector $z$ and selling to country $l$ is given by $R_{jl}(z) \equiv N_{jl}(z)\int_0^{c_{jl}^*(z)} r_{jl}(c;z) dG_j(c;z)/G_j(c_{jl}^*(z);z)$ and the corresponding aggregate profit is given by $\Pi_{jl}(z) \equiv N_{jl}(z) \int_0^{c^*_{jl}(z)} \pi_{jl}(c;z) dG_j(c;z)/G_j(c^*_{jl}(z);z)$,

$$
\begin{aligned}
R_{jl}(z) &= \underbrace{N_j^E(z)[\tau_{jl}(z) w_j c_j^{max}(z)]^{-k(z)} \hat{p}_l(z)^{k(z)}}_{\text{mass of exporters}} \times \underbrace{\zeta_X(z)\hat{p}_l(z) L_l}_{\text{average revenue}} \\
\Pi_{jl}(z) &= \underbrace{N_j^E(z)[\tau_{jl}(z) w_j c_j^{max}(z)]^{-k(z)} \hat{p}_l(z)^{k(z)}}_{\text{mass of exporters}} \times \underbrace{\zeta_\Pi(z)\hat{p}_l(z) L_l}_{\text{average profit}}
\end{aligned}
$$

with $\zeta_X(z) \equiv k(z)\left(\frac{1}{k(z)} - \frac{1}{k(z)+2}\right)\frac{\alpha}{4\gamma}$ and $\zeta_\Pi(z) \equiv k(z)\left(\frac{1}{k(z)} - \frac{2}{k(z)+1} + \frac{1}{k(z)+2}\right)\frac{\alpha}{4\gamma}$. To obtain these expressions, we have substituted for the fraction of entrants in country $j$ that become exporters to country $l$, $N_{jl}(z) = (c^*_{jl}(z)/c_j^{max}(z))^{k(z)} N_j^E(z)$, and then for the corresponding export cutoff $c^*_{jl}(z) = \frac{\hat{p}_l(z)}{\tau_{jl}(z) w_j}$. Moreover, we have followed equation (20) in Melitz and Redding (2013) to decompose extensive and intensive margins.

The implication is that aggregate profits are a constant fraction $\Pi_{jl}(z) = \delta(z) R_{jl}(z)$ of aggregate revenue, as $\delta(z) \equiv \zeta_\Pi(z)/\zeta_X(z)$ is fixed by the exogenous concentration parameter of the technological distribution and does not vary by country. Not only $\zeta_X(z)$ and $\zeta_\Pi(z)$ but also $\delta(z)$ are decreasing functions of $k(z)$.

### B.2 Equilibrium with diversification

An equilibrium with diversification is characterized by a strictly positive entry of firms in every country and sector pair, such that $N_j^E(z) > 0$ for all $j = 1, ..., J$ and $z = 1, ..., Z$.

**Autarky.** The special case of countries that do not trade is an equilibrium with diversification, since in every country there is positive demand for every sector and this cannot be satisfied by imports by definition. Therefore, there exists a set of trade costs $\{\tau_{jl}(z) \geq 1 : j, l = 1, ..., J,\ z = 1, ..., Z\}$ for which an equilibrium with diversification exists.

Moreover, this equilibrium is unique. Since countries are disconnected, there is no relationship between their nominal wages, therefore, the wage in every country can arbitrarily be set to 1, as the numeraire in its own market. This can be seen by replacing prohibitive trade costs in the equilibrium conditions FEC* and OMC*, then noticing that any nominal wage drops from the equations. The solution is:

$$
w_j = 1 \ \ \forall j\,, \ \ c_j^{aut}(z) = \left(\frac{c_j^{max}(z)^{k(z)}}{\zeta_\Pi(z) L_j / f_j}\right)^{\frac{1}{1+k(z)}} \ \ , \ \ N_j^{E\ aut}(z) = \theta(z)\delta(z)\frac{L_j}{f_j} \ \ \forall (j,z).
$$

**Uniqueness in open economy.** Assume that an open economy equilibrium with diversification exits and there are two different vectors of relative wages, $\boldsymbol{a} = (1, a_2, ..., a_J)$ and $\boldsymbol{b} = (1, b_2, ..., b_J)$, that are both an equilibrium. This means that the labor market clearing conditions must hold in each country. Let $f(\cdot; z) : \Re_+^{J-1} \to \Re_+$ be the continuous function describing the relative labor demand (left hand side of LMC**) after substituting for the measures of entrants $(y_1(z), y_2(z), ..., y_J(z))$ implied by FEC** and OMC**. Highlight with superscript $^{(+)}$ or $^{(-)}$ the sign of functional dependence on the corresponding relative wage.

Consider first the labor market equilibrium conditions with $J = 2$ countries:

$$\text{LMC 1:} \quad \sum_{z=1}^{Z} f_1\left(1, a_2^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_1\left(1, b_2^{(+)}; z\right) = 1$$
$$\text{LMC 2:} \quad \sum_{z=1}^{Z} f_2\left(1, a_2^{(-)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_2\left(1, b_2^{(-)}; z\right) = 1$$

If $\boldsymbol{a}$ is an equilibrium and $b_2 < a_2$ applies, then LMC in country 1 does not hold (demand is too low) and LMC in country 2 also fails (demand is too high).
Consider now the labor market equilibrium with $J = 3$ countries:

$$\text{LMC 1:} \quad \sum_{z=1}^{Z} f_1\left(1, a_2^{(+)}, a_3^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_1\left(1, b_2^{(+)}, b_3^{(+)}; z\right) = 1$$
$$\text{LMC 2:} \quad \sum_{z=1}^{Z} f_2\left(1, a_2^{(-)}, a_3^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_2\left(1, b_2^{(-)}, b_3^{(+)}; z\right) = 1$$
$$\text{LMC 3:} \quad \sum_{z=1}^{Z} f_3\left(1, a_2^{(+)}, a_3^{(-)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_3\left(1, b_2^{(+)}, b_3^{(-)}; z\right) = 1.$$

If $\boldsymbol{a}$ is an equilibrium and $b_2 < a_2$ applies, then $b_3 > a_3$ is necessary, otherwise LMC in country 1 does not hold. However, if $b_2 < a_2$ and $b_3 > a_3$ apply, labor demand in country 2 is too high while labor demand in country 3 is too small.
Consider finally the labor market equilibrium with $J = 4$ countries:

$$\text{LMC 1:} \quad \sum_{z=1}^{Z} f_1\left(1, a_2^{(+)}, a_3^{(+)}, a_4^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_1\left(1, b_2^{(+)}, b_3^{(+)}, b_4^{(+)}; z\right) = 1$$
$$\text{LMC 2:} \quad \sum_{z=1}^{Z} f_2\left(1, a_2^{(-)}, a_3^{(+)}, a_4^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_2\left(1, b_2^{(-)}, b_3^{(+)}, b_4^{(+)}; z\right) = 1$$
$$\text{LMC 3:} \quad \sum_{z=1}^{Z} f_3\left(1, a_2^{(+)}, a_3^{(-)}, a_4^{(+)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_3\left(1, b_2^{(+)}, b_3^{(-)}, b_4^{(+)}; z\right) = 1.$$
$$\text{LMC 4:} \quad \sum_{z=1}^{Z} f_4\left(1, a_2^{(+)}, a_3^{(+)}, a_4^{(-)}; z\right) = 1 \text{ and } \sum_{z=1}^{Z} f_4\left(1, b_2^{(+)}, b_3^{(+)}, b_4^{(-)}; z\right) = 1.$$

If $\boldsymbol{a}$ is an equilibrium and $b_2 < a_2$ applies, then at least $b_3 > a_3$ or $b_4 > a_4$ is necessary, otherwise LMC in country 1 does not hold. However, in both cases, labor demand in country 2 is too high, if no other change - of the opposite sign - occurs. Therefore, either $b_3 > a_3$ and $b_4 < a_4$ apply or $b_3 < a_3$ and $b_4 > a_4$ apply. If $b_2 < a_2$, $b_3 > a_3$ and $b_4 < a_4$ apply, then labor demand in country 3 is too small. Otherwise, if $b_2 < a_2$, $b_3 < a_3$ and $b_4 > a_4$ apply, then labor demand in country 4 is too small.

By induction, we conclude that, regardless of the number of countries, if $\boldsymbol{a}$ is an equilibrium, then any other vector $\boldsymbol{b} \neq \boldsymbol{a}$ is not an equilibrium since the labor market in at least one country does not clear.

For a more formal discussion of uniqueness, consider the equilibrium conditions:

$$\begin{aligned}
\text{FEC**} &: \quad \sum_{l=1}^{J} \frac{K_{jl}(z)}{T_{jl}(z)} \left(\frac{w_l}{w_j}\right)^{1+k(z)} x_l(z) = 1 && \forall (j,z) \\
\text{OMC**} &: \quad \sum_{j=1}^{J} \frac{K_{jl}(z)E_{jl}(z)}{T_{jl}(z)} \left(\frac{w_l}{w_j}\right)^{k(z)} x_l(z)y_j(z) = 1 && \forall (l,z) \\
\text{LMC**} &: \quad \sum_{z=1}^{Z} \theta(z)y_j(z) = 1 && \forall j.
\end{aligned}$$

Write the running index as $n$ instead of either $j$ or $l$

$$\begin{aligned}
\text{FEC**} &: \quad \sum_{n=1}^{J} \frac{K_{jn}(z)}{T_{jn}(z)} \left(\frac{w_n}{w_j}\right)^{1+k(z)} x_n(z) = 1 && \forall (j,z) \\
\text{OMC**} &: \quad \sum_{n=1}^{J} \frac{K_{nl}(z)E_{nl}(z)}{T_{nl}(z)} \left(\frac{w_l}{w_n}\right)^{k(z)} x_l(z)y_n(z) = 1 && \forall (l,z) \\
\text{LMC**} &: \quad \sum_{z=1}^{Z} \theta(z)y_j(z) = 1 && \forall j.
\end{aligned}$$

Now all the equilibrium conditions can be stated for the same country, say $j$

$$\begin{aligned}
\text{FEC**} &: \quad \sum_{n=1}^{J} \frac{K_{jn}(z)}{T_{jn}(z)} \left(\frac{w_n}{w_j}\right)^{1+k(z)} x_n(z) = 1 && \forall (j,z) \\
\text{OMC**} &: \quad \sum_{n=1}^{J} \frac{K_{nj}(z)E_{nj}(z)}{T_{nj}(z)} \left(\frac{w_j}{w_n}\right)^{k(z)} x_j(z)y_n(z) = 1 && \forall (j,z) \\
\text{LMC**} &: \quad \sum_{z=1}^{Z} \theta(z)y_j(z) = 1 && \forall j.
\end{aligned}$$

Multiply both sides of FEC and OMC by the fraction $\frac{k(z)}{\sum_{z=1}^{Z} k(z)}$ and take the sum over sectors. This yields equilibrium conditions at the country level only

$$\begin{aligned}
\text{FEC**} &: \quad \sum_{z=1}^{Z} \left( \frac{k(z)}{\sum_{z=1}^{Z} k(z)} \sum_{n=1}^{J} \frac{K_{jn}(z)}{T_{jn}(z)} \left(\frac{w_n}{w_j}\right)^{1+k(z)} x_n(z) \right) = 1 && \forall j \\
\text{OMC**} &: \quad \sum_{z=1}^{Z} \left( \left[ \frac{k(z)}{\sum_{z=1}^{Z} k(z)} \sum_{n=1}^{J} \frac{K_{nj}(z)E_{nj}(z)}{T_{nj}(z)} \left(\frac{w_j}{w_n}\right)^{k(z)} x_j(z) \right] y_n(z) \right) = 1 && \forall j \\
\text{LMC**} &: \quad \sum_{z=1}^{Z} \theta(z)y_j(z) = 1 && \forall j.
\end{aligned}$$

An inspection of the system of OMC and LMC shows that the following is a solution, in particular a solution in which a strictly positive vector of wages implies entry in every country and every sector:

$$x_j(z) = \frac{\theta(z)}{\frac{k(z)}{\sum_{z=1}^{Z} k(z)} \sum_{n=1}^{J} \frac{K_{nj}(z)E_{nj}(z)}{T_{nj}(z)} \left(\frac{w_j}{w_n}\right)^{k(z)}} \qquad \forall j. \tag{45}$$

Substituting back in FEC yields the system of $J-1$ conditions that implicitly solve for as many relative wages:

$$\sum_{z=1}^{Z}\theta(z)\left[\sum_{n=1}^{J}\left(\frac{\frac{K_{jn}(z)}{T_{jn}(z)}\frac{w_n}{w_j}}{\sum_{m=1}^{J}\frac{K_{mn}(z)E_{mn}(z)}{T_{mn}(z)}\left(\frac{w_j}{w_m}\right)^{k(z)}}\right)\right]=1 \qquad \forall j \tag{46}$$

where, given LMC, the expression within squared brackets is the relative measure of entrants in open economy relative to autarky:

$$y_j(z)=\sum_{n=1}^{J}\left(\frac{\frac{K_{jn}(z)}{T_{jn}(z)}\frac{w_n}{w_j}}{\sum_{m=1}^{J}\frac{K_{mn}(z)E_{mn}(z)}{T_{mn}(z)}\left(\frac{w_j}{w_m}\right)^{k(z)}}\right) \qquad \forall j \tag{47}$$

which illustrates the relative labor demand in open economy. Condition (45) and (47) show that relative cost cutoffs $x_j(z)$ and relative measure of entrants are ultimately a decreasing functions of the wage $w_j$. The left hand side in (46) is monotonically decreasing in $w_j$ while the right hand side is constant, for every $j$.

**Existence in open economy.** An open economy equilibrium with diversification exists only if, for all $(m,z)$, we have:

$$x_m(z)>0 \iff \frac{w_m}{w_1}>\left[\sum_{l\neq m}^{J}\frac{K_{ml}(z)}{T_{ml}(z)}\left(\frac{w_l}{w_1}\right)^{1+k(z)}x_l(z)\right]^{\frac{1}{1+k(z)}}>0 \tag{48}$$

$$y_m(z)>0 \iff \frac{w_m}{w_1}<\left[x_m(z)\sum_{j\neq m}^{J}\frac{K_{jm}(z)E_{jm}(z)}{T_{jm}(z)}\left(\frac{w_1}{w_j}\right)^{k(z)}y_j(z)\right]^{-\frac{1}{k(z)}}. \tag{49}$$

Since existence of an equilibrium postulates that FEC** and OMC** are satisfied, then the previous conditions are equivalent to:

$$x_m(z)>0 \iff \frac{w_m}{w_1}>[1-x_m(z)]^{\frac{1}{1+k(z)}} \qquad \forall(m,z) \tag{50}$$

$$y_m(z)>0 \iff \frac{w_m}{w_1}<[1-x_m(z)y_m(z)]^{-\frac{1}{k(z)}} \qquad \forall(m,z) \tag{51}$$

which are the expressions (28) and (29). Several special cases can be discussed.

*Factor price equalization*. Consider the special case in which countries pay the same wage, i.e. $w_j=w_1=1$ for all $j$, and there is no sector in which a country experiences more entry in open economy relative to autarky, therefore $y_j(z)=1$ for all $(j,z)$. The necessary and sufficient condition (30) is satisfied for $0<x_j(z)<1$ for all countries and sectors $(j,z)$.

In this case, if an equilibrium exists, then, by definition, wages are the same across countries. Furthermore, condition (26) implies that it is an equilibrium with diversification $0<x_j(z)<1$ for all countries and sectors $(j,z)$. The system of necessary conditions (28) and (29) simplifies to:

$$\sum_{l\neq m}^{J}\frac{K_{ml}(z)}{T_{ml}(z)}<\frac{w_m}{w_1}=1<\left[\sum_{j\neq m}^{J}\frac{K_{jm}(z)E_{jm}(z)}{T_{jm}(z)}\right]^{-1}\qquad\forall(m,z),$$

which can be assessed based on the model's parameters only. Clearly as technological differences attenuate and trade costs increase while remaining finite (i.e. $K_{ml}(z)/T_{ml}(z)\to 0_+$ and $K_{jm}(z)/T_{jm}(z)\to 0_+$ for all $z$), the feasible support for each relative wage widens.

*Symmetric countries*. A further special case is one in which countries are symmetric in their characteristics and face a common bilateral trade cost $\tau>1$. Due to symmetry, the system of necessary conditions (28) and (29) simplifies to:

$$\frac{J-1}{\tau}<\frac{w_m}{w_1}=1<\frac{\tau}{J-1}\qquad\forall m.$$

Therefore, $\tau>J-1$ is the finite (hence not prohibitive) level of trade cost such that the sufficient condition for existence of an open economy equilibrium with diversification is satisfied. Note that, for the classical example with $J=2$ countries, the presence of any trade cost $\tau>1$ is sufficient.

*One sector open economy*. In a one-sector economy, a fixed labor supply trivially fixes labor demand in the one sector. This simplifies OMC** which is now a condition on relative wage and cutoff costs only as FEC**:

$$\begin{aligned}
\text{FEC**}\quad &:\quad \sum_{l=1}^{J}\frac{K_{jl}}{T_{jl}}\left(\frac{w_l}{w_j}\right)^{1+k}x_l=1 &&\forall j,\\
\text{OMC**}\quad &:\quad x_l\sum_{j=1}^{J}\frac{K_{jl}E_{jl}}{T_{jl}}\left(\frac{w_l}{w_j}\right)^{k}=1 &&\forall l,\\
\text{LMC**}\quad &:\quad y_j=1 &&\forall j.
\end{aligned}$$

The system reduces to

$$w_jL_j=\sum_{l=1}^{J}\left(\frac{\frac{K_{jl}}{\tau_{jl}^k}\left(\frac{1}{w_j}\right)^k}{\sum_{j=1}^{J}\frac{K_{jl}}{\tau_{jl}^k}\left(\frac{1}{w_j}\right)^k}\right)w_lL_l\quad\forall j.$$

Using labor in country 1 as the numeraire, such that $w_1=1$ yields

$$L_1=\sum_{l=1}^{J}\left(\frac{\frac{K_{1l}}{\tau_{1l}^k}}{\sum_{j=1}^{J}\frac{K_{jl}}{\tau_{jl}^k}\left(\frac{1}{w_j}\right)^k}\right)w_lL_l.$$

Note that $K_{jl}/K_{1l}=K_{j1}$ does not depend on $l$, and a decomposition of trade costs $\tau_{jl}=\tau_j\tau_l$ implies that $\tau_{1l}/\tau_{jl}=\tau_1/\tau_j$, also does not depend on $l$. Under this parametrization, the wage can be obtained in closed form:

$$w_j = \left( \frac{\tau_1^k K_{j1}}{\tau_j^k} \frac{L_1}{L_j} \right)^{\frac{1}{1+k}} = \left( \left( \frac{\tau_1 c_1^{max}}{\tau_j c_j^{max}} \right)^k \frac{f_1}{f_j} \right)^{\frac{1}{1+k}} \quad \forall j,$$

and it is a decreasing function of the upper bound of the cost support $c_j^{max}$, of fixed cost $f_j$ and of the trade cost $\tau_j$.

**Outside of diversification.** Assume that, given the fundamentals of the economy, there are some pairs of country and sector that do not feature production. Given free entry and a continuous distribution of cutoff costs, this means that there is no positive measure of entrants for a feasible cost cutoff in at least one country-sector pair, i.e. $N_j^E(z) = 0$ for every $c_j^*(z) > 0$.

Since no entry means also a null measure of incumbent firms, country-sector pairs with in which there is no production do not generate income. Yet, the labor market clearing condition must hold, irrespective of the fact that the equilibrium features diversification or not. The system (12)-(14) "fails" to characterize an equilibrium without diversification because the free entry condition and the output market clearing condition are not well-posed.

More precisely, the free entry condition is misspecified because it postulates the existence of a full matrix of strictly positive country-sector cutoff costs, but this is true if and only if there is a positive mass of entrants. The output market clearing condition fails because it is "coupled" with the free entry condition through - again - a postulated matrix of cutoff costs. Therefore, it is not enough to simply replace $N_j^E(z) = 0$ in the output market clearing condition. To this condition should correspond a missing cost cutoff $c_j^*(z)$, hence, the free entry condition for country $j$'s sector $z$ should be removed from the system (12)-(14), and this implies solving a different problem than the original one with a zero measure of entrants for the pair $(j, z)$.

To solve the equilibrium of the model without assuming diversification, one needs to assess when entry fails "before" writing the system of free entry conditions. For this assessment, note that free entry with a continuous distribution of costs over the positive support (so that $c_j^*(z)$ can be arbitrarily close to zero) implies that the output of every country-sector pair is sold to every country. Consequently, the matrix of trade flows must also be full. If one knows which trade flows should be zero (both in the observation and in the model's predictions), then one also knows which free entry conditions to remove and which measure of entrants to set to zero. The resulting "truncated" version of (12)-(14) is still characterized by as many equations as unknowns.

*An interesting restriction.* Any pair of sub-utility bundles $u_{jl}(z)$ and $u_{kl}(z)$ are perfect substitutes in the utility (1) of consumers in country $l$. The prices of these sub-utility bundles are, respectively, $e_{jl}(z)/u_{jl}(z) = \frac{\alpha}{2} \frac{\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)}{1 - \bar{p}_{jl}(z)} \hat{p}_l(z)$ and $e_{kl}(z)/u_{kl}(z) = \frac{\alpha}{2} \frac{\bar{p}_{kl}(z) - \bar{\bar{p}}_{kl}(z)}{1 - \bar{p}_{kl}(z)} \hat{p}_l(z)$. Therefore, $\frac{\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)}{1 - \bar{p}_{jl}(z)} = \frac{\bar{p}_{kl}(z) - \bar{\bar{p}}_{kl}(z)}{1 - \bar{p}_{kl}(z)}$ is a necessary condition for country $l$ to source goods from sector $z$ from both origins $j$ and $k$, which must be true if the matrix of trade flows should be full. Interestingly, the assumption of an Inverse Pareto distribution of technology implies that this restriction is satisfied mechanically, since moments of the relative price distribution only depend on the sector.

**Bilateral trade balance.** $\sum_{z=1}^{Z} R_{jl}(z)$ equals total import of country $l$ from country $j$. Then, total import of country $l$ from country $j$, and total export of country $l$ to country $j$ are equal if and only if the bilateral trade balance condition holds:

$$\text{BTB}: \quad \sum_{z=1}^{Z} R_{jl}(z) = \sum_{z=1}^{Z} R_{lj}(z) \quad \forall (j,l). \tag{52}$$

Two remarks are important. First, output market clearing in each country and sector pair implies that the country-level budget constraint is satisfied. To see this, write a sectoral output market clearing condition for a country $j$

$$\sum_{m=1}^{J} N_{mj}(z) \int_0^{c^*_{mj}(z)} r_{mj}(c;z) \frac{dG_m(c;z)}{G_m(c^*_{mj}(z);z)} = \theta_j(z) w_j L_j \ ,$$

and sum over sectors:

$$\underbrace{\sum_{z=1}^{Z} \sum_{m=1}^{J} N_{mj}(z) \int_0^{c^*_{mj}(z)} r_{mj}(c;z) \frac{dG_m(c;z)}{G_m(c^*_{mj}(z);z)}}_{\text{total expenditure of country } j} = w_j L_j \ .$$

Second, output market clearing, free entry and bilateral trade balance imply labor market clearing at the country level. To see this, write a sectoral output market clearing condition for a country $j$, sum over sectors and recognize the expression for total imports from a certain origin:

$$\sum_{m=1}^{J} \underbrace{\left( \sum_{z=1}^{Z} N_{mj}(z) \int_0^{c^*_{mj}(z)} r_{mj}(c;z) \frac{dG_m(c;z)}{G_m(c^*_{mj}(z);z)} \right)}_{\text{total import of country } j \text{ from country } m} = w_j L_j \ .$$

If there is trade balance between countries, then total import of country $j$ from country $m$ must be equal to total export of country $j$ to country $m$

$$\sum_{m=1}^{J} \underbrace{\left( \sum_{z=1}^{Z} N_{jm}(z) \int_0^{c^*_{jm}(z)} r_{jm}(c;z) \frac{dG_j(c;z)}{G_j(c^*_{jm}(z);z)} \right)}_{\text{total export of country } j \text{ to country } m} = w_j L_j \ .$$

Thus, substituting for the measure of exporters $N_{jm}(z) = G_j(c^*_{jm}(z);z) N^E_j(z)$, yields the labor market clearing condition in country $j$:

$$\sum_{z=1}^{Z} N^E_j(z) \left( \sum_{l=1}^{J} \int_0^{c^*_{jl}(z)} r_{jl}(c;z) dG_j(c;z) \right) = w_j L_j \ .$$

Hence, in autarky the bilateral trade balance condition is redundant by definition. In open economy with aggregate trade balance, either the bilateral trade balance condition or the labor market clearing condition is redundant because it is implied by the other equilibrium conditions.

### B.3 Comparative statics of a resource shock holding the vector of wages constant

Given the following representation of FEC**

$$\sum_{l=1}^{J} a_{hl}^0(s)\left(\frac{x_l^1(s)}{x_l^0(s)}\right) = \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)}\right)^{k(s)} < 1,$$

$$\text{where } a_{hl}^0(s) \equiv \frac{K_{hl}^0(s)}{T_{hl}^0(s)}\left(\frac{w_l^0}{w_h^0}\right)^{1+k(s)} x_l^0(s) \text{ and } \sum_{l=1}^{J} a_{hl}^0(s) = 1,$$

define $A^0(s)$ to be the $J$-dimensional matrix with row-$j$ and column-$l$ element $a_{jl}^0(s)$, whose entries sum to one on each row. Call $A_l^0(s)$ the matrix constructed from $A^0(s)$ by replacing the $l$-th column with a vector of $J$ entries all equal to 1 and call $A_l^1(s)$ the matrix constructed from $A^0(s)$ by replacing the $l$-th column with a vector of $J$ entries all equal to $\left(c_h^{max\ 1}(s)/c_h^{max\ 0}(s)\right)^{k(s)} < 1$. Define the column vectors $\boldsymbol{z}^i(s) = \{z_l^i(s) = x_l^i(s)/x_l^0(s) : l = 1, ..., J\}$ and $\boldsymbol{c}^i(s) = \{c_l^i(s) = (c_h^{max\ i}(s)/c_h^{max\ 0}(s))^{k(s)} : l = 1, ..., J\}$, for the two regimes $i = \{0, 1\}$ before and after the shock, then the FEC** takes the form of a linear system:

$$A^0(s)\boldsymbol{z}^i(s) = \boldsymbol{c}^i(s)$$

whose solution is obtained by Cramer's rule

$$z_l^i(s) = \frac{detA_l^i(s)}{detA^0(s)}$$

and note that $detA_l^0(s) = detA^0(s)$ by construction. Call $TA_l^0(s)$ the transpose of $A_l^0(s)$ and $TA_l^1(s)$ the transpose of $A_l^1(s)$. Since $TA_l^1(s)$ is the matrix which results from multiplying one row of $TA_l^0(s)$ by the scalar $\left(c_h^{max\ 1}(s)/c_h^{max\ 0}(s)\right)^{k(s)}$ then the determinant satisfies $detTA_l^1(s) = \left(c_h^{max\ 1}(s)/c_h^{max\ 0}(s)\right)^{k(s)} detTA_l^0(s)$. Since the determinant of the transpose is equal to the determinant of the original matrix then $detA_l^1(s) = \left(c_h^{max\ 1}(s)/c_h^{max\ 0}(s)\right)^{k(s)} detA_l^0(s)$. This shows that:

$$z_l^1(s) = \frac{x_l^1(s)}{x_l^0(s)} = \frac{detA_l^1(s)}{detA^0(s)} = \frac{detTA_l^1(s)}{detTA_l^0(s)} = \left(\frac{c_h^{max\ 1}(s)}{c_h^{max\ 0}(s)}\right)^{k(s)} < 1\ \forall l.$$

Substituting for the definition of $x_l(s)$

$$\frac{x_l^1(s)}{x_l^0(s)} = \left(\frac{c_l^{*1}(s)}{c_l^{*0}(s)}\right)\left(\frac{c_l^{max\ 0}(s)}{c_l^{max\ 1}(s)}\right)^{k(s)}$$

yields (36).

Given the following representation of the OMC**

$$\sum_{j=1}^{J} b_{jh}^{0}(s)\frac{y_j^1(s)}{y_j^0(s)} = \frac{x_h^0(s)}{x_h^1(s)}\left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{k(s)} = \left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{2k(s)} > 1$$

where $b_{jh}^{0}(s) \equiv \frac{K_{jh}^{0}(s)E_{jh}^{0}(s)}{T_{jh}^{0}(s)}\left(\frac{w_h^0}{w_j^0}\right)^{k(s)} x_h^0(s)y_j^0(s)$ and $\sum_{j=1}^{J} b_{jh}^{0}(s) = 1$,

define $TB^0(s)$ the $J$-dimensional matrix with row-$l$ and column-$j$ element $b_{jl}^{0}(s)$, whose entries sum to one on each row. Call $TB_j^0(s)$ the matrix constructed from $TB^0(s)$ by replacing the $j$-th column with a vector of $J$ entries all equal to 1 and call $TB_j^1(s)$ the matrix constructed from $TB^0(s)$ by replacing the $j$-th column with a vector of $J$ entries all equal to $\left(c_h^{max\ 0}(s)/c_h^{max\ 1}(s)\right)^{2k(s)} > 1$. Define the column vectors $\boldsymbol{t}^i(s) = \{t_j^i(s) = y_j^i(s)/y_j^0(s) : j = 1, ..., J\}$ and $\boldsymbol{d}^i(s) = \{d_j^i(s) = (c_h^{max\ 0}(s)/c_h^{max\ i}(s))^{2k(s)} : j = 1, ..., J\}$, for the two regimes $i = \{0, 1\}$ before and after the shock, then the OMC** takes the form of the linear system

$$TB^0(s)\boldsymbol{t}^i(s) = \boldsymbol{d}^i(s).$$

Cramer's rule yields the solution

$$t_j^i(s) = \frac{detTB_j^i(s)}{detTB^0(s)}$$

and note that $detTB_j^0(s) = detTB^0(s)$ by construction. Since the determinant of the transpose is equal to the determinant of the original matrix then $detTB_j^0(s) = detB_j^0(s)$ and $detTB_j^1(s) = detB_j^1(s)$, where $B_j^i(s)$ is the transpose of $TB_j^i(s)$. Note that $B_j^1(s)$ is the matrix which results from multiplying one row of $B_j^0(s)$ by the scalar $(c_h^{max\ 0}(s)/c_h^{max\ i}(s))^{2k(s)}$ then the determinant satisfies $detB_j^1(s) = (c_h^{max\ 0}(s)/c_h^{max\ i}(s))^{2k(s)} detTB_j^0(s)$. This shows that:

$$\frac{y_j^1(s)}{y_j^0(s)} = \frac{detTB_j^1(s)}{detTB^0(s)} = \frac{detB_j^1(s)}{detB_j^0(s)} = \left(\frac{c_h^{max\ 0}(s)}{c_h^{max\ 1}(s)}\right)^{2k(s)} > 1\ \forall j.$$

Substituting for the definition of $y_j(s)$ yields (37).

# C Relationship to ACR (2012) and ACDR (2018)

In this section we clarify the relationship between our model and the class of models discussed in Arkolakis et al. (2012) and extended in Arkolakis et al. (2019).

## C.1 The class of models considered in ACR (2012)

It is immediate to conclude that the so-called macro restrictions in Arkolakis et al. (2012) hold in our framework: total value of imports is equal to total value of exports (R1); in each sector aggregate profits are a constant fraction of aggregate revenue (R2); the gravity equation implied by the model, once written in terms of the measure of potential entrants, has a canonical structure (R3′ that is the stronger form of R3). In particular, the latter applies thanks to the adoption of a Pareto distribution of technology that makes the moments of the relative price distribution depend only on the concentration parameter. In our model, as in their analysis, (i) the cost function at the firm level is linear, (ii) labor is the only factor of production, (iii) the labor market is competitive, while (iv) the output market has a monopolistically competitive structure.

Indeed, there is only one primitive of the theory in which our model deviates from the class of models considered in Arkolakis et al. (2012): preferences across varieties are represented by an additive-separable utility function that features a variable elasticity of substitution.

## C.2 The class of models considered in ACDR (2018)

The existence of a finite choke price and the adoption of the Pareto distribution place the model within the class of those discussed in Arkolakis et al. (2019). To show this, in what follows we rewrite the salient feature of our framework using their approach.

With reference to the group of firms producing in country $j$ sector $z$ and selling to a destination $l$, call $v \equiv \hat{p}_l(z)/[\tau_{jl}(z)w_j c] = c^*_{jl}(z)/c \geq 1$ the measure of efficiency of a firm endowed with productivity $1/c$ relative to the other firms in the group. Call $\mu_{jl}(v;z) = p_{jl}(c;z)/[\tau_{jl}(z)w_j c]$ the function describing the markup factor as a function of relative efficiency. After substituting for $c = c^*_{jl}(z)/v$, the markup factor loses its dependence on the origin, destination, or sector.

$$\mu(v) = \frac{1}{2}\left(1 + \frac{c^*_{jl}(z)}{c}\right) = \frac{1+v}{2},$$

as the three channels of dependence are captured by the cutoff cost, and the same holds true for the elasticity of the markup factor with respect to relative efficiency:

$$\frac{d\ln \mu(v)}{d\ln(v)} = \frac{v}{1+v}.$$

The individual Marshallian demand function is described by a demand shifter $Q \equiv \frac{\alpha}{\gamma}$ and a decreasing function $D\left(\mu(v)/v\right) \equiv 1 - \mu(v)/v = (v-1)/(2v)$ such that total sales and profits are given by:

$$r_{jl}\left(c,v;z\right)=L_{l}Q\tau_{jl}\left(z\right)w_{j}c\mu(v)D\left(\mu(v)/v\right)$$
$$\pi_{jl}\left(c,v;z\right)=\left[\left(\mu(v)-1\right)/\mu(v)\right]r_{jl}\left(c;z\right).$$

### C.3 A foreign trade shock

The implications of incomplete pass-through for welfare following a foreign trade shock can be illustrated by replicating the "ex-ante" conjecture in Arkolakis et al. (2012), that consists of the limit counterfactual exercise of "*moving to autarky*" given the same fixed entry cost, i.e. $f_j$, and labor endowment, i.e. $L_j$, and support of the technological distribution $c_j^{max}(z)$. By definition $\lambda_{jj}^{aut}(z)\equiv 1$ while the allocation of labor across sectors is proportional to the measure of firms $\rho_j^{aut}(z)=f_jN_j^{E\ aut}(z)/[\delta(z)L_j]=\theta(z)$. Thus, welfare in autarky is given by

$$W_j^{aut}\;=\;\prod_{z=1}^{Z}B(z)\left(\frac{f_jc_j^{max}(z)^{k(z)}}{L_j}\frac{1}{\theta(z)}\right)^{-\frac{\beta(z)}{k(z)}\frac{k(z)}{1+k(z)}}$$

and the measured welfare cost for country $j$ of a shock that all-and-only shuts down trade linkages is given by:

$$\frac{W_j^{aut}}{W_j}=\prod_{z=1}^{Z}\left(\frac{\lambda_{jj}(z)}{\rho_j(z)/\theta(z)}\right)^{\frac{\beta(z)}{k(z)}\frac{k(z)}{1+k(z)}}. \tag{53}$$

Note that expenditure shares matter for an accounting of the cost of moving to autarky. Expenditure shares, e.g. $\theta(z)$, are lower than the corresponding Cobb-Douglas shares, e.g. $\beta(z)$, for sector with lower technological concentration, hence with lower pass-through. Therefore, sectors characterized by lower technological concentration attenuate the measurement of autarky-induced welfare changes, both due to lower pass-true and a comparatively smaller expenditure share.

These differential effects would be absent if technological concentration was the same across sectors, and the closed results provided in Arkolakis et al. (2019) are confined to that scenario. More precisely, in their paper the derivation of Proposition 2 holds under the assumption that the average elasticity of markup to relative efficiency and is the same across sectors. The authors are explicit about it, as in page 66 they write: "*These cross-sector effects are ruled out by the assumption that $\eta^k=\eta$ and $\zeta_k=\zeta$ for all k, implying that the focus of Proposition 2 is on within rather than between-sector distortions*".

Thus, the tractability of the present framework sheds light on the role played by heterogeneity of technological concentration across sectors on the measurement of autarky-induced welfare changes.

## D BP preferences and Beta distribution of firm productivity

**Proposition**. *Let the exogenous distribution of technological coefficients among potential entrants in country $j$ sector $z$ be a 3-parameter Beta, with shape parameters $\kappa_1 > 0$ and $\kappa_2 > 0$ and location parameter $c_j^{max}(z) > 0$, such that the p.d.f. is given by*

$$g_j(c;z) = \frac{c^{\kappa_1-1}(c_j^{max}(z) - c)^{\kappa_2-1}}{\frac{\Gamma(\kappa_1)\Gamma(\kappa_2)}{\Gamma(\kappa_1+\kappa_2)} c_j^{max}(z)^{\kappa_1+\kappa_2-1}}, \quad c \in [0, c_j^{max}(z)].$$

*Consider aggregate variables (i.e. expenditure, utility, average revenue and average profit) in a market segment $(j,l,z)$. The following results hold:*

(i) *If there is no selection, i.e. $c_{jl}^*(z) = c_j^{max}(z)$, then aggregate variables are determined in closed form.*

(ii) *If there is selection, then determining aggregate variables only requires integration of a continuous and smooth (i.e. with continuous derivatives) function on the given compact support $[(-\sigma)/(1-\sigma), 1]$, and for which the limit of integration is bounded; thus, standard numerical integration applies.*

(iii) *If there is selection, then, under the assumption that $\kappa_2$ is a natural number, the expected profit unconditional on entry that a firm producing in country $j$ sector $z$ earns from sales in country $l$ can be computed in closed form:*

$$\bar{\pi}_{jl}(z) = \frac{w_l L_l \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} (-\sigma)}{\omega_{jl}(z)^{\kappa_1+\kappa_2-1}} c_l^*(z)^{\kappa_1+\kappa_2} \sum_{k=0}^{\kappa_2-1} a(\kappa_1, \kappa_2, \sigma; k) \left(\frac{\omega_{jl}(z)}{c_l^*(z)} - \frac{1}{1-\sigma}\right)^{\kappa_2-1-k}. \quad (54)$$

*where $\omega_{jl}(z) \equiv [w_j c_j^{max}(z)\tau_{jl}(z)]/[w_l(1-\sigma)]$ and the coefficients $a(\kappa_1, \kappa_2, \sigma; k)$ are positive real numbers, determined by parameters only.*
(iv) *For a given vector of wages $\{w_j\}_{j=1}^J$, the sector-$z$ specific non-linear system of $J$ free entry conditions in $J$ domestic cutoffs $\{c_j^*(z)\}_{j=1}^J$ has a solution in $\Re^J$ and it is unique.*

**Proof**. The optimal pricing rule with monopolistic competition and linear technology and under BP yields a constant absolute pass-through between price relative to the choke price and marginal cost relative to the cutoff for market entry. Let $x = c/c_{jl}^*(z)$ and $y = p_{jl}(c;z)/\hat{p}_l(z)$, then BP implies a linear relationship between the optimal price relative to the choke price $y$, the cost relative to the cost cutoff $x$, and the technological coefficient $c$

$$y = \frac{x-\sigma}{1-\sigma} \implies \frac{y(x)}{dx} = \frac{1}{1-\sigma}, \quad \frac{dy(x(c))}{dc} = \frac{1}{(1-\sigma)c_{jl}^*(z)}$$

where it shall be stressed that if there is selection at entry in the market segment $(j,l,z)$ then the auxiliary variables $x$ and $y(x)$ are specific of origin-$j$, destination-$l$ and sector-$z$.

Let $T_{jl}(y;z)$ be the endogenous c.d.f. of prices relative to the choke price in the market segment $(j,l,z)$, let $F_{jl}(x;z)$ be the endogenous c.d.f. of marginal costs relative to the cost cutoff in the market segment $(j,l,z)$ and recall that $G_j(c;z)$ is the exogenous c.d.f. of technological coefficients among potential entrants in country $j$ sector $z$ for $c \in [0, c_j^{max}(z)]$. Then, the following relationship between the respective probability densities holds:

$$T_{jl}(y(x(c));z) = F_{jl}(x(c);z) = G_j(c;z) \implies$$
$$t_{jl}(y(x(c));z) = (1-\sigma)f_{jl}(x(c);z) = (1-\sigma)c^*_{jl}(z)g_j(c;z).$$

Without loss of generality, let varieties $i \in [0, N_j^E(z)]$ produced in a certain country $j$ and sector $z$ be sorted in increasing order by technological coefficient, such that $i = G_j(c;z)N_j^E(z)$, given $G_j(0;z) = 0$ and $G_j(c^*_{jl}(z);z)N_j^E(z) = N_{jl}(z)$. Consider the Marshallian demand implied by BP preferences:

$$q_l^*\left(p(i);z\right) = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}}\left(1-\frac{p(i)}{\hat{p}_l(z)}\right)^{\frac{1}{-\sigma}} = \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}}(1-y)^{\frac{1}{-\sigma}}.$$

A general equilibrium analysis requires three aggregate statistics of a market segment $(j,l,z)$ and a selection equation. First, individual expenditure:

$$\begin{aligned}
e_{jl}(z) &\equiv \int_0^{N_{jl}(z)} p(i)q_l^*\left(p(i);z\right)di\,, \\
&= \hat{p}_l(z)\left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}}\int_0^{N_{jl}(z)}\frac{p(i)}{\hat{p}_l(z)}\left(1-\frac{p(i)}{\hat{p}_l(z)}\right)^{\frac{1}{-\sigma}}di, \\
&= \hat{p}_l(z)\left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}}N_j^E(z)\int_{y_0}^1 y\,(1-y)^{\frac{1}{-\sigma}}\,t_{jl}(y;z)dy,
\end{aligned}$$

second, individual utility:

$$\begin{aligned}
u_{jl}(z) &\equiv \int_0^{N_{jl}(z)} \alpha q_l^*\left(p(i);z\right) - \frac{\gamma}{1-\sigma}q_j^*\left(p(i);z\right)^{1-\sigma}di, \\
&= \frac{\gamma}{1-\sigma}\left(\frac{\alpha}{\gamma}\right)^{\frac{1-\sigma}{-\sigma}}\int_0^{N_{jl}(z)}\left(-\sigma+\frac{p(i)}{\hat{p}_l(z)}\right)\left(1-\frac{p(i)}{\hat{p}_l(z)}\right)^{\frac{1}{-\sigma}}di, \\
&= \frac{\gamma}{1-\sigma}\left(\frac{\alpha}{\gamma}\right)^{\frac{1-\sigma}{-\sigma}}N_j^E(z)\int_{y_0}^1(-\sigma+y)\,(1-y)^{\frac{1}{-\sigma}}\,t_{jl}(y;z)dy,
\end{aligned}$$

third, expected profit per firm unconditional on entry:

$$\begin{aligned}
\bar{\pi}_{jl}(z) &\equiv \frac{L_l}{N_j^E(z)}\int_0^{N_{jl}(z)}\left(p(i)-w_j\tau_{jl}(z)c\right)q_l^*\left(p(i);z\right)di\,, \\
&= \frac{\hat{p}_l(z)L_l}{N_j^E(z)}\left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}}(-\sigma)\int_0^{N_{jl}(z)}\left(1-\frac{p(i)}{\hat{p}_l(z)}\right)^{\frac{1-\sigma}{-\sigma}}di, \\
&= \hat{p}_l(z)L_l\left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}}(-\sigma)\int_{y_0}^1(1-y)^{\frac{1-\sigma}{-\sigma}}\,t_{jl}(y;z)dy,
\end{aligned}$$

where $y_0 = (x_0-\sigma)/(1-\sigma)$ and $x_0 = 0$, as implied by the change of variable $y = (x-\sigma)/(1-\sigma)$ and $x = c/c^*_{jl}(z)$ over the support $c \in [0, c_j^{max}(z)]$, and it shall be remarked that the upper bound of the relative price distribution $y_{jl} \geq 1$ exceeds one if and only if there is selection, i.e. $c_j^{max}(z) \geq c^*_{jl}(z)$.

Assume that the exogenous distribution of technological coefficients among potential entrants in country $j$ sector $z$ is a 3-parameter Beta with parameters $(\kappa_1, \kappa_2, c_j^{max}(z))$, thus with p.d.f. given by

$$g_j(c;z) = \frac{c^{\kappa_1-1}(c_j^{max}(z)-c)^{\kappa_2-1}}{\frac{\Gamma(\kappa_1)\Gamma(\kappa_2)}{\Gamma(\kappa_1+\kappa_2)} c_j^{max}(z)^{\kappa_1+\kappa_2-1}}, \quad c \in [0, c_j^{max}(z)].$$

Then, the p.d.f. of the hypothetical distribution of prices across varieties produced in country $j$ sector $z$ eventually served in country $l$ is also a 4-parameter Beta with the same shape parameters $(\kappa_1, \kappa_2)$ and endogenous support, whose p.d.f. is given by:

$$t_{jl}(y;z) = \frac{(y-y_0)^{\kappa_1-1}\left(y_{jl}-y\right)^{\kappa_2-1}}{\frac{\Gamma(\kappa_1)\Gamma(\kappa_2)}{\Gamma(\kappa_1+\kappa_2)}(y_{jl}-y_0)^{\kappa_1+\kappa_2-1}}, \quad y \in [y_0, y_1],\ y_0 = \frac{-\sigma}{1-\sigma},\ y_{jl} = \frac{\frac{c_j^{max}(z)}{c_{jl}^*(z)}-\sigma}{1-\sigma}$$

where we have substituted for $y - y_0 = (x - x_0)/(1-\sigma) = (c - c_0)/[(1-\sigma)c_{jl}^*(z)]$ and $y_{jl} - y = (c_j^{max}(z) - c)/[(1-\sigma)c_{jl}^*(z)]$.

**(i) If there is no selection**, i.e. $c_{jl}^*(z) = c_j^{max}(z)$, then $y_1 = 1$ and, the integral expressions in expenditure, utility and average profit are given in closed form:

$$\int_{y_0}^{1} y\,(1-y)^{\frac{1}{-\sigma}}\,t_{jl}(y;z)dy = E[Y]$$

$$\int_{y_0}^{1} (-\sigma+y)\,(1-y)^{\frac{1}{-\sigma}}\,t_{jl}(y;z)dy = -\sigma\frac{\frac{\Gamma(\kappa_1)\Gamma(\kappa_2+\frac{1}{-\sigma})}{\Gamma(\kappa_1+\kappa_2+\frac{1}{-\sigma})}}{\frac{\Gamma(\kappa_1)\Gamma(\kappa_2)}{\Gamma(\kappa_1+\kappa_2)}}\,(1-y_0)^{\frac{1}{-\sigma}} + E[Y]$$

$$\int_{y_0}^{1} (1-y)^{\frac{1-\sigma}{-\sigma}}\,t_{jl}(y;z)dy = \frac{\frac{\Gamma(\kappa_1)\Gamma(\kappa_2+\frac{1-\sigma}{-\sigma})}{\Gamma(\kappa_1+\kappa_2+\frac{1-\sigma}{-\sigma})}}{\frac{\Gamma(\kappa_1)\Gamma(\kappa_2)}{\Gamma(\kappa_1+\kappa_2)}}\,(1-y_0)^{\frac{1-\sigma}{-\sigma}}$$

where $\Gamma$ is the Gamma function, $E[\cdot]$ is the expectation operator and $Y$ is a random variable that follows a 4-parameter Beta distribution with parameters $(\kappa_1, \kappa_2 + \frac{1}{-\sigma}, y_0, y_1)$ and $E[Y]$ is known positive real value.

To see this, note that the 4-parameter Beta distribution can be defined in terms of the expected value and the mode, instead of $\kappa_1$ and $\kappa_2$. Under this parametrization, we obtain the PERT distribution. Since there is a unique mapping from $\kappa_1$, $\kappa_2$ and expected value, mode and extremes of the support, the expected value $E[Y]$ is known given the shape parameters of the exogenous distribution $G_j(c;z)$ and the support $[y_0, y_1]$.

**(ii) If there is selection**, i.e. $c_{jl}^*(z) < c_j^{max}(z)$, then $T_{jl}(1;z) < 1$ the integral expressions in expenditure, utility and average profit are proportional to moments of continuous transformations of 4-parameter Beta random variable defined on $[y_0, y_1]$, but computed on the truncated support

$[y_0, 1]$, with $y_1 > 1$. To see this, note that aggregate variables are proportional to, respectively:

$$I_e(\bar{y}) \equiv \int_{y_0}^{\bar{y}} y\,(1-y)^{\frac{1}{-\sigma}}\,(y-y_0)^{\kappa_1-1}(y_1-y)^{\kappa_2-1}dy,$$

$$I_u(\bar{y}) \equiv \int_{y_0}^{\bar{y}} (-\sigma+y)\,(1-y)^{\frac{1}{-\sigma}}\,(y-y_0)^{\kappa_1-1}(y_1-y)^{\kappa_2-1}dy,$$

$$I_\pi(\bar{y}) \equiv \int_{y_0}^{\bar{y}} (1-y)^{\frac{1-\sigma}{-\sigma}}\,(y-y_0)^{\kappa_1-1}(y_1-y)^{\kappa_2-1}dy,$$

for $y_0 < \bar{y} = 1 < y_1$. However, in all cases the integrand function is positive, such that $I_v(\bar{y}) \geq 0$ and the integration on the continuous probability density on a subset of the support implies $I_v(\bar{y}) \leq I_v(y_1)$, for every variable $v = \{e, u, \pi\}$. This shows that the determination of aggregate variables only requires integration of continuous and smooth (i.e. with continuous derivatives) functions, on a given compact support $[y_0, 1]$, and for which the limit of integration is bounded. ■

**(iii) If there is selection, and $\kappa_2$ is a natural number**, given $y_{jl} > 1$, and $y \in (0, 1)$. We aim to express $(y_{jl} - y)^{\kappa_2 - 1}$ as proportional to $(1-y)^{\kappa_2-1}$. Rearrange $y_{jl} - y = (y_{jl} - 1) + (1 - y)$, then raise this expression to the power of $\kappa_2 - 1$ and, using the Binomial Theorem, we obtain:

$$(y_{jl} - y)^{\kappa_2-1} = \sum_{k=0}^{\kappa_2-1} \binom{\kappa_2 - 1}{k} (y_{jl} - 1)^{\kappa_2-1-k}(1-y)^k.$$

Now it is sufficient to substitute in the original expressions, and apply Fubini's Theorem to factor the sum operator out of the integral, to obtain

$$I_e(1) = \sum_{k=0}^{\kappa_2-1} \binom{\kappa_2 - 1}{k} (y_{jl} - 1)^k \int_{y_0}^{1} y(y-y_0)^{\kappa_1-1}\,(1-y)^{\frac{1}{-\sigma}+k}\,dy$$

$$I_u(1) = \sum_{k=0}^{\kappa_2-1} \binom{\kappa_2 - 1}{k} (y_{jl} - 1)^k \int_{y_0}^{1} (-\sigma+y)(y-y_0)^{\kappa_1-1}\,(1-y)^{\frac{1}{-\sigma}+k}\,dy$$

$$I_\pi(1) = \sum_{k=0}^{\kappa_2-1} \binom{\kappa_2 - 1}{k} (y_{jl} - 1)^k \int_{y_0}^{1} (y-y_0)^{\kappa_1-1}\,(1-y)^{\frac{1-\sigma}{-\sigma}+k}\,dy.$$

where it shall be noted that the integral expressions are positive real numbers. With specific application to the expected profit unconditional on entry we obtain

$$\bar{\pi}_{jl}(z) = \frac{\hat{p}_l(z) L_l \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} (-\sigma)}{(y_{jl} - y_0)^{\kappa_1+\kappa_2-1}} \sum_{k=0}^{\kappa_2-1} a(\kappa_1, \kappa_2, \sigma; k)(y_{jl} - 1)^{\kappa_2-1-k}$$

where $b(\kappa_1, \sigma; k) = \int_{y_0}^{1} (y-y_0)^{\kappa_1-1}\,(1-y)^{\frac{1-\sigma}{-\sigma}+k}\,dy = B(\kappa_1, \frac{1-\sigma}{-\sigma} + k + 1)(1-y_0)^{\kappa_1+\frac{1-\sigma}{-\sigma}+k}$ and $a(\kappa_1, \kappa_2, \sigma; k) \equiv \binom{\kappa_2-1}{k} b(\kappa_1, \sigma; k) / \frac{\Gamma(\kappa_1)\Gamma(\kappa_2)}{\Gamma(\kappa_1+\kappa_2)}$ are positive real numbers. Note that $c^*_{jl}(z) = \frac{c_j^{max}(z)}{[(1-\sigma)y_{jl}+\sigma]}$

hence $\hat{p}_l(z) = w_l c_l^*(z) = \frac{w_j \tau_{jl}(z) c_j^{max}(z)}{(1-\sigma)[y_{jl} - (-\sigma)/(1-\sigma)]}$ and implies

$$y_{jl} = y_0 + \frac{\omega_{jl}(z)}{c_l^*(z)}, \text{ for } \omega_{jl}(z) \equiv \frac{w_j c_j^{max}(z) \tau_{jl}(z)}{w_l(1-\sigma)},$$

where we have substituted for $(1-\sigma)c_{jl}^*(z) = \frac{c_j^{max}(z)}{\omega_{jl}(z)} c_l^*(z)$. This allows to substitute for

$$y_{jl} - y_0 = \frac{\omega_{jl}(z)}{c_l^*(z)}$$
$$y_{jl} - 1 = \frac{\omega_{jl}(z)}{c_l^*(z)} - \frac{1}{1-\sigma}$$

thus, the expected profit unconditional on entry in country $l$ for firms producing in country $j$ sector $z$ is given by:

$$\bar{\pi}_{jl}(z) = \frac{w_l L_l \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} (-\sigma)}{\omega_{jl}(z)^{\kappa_1+\kappa_2-1}} c_l^*(z)^{\kappa_1+\kappa_2} \sum_{k=0}^{\kappa_2-1} a(\kappa_1, \kappa_2, \sigma; k) \left(\frac{\omega_{jl}(z)}{c_l^*(z)} - \frac{1}{1-\sigma}\right)^{\kappa_2-1-k}. \blacksquare$$

**(iv) Free entry condition**. In the case of selection under free entry, the dependence of the average profit unconditional on entry on the cutoff $c_{jl}^*(z)$ is characterized by a continuous and strictly monotonic function. To see this, recall that $y(c) = (c/c_{jl}^*(z) - \sigma)/(1-\sigma)$ and $t_{jl}(y(c); z) = (1-\sigma)c_{jl}^*(z) g_j(c; z)$. Therefore, simple substitution in the expression of average profit unconditional on entry in terms of technological coefficients yields:

$$\bar{\pi}_{jl}(z) \quad = \quad w_j \tau_{jl}(z) L_l \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} \frac{(-\sigma)}{(1-\sigma)^{\frac{1-\sigma}{-\sigma}}} \left[ c_{jl}^*(z) \int_0^{c_{jl}^*(z)} \left(1 - \frac{c}{c_{jl}^*(z)}\right)^{\frac{1-\sigma}{-\sigma}} g_j(c; z) dc \right].$$

The derivative of the expression in squared brackets with respect to $c_{jl}^*(z)$

$$\int_{c_0}^{c_{jl}^*(z)} \left(1 - \frac{c}{c_{jl}^*(z)}\right)^{\frac{1-\sigma}{-\sigma}} g_j(c; z) dc + \frac{1-\sigma}{-\sigma} \int_0^{c_{jl}^*(z)} \left(1 - \frac{c}{c_{jl}^*(z)}\right)^{\frac{1}{-\sigma}} \frac{c}{c_{jl}^*(z)} g_j(c; z) dc > 0$$

shows that the average profit unconditional on entry is an increasing function of the cutoff $c_{jl}^*(z)$. Recall that $w_j \tau_{jl}(z) c_{jl}^*(z) = w_l c_l^*(z)$, then, for a given vector of wages, the average profit unconditional on entry in a market segment $(j, l, z)$ is an increasing function of the cutoff $c_l^*(z)$ for every origin $j$.

For a given vector of wages, the system of $J$-many free entry conditions (12) of a given sector $z$ can be written as a vector mapping $F : \Re^J \to \Re^J$ such that $F(\boldsymbol{c}; z) = \boldsymbol{0}$ in which the domain is a compact subset in $\Re^J$ and the image is a constant $J$-dimensional null vector. Every function $F_j(\boldsymbol{c}) = \sum_{l=1}^J \bar{\pi}_{jl}(z) - 1$ is continuous and increasing in each $c_j^*(z) \in [0, c_j^{max}(z)]$ for every $j = 1, ..., J$. Therefore, by Brouwer Fixed-Point Theorem a solution in $\Re^J$ exists.

Then, continuity and strict monotonicity of $F$ are sufficient to imply uniqueness of the solution. To see this, assume that both $\boldsymbol{a} \in \Re^J$ and $\boldsymbol{b} \in \Re^J$ are solutions, such that $F(\boldsymbol{a}; z) = \boldsymbol{0}$ and

$F(\boldsymbol{b};z) = \mathbf{0}$. If $\boldsymbol{a} \neq \boldsymbol{b}$ then there exists at least one dimension $k$ in which $a_k \neq b_k$, say $a_k < b_k$ with no loss of generality. But, then by strict monotonicity of the continuous function $F$ implies $\mathbf{0} = F(\boldsymbol{a};z) < F(\boldsymbol{b};z) = \mathbf{0}$ which is a contradiction. ■

This concludes the proof. For completeness of exposition consider the expected profit unconditional on entry in the special case of $\kappa_2 = 1$. We obtain:

$$\bar{\pi}_{jl}(z) = \frac{w_l L_l \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{-\sigma}} (-\sigma)}{\omega_{jl}(z)^{\kappa_1}} a_0^1 c_l^*(z)^{\kappa_1+1}.$$

where we have used the short-hand notation $a_0^{k_2} \equiv a(\kappa_1, \kappa_2, \sigma; 0)$. This is the case in which the distribution of technological coefficients is an Inverse Pareto with shape parameter $\kappa_1$, as it can be seen by noticing that $\Gamma(\kappa_1 + 1) = \kappa_1 \Gamma(\kappa_1)$ and $\Gamma(1) = 1$ and substituting back in $G_j(c;z)$. Furthermore, if $\sigma = -1$ then $B(\kappa_1, 3) = \frac{\Gamma(\kappa_1)\Gamma(3)}{\Gamma(\kappa_1+3)} = \frac{\Gamma(\kappa_1)2}{(\kappa_1+2)(\kappa_1+1)\kappa_1\Gamma(\kappa_1)} = \frac{2}{(\kappa_1+2)(\kappa_1+1)\kappa_1}$ and $\frac{\Gamma(\kappa_1)\Gamma(1)}{\Gamma(\kappa_1+1)} = \frac{1}{\kappa_1}$ yield $a_0^1 = \frac{2(1/2)^{\kappa_1+2}}{(\kappa_1+2)(\kappa_1+1)}$, that, ultimately, leads to the expression of expected profit unconditional on entry

$$\bar{\pi}_{jl}(z) = \frac{\alpha w_l L_l}{\gamma(\kappa_1 + 2)(\kappa_1 + 1)} \left( \frac{w_l / w_j}{c_j^{max}(z) \tau_{jl}(z)} \right)^{\kappa_1} c_l^*(z)^{\kappa_1+1}.$$

that we will be using for a qualitative analysis of model's predictions, i.e. quadratic preferences and Pareto-distributed productivity. The attractive feature of this parametrization is that $\bar{\pi}_{jl}(z)$ is linear in $c_l^*(z)^{\kappa_1+1}$, which implies that the system of free entry conditions is also linear in the same power function of cost cutoffs. However, it shall be stressed that this result holds for an arbitrary value of $\sigma < 0$, thus, quadratic preferences are not necessary.