Fight or Flight? How Do Firms Adapt their Product Mix in Response to Demand and Competition*

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Abstract

We propose a new model of multi-product firms in international trade, where firms choose their product mix based on the products' attractiveness and endogenous competition. The model is motivated by two novel stylized facts using Danish manufacturing data, which demonstrate the importance of product-specific characteristics in understanding firms' product mix choices. The model predicts that as a larger number of firms want to supply products with high attractiveness, these products also feature the toughest competition. Depending on the strength of competition, two sorting patterns are possible: one in which only the most productive firms produce the most attractive products and another in which all firms produce the most attractive products. Our model can generate both sorting patterns depending on the value of a key preference parameter. By quantifying our model, we find that product-specific differences in attractiveness and competition explain a quarter of the variation in sales. Furthermore, we find that the most attractive products tend to be produced by all firms, while the least attractive products are made only by the most productive firms.

Keywords: Multi-product firms, competition, product attractiveness, sorting.

JEL Code: F12, F14, L11 L25.

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1 Introduction

The growing availability of firm-level data from multiple countries has spurred the development of an extensive body of literature that explores diverse aspects of firm-level responses to demand and competitive shocks. For example, international competition resulting from trade liberalization can lead to changes in firm-level pricing, quality, labor demand, and scope (Bernard et al., 2011; Edmond et al., 2015; De Loecker et al., 2016; Fieler and Harrison, 2023; Piveteau and Smagghue, 2024). Often, shocks manifest at the product level, where shifts in consumer preferences may increase demand for a specific product, or changes in product-specific tariffs or regulations may modify the competitive landscape for a singular product. Yet, the workhorse trade model of multi-product firms is ill-equipped to deal with these product-level shocks. Standard models cannot accommodate differences in shocks across core and non-core varieties of a firm as the implicit assumption of these models is that shocks are occurring at the firm-level (Bernard et al., 2011; Mayer et al., 2014, 2021). In this paper, we challenge this assumption and investigate the consequences of incorporating product-specific shocks on firms' product mix decisions and their responses to shifting competitive conditions.

Using Danish manufacturing data, we present two novel stylized facts demonstrating the significance of product-specific characteristics in understanding firm behavior. In our first stylized fact, we find that core competence and firm-product characteristics only partially explain the variance of sales across the products within a firm. In fact, we find that core products typically face less competition and higher demand than non-core products, suggesting product-specific attributes significantly impact a firm's product mix decisions. In our second stylized fact, we discover that product-specific shocks have an explanatory power equivalent to firm-specific shocks in influencing domestic sales at the firm-product level, challenging the assumption that firm-specific shocks dominate variations in firm-product sales. Although these stylized facts are not surprising, they challenge the standard models where demand and competition are firm-level attributes.

Motivated by these stylized facts, we build a multi-country, general equilibrium model of multi-product firms, which differ in their firm-level marginal costs. Our model diverges from standard multi-product firm models, such as those of Eckel and Neary (2010), Bernard et al. (2011), and Mayer et al. (2014), by including a discrete set of products of which any firm can offer a unique variety. These "products" can be thought of as product categories or nests. This structure aligns with real-world data where firms produce in multiple product codes and different firms manufacture varieties under the same code. For example, a product could be toasters, with several firms each producing their distinct varieties. Another product could be mobile phones, with a large number of firms each creating a unique version.[1]

In this framework, the key question is how firms sort into different products - which firms produce a variety of a specific product? For instance, consider toasters and mobile phones. Hitachi, Toshiba, Mitsubishi, Panasonic, and Sharp were all active in both product categories, offering their unique varieties. While they continue to produce toaster varieties, they have all exited the mobile phone product market.[2] Our model provides an explanation for this sorting pattern, where numerous firms produce toasters and fewer firms produce mobile phones, even though the latter has a larger market demand.

Product categories differ in their exogenous "attractiveness", which is modeled as a product-specific demand shifter. For example, if the mobile phone category is more attractive than toasters, it implies that, conditional on the same level of (endogenous) competition, firms can generate higher profits by producing mobile phone varieties rather than toaster varieties. This attractiveness can be attributed to differences in demand characteristics (such as a higher willingness to pay for a smartphone compared to a toaster) or unmodeled supply characteristics like R&D fixed costs. Firms tend to be drawn to products with higher attractiveness, which in turn leads to increased competition in those categories in the equilibrium. Consequently, a firm's profitability in a product depends on the extent to which competition intensifies with product attractiveness.

There are two possible sorting patterns. First, if greater attractiveness results in a high increase in competition, only the most productive firms will produce the most attractive products (e.g., mobile phones), and all firms will produce the least attractive products (e.g., toasters).[3] Second, if competition does not rise significantly as attractive-

[1] Firms can offer several unique "sub-varieties" under a single product code. For example, Apple produces the iPhones 14, 15, and 16 at the same time. Due to data limitation, we cannot observe the information of different sub-varieties offered by a given firm within a product code. Hence, in our model, a variety offered by a firm can be thought of as a bundling of all its sub-varieties under the same product code.

[2] See https://www.ft.com/content/f2876863-6f59-487e-8e9e-95ce6f82b0b6.

[3] This second sorting pattern is also present in the model of firm locating in different cities by Nocke (2006). In fact, the largest cities tend to attract the largest number of firms, but because of the tougher competition, only the most productive firms can survive in the equilibrium.

ness increases, all firms will produce the most attractive products, while only the most productive firms will manufacture the least attractive ones.

The sorting patterns of firms into different products are determined by the elasticity of the cost cutoff with respect to the "remoteness" of a product in a destination (i.e., the multilateral resistance term in the gravity equation). Two competing forces influence this elasticity. First, lower remoteness increases demand for a product, leading to higher markups and weaker selection pressures on firms. Second, reduced remoteness enhances competition, driving down markups and enforcing stronger selection. The dominant force depends on whether the elasticity of the cost cutoff (and thus markups) with respect to product remoteness is positive or negative.

A contribution of our model is to allow this elasticity to assume either positive or negative values, depending on the value of a key parameter. This flexibility enables our model to generate both sorting patterns: one where only the most productive firms manufacture the most attractive products, and one in which all firms produce the most attractive products. The key parameter determining the elasticity's sign is a preference parameter within the Generalized Translated Power (GTP) preferences, as proposed by Bertoletti and Etro (2020). Depending on this parameter's value, these preferences cover a range of common models used in the literature, including indirectly additive (Bertoletti et al., 2018), directly additive (Melitz and Ottaviano, 2008), and homothetic preferences. Thus, our choice of preferences supports both sorting patterns, whereas less flexible preferences would limit their prediction to a single pattern.

In our model, each firm's decision regarding its product mix is solely determined by the product's attractiveness level, the endogenous competitive responses of firms, and the firm's marginal costs, which remains constant across products. To incorporate additional sources of differences in sales across the products of a firm, we introduce idiosyncratic product-specific demand shocks, following the approach of Arkolakis et al. (2021). These shocks enable us to capture all other forces that affect firm-product sales, such as differences in marginal costs and product appeal.

We carry out a quantification of our model to achieve two main objectives. First, we aim to determine the significance of the novel aspects of our model. Second, we seek to measure the (unobserved) product attractiveness and associate it with observable characteristics of product codes. Our model requires calibrating four key parameters. The first three parameters are commonly used in the literature. In particular, we calibrate the parameter that shapes the Pareto distribution of marginal costs by targeting the trade elasticity. Second, we calibrate the parameter controlling demand curvature to target the domestic sales advantage of exporters compared to non-exporters. Lastly, we calibrate the standard deviation of the firm-product specific demand shock by matching the coefficient of variation of revenues.

A key contribution of this paper is the calibration of the parameter that governs the sorting of firms into different products, which controls whether preferences are homothetic, indirectly additive, directly additive or in between these. We calibrate this parameter by estimating the effects of changes in trade openness, measured by the domestic expenditure share of a product, on the number of domestic firms selling that product. This relationship is influenced by the parameter that controls the elasticity of the cost cutoff with respect to the remoteness of a product discussed above. To identify the causal effect of a change in the domestic expenditure share on the number of active domestic firms at the product level, we adopt an instrumental variable approach, where we use the total exports of countries other than Denmark as the instrument, following the recent literature that investigates the labor market effects of international trade (Autor et al., 2013; Hummels et al., 2014).

With our calibrated parameters, we assess the significance of allowing firms to sort into different products depending on their attractiveness and level of competition. First, we quantify the relative importance of product characteristics on the sales variance across firms and products. In our model, firm-product sales differ because of the product-specific attractiveness and competition and because of the idiosyncratic firm-product shock that captures all other possible mechanisms. We find that product-level attributes explain a quarter of the sales variance across firms. Hence, product-level characteristics are crucial drivers of sales across firms and products, an important feature however missed in the current models of multi-product firms.

Second, we compare our model to one that does not incorporate heterogeneity in attractiveness and competition across products, i.e., a version of the existing standard model of multi-product firms. We find that neglecting the product-specific levels of attractiveness and competition overstates the explanatory power of firm-specific shocks by 10 percentage points.

Our calibration results support a sorting pattern where the most attractive products are produced by all firms, while only the largest firms produce the least attractive products. Using our model's structure, we can estimate a value for product attractiveness and relate it to product characteristics. Specifically, the most attractive products generally have the highest production values and the largest export and import values. Moreover, we can infer that our measure of product attractiveness encompasses both demand and supply features: on the demand side, attractive products tend to be differentiated, with limited scope for quality differentiation. On the supply side, attractiveness correlates with increased capital intensity, indicating higher investment requirements.

Related Literature. Our study contributes to the expanding field of research on multi-product firms, which has recently been summarized by Irlacher (2022). Previous research has explored how firms modify the number and distribution of their products in reaction to changes in foreign market access and trade costs (Feenstra and Ma, 2007; Baldwin and Gu, 2009; Eckel and Neary, 2010; Bernard et al., 2011; Qiu and Zhou, 2013; Dhingra, 2013; Mayer et al., 2014; Nocke and Yeaple, 2014; Eckel et al., 2015; Lopresti, 2016; Mayer et al., 2021; Macedoni, 2022; Macedoni and Xu, 2022; Boehm et al., 2022).[4] For instance, Bernard et al. (2011) show that multi-product firms tend to concentrate on their core products following trade liberalization. However, these studies generally assume that demand or competition changes impact all of a firm's products uniformly. Contrary to this assumption, our paper aims to open the black box of multi-product production and allow firms to face attractiveness and competition that differ by product.

Our paper is closely related to two prior studies on multi-product firms and their responses to competition: Mayer et al. (2021) and Fieler and Harrison (2023). Mayer et al. (2021) empirically demonstrates that positive demand shocks lead to an expansion in scope and a reallocation of sales towards better-performing products. These findings are explained using a model in which an increase in market size triggers both scope expansion and intensified competition. This heightened competition, in turn, results in firms reallocating sales towards their most successful products. There are two primary distinc-

[4] There have also been several studies investigating how changes in exchange rates, deunionization, taxation, trade liberalization, and horizontal mergers and acquisitions impact a firm's product scope (Chatterjee et al., 2013; Freitag and Lein, 2023; Flach and Irlacher, 2018; Egger and Koch, 2012; Flach et al., 2021; Qiu and Yu, 2020; Chan et al., 2022).

tions between their model and ours. First, in our model, demand shocks are modeled as product-specific attractiveness, whereas in Mayer et al. (2021), the demand shocks are uniform across all products. Second, while our model encompasses the type of sorting predicted by Mayer et al. (2021)–where demand shocks lead to more firms entering the market–it also considers the reverse scenario, where firms exit the market when markups are more sensitive to increased competition.

Fieler and Harrison (2023) investigate the impact of import competition on firms' product choices outside the conventional model of multi-product firms. While both our study and Fieler and Harrison (2023) investigate firms' responses to heightened competition, their model permits firms to venture into producing more diversified varieties in response to increased competition. In contrast, in our model, firms cannot launch new product categories. Instead, they decide which product categories to engage in based on product-level competition intensity and product attractiveness.

Our paper also complements the growing body of literature on how firms employ different strategies to escape competition. Piveteau and Smagghue (2024) propose a model where competition from low-cost exporters affects low-quality goods more than high-quality goods, causing firms to evade competition by enhancing their quality. Unlike our study, they do not consider the selection of firms into different products. Lim et al. (2022) also consider the interplay between product attractiveness (driven by scale in their case) and the subsequent rise in competition. While our study focuses on firms' selection of product categories, they examine how increased foreign market access influences firms' quality upgrading along the quality distribution. In their research, quality upgrading serves as a strategy for firms to sidestep future competition.[5]

The paper is organized as follows. Section 2 presents the stylized facts that motivate the theory of the paper. Section 3 illustrates the model. Section 4 describes our calibration strategy and the estimated parameters. Section 5 discusses the results of the quantification of the model using the calibrated parameters. Section 6 concludes.

[5] The effects of competition on innovation at the firm level, including the escaping competition mechanism, was initially examined in the seminal work of Aghion et al. (2001) and Aghion et al. (2005). Melitz and Redding (2021) and Akcigit and Melitz (2022) provide comprehensive surveys of the recent development of this literature from the perspective of trade-induced competition.

2 Stylized Facts

2.1 Data

We use data from Denmark Statistics (DST) from 2000 to 2015. The data is collected from the production statistics (VARS) and the trade statistics (UHDI). VARS is a survey that focuses on manufacturing firms with a minimum of 10 employees. It provides data on the total sales value and quantity of each product manufactured by these firms, regardless of whether the product is sold domestically or exported. To align this dataset with the trade statistics, a product is defined as a Combined Nomenclature (CN) eight-digit code. The trade statistics provide information on the exports and imports of each eight-digit CN code by destination.

The literature consistently shows that multi-product firms dominate production. We confirm this fact by showing that in 2000, while single-product firms constituted 45% of all firms manufacturing firms in the survey, they only accounted for 25% of domestic sales, with the remaining 75% generated by multi-product firms. The average product scope of these firms was around 3, albeit with a considerable standard deviation of 4.92. Detailed summary statistics on the distribution of product scope, its sector-wise averages, and its evolution over time are provided in Appendix A.1. Sector analysis reveals that textiles, animal products, and foodstuffs exhibit the highest average scopes of 6.5, 5.2, and 4.1, respectively. In contrast, transportation, metals, and plastics and rubbers sectors have the smallest scopes, with averages of 1.4, 1.6, and 1.7 respectively. The average scope of products initially increased from 2000 to 2005, declined during the Great Recession, but began a steady climb after 2010, reaching a peak of 3.2 in 2014.

2.2 Rank within Firms

In conventional models of multi-product firms, the within-firm ranking of sales across products only depend on the firm-product specific appeal (Bernard et al., 2011) or marginal costs (Eckel and Neary, 2010), and any product-specific attributes bear no relationship with the within-firm ranking. This is because all of a firm's products are assumed to face the same level of competition and share the same set of customers.[6] In our first stylized fact, we test this prediction by evaluating the correlation between product-specific characteristics and within-firm ranking, such as the level of competition and market size that are product-specific, on the ranking of sales across the products offered by a firm.

We compute domestic sales per firm-product in each year by subtracting the value of exports reported in UHDI from the value of total sales reported in VARS. We denote as Rank_{mf} the within-firm ranking of a firm f's product sales in a particular product category m. A Rank_{mf} value of 1 indicates that product m is the top-selling or core product of firm f and hence generates the largest sales revenue for firm f. As sales decrease for other products, the Rank_{mf} value increases. We consider the following regression:

$$\begin{aligned}\text{Rank}_{mft} = {} & \beta_1 \text{Log \# Firms}_{mt} + \beta_2 \text{Log Domestic Sales}_{mt} + \\ & + \beta_3 \text{Market Share}_{mft} + \beta_4 \text{Log Import Competition}_{mt} FE_{ft} + \epsilon_{mft}. \end{aligned} \tag{1}$$

where $\text{Log \# Firms}_{mt}$ is the log of the number of Danish firms included in the production statistics with positive domestic sales in product m and year t, $\text{Log Domestic Sales}_{mt}$ is the aggregate level of domestic sales of Danish firms in product m and year t, $\text{Market Share}_{mft}$ is computed as the ratio of product-firm mf domestic sales over total domestic sales of product m (and multiplied by 100), and $\text{Log Import Competition}_{mt}$ is the log of total imports of product m in year t minus the log of the total sales value (which includes both domestic sales and exports) of product m in year t. We control for unobservable firm characteristics using firm-year fixed effects FE_{ft} to make sure that the comparison is between products within a firm-year combination. The error term in the specification is ϵ_{mft}.

Table 1 presents the results of our analysis, where a negative coefficient indicates a correlation between the variable and a higher ranking within a firm (since the core product has a ranking of one). We find a negative correlation between the number of firms and product ranking, which suggests that the core products of firms typically face more competition. However, after controlling for total domestic sales, the coefficient on the number of firms turns positive, indicating a case of omitted variable bias. Our results show that the core varieties of a firm tend to have a large market size, as measured by

[6] Fontagné et al. (2018) show that firms export different combinations of products across markets, suggesting that some product-specific characteristics are important in determining firm's choices and that these vary across countries.

Table 1: Within Firm Ranking

	Dependent Variable: Within firm rank of product sales			
	(1)	(2)	(3)	(4)
Log Number of Firms	-8.765***	28.207***	12.169***	11.920***
	(0.287)	(0.380)	(0.385)	(0.396)
Log Domestic Sales		-19.630***	-24.066***	-23.763***
		(0.146)	(0.143)	(0.182)
Product Market Share			-1.038***	-1.038***
			(0.009)	(0.009)
Log Import Competition				0.426***
				(0.158)
Firm FE	Yes	Yes	Yes	Yes
R^2	0.69	0.74	0.77	0.77
# Obs.	114454	114454	114454	114454

Results from OLS of (1). Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

total domestic sales, and fewer competitors. Furthermore, we find that a larger market share within a product is also associated with a higher product ranking within a firm. However, this variable does not affect the economic and statistical significance of market size and competition. Finally, we observe that larger imports are associated with a lower ranking. Results are robust to fixing a year: see Table A.3 in the appendix.

While our results are intuitive, they are often overlooked in traditional frameworks of multi-product firms, which assume that the size of the market and the level of competition are uniform across all products. Our first stylized fact challenges this assumption, as we find that firms flee from product-specific competition from other Danish firms and from foreign firms and instead chase product-specific market size and market share.

2.3 Firm-product sales decomposition

Our study also investigates the contribution of firm-year and product-year shocks to the evolution of firm sales. To achieve this, we begin by regressing the log domestic sales of a firm f in product m and year t on a firm-product fixed effect. We record the residual ϵ_{mft}, which is the log of domestic sales adjusted for the average sales of firm f in product m:

$$\text{Log Domestic Sales}_{fmt} = FE_{fm} + \epsilon_{mft}. \tag{2}$$

To investigate the contribution of firm-year and product-year shocks to the variance of the adjusted sales ϵ_{mft}, we use the variance decomposition approach utilized by Hottman et al. (2016) and Bernard et al. (2022). We estimate the following regression:

$$\epsilon_{mft} = FE_{ft} + FE_{mt} + \nu_{fmt}. \tag{3}$$

Next, we regress the two estimated fixed effects FE_{ft} and FE_{mt} on ϵ_{mft} without a constant to conduct a variance decomposition. Using OLS properties, the coefficient of a regression of the estimated fixed effect on ϵ_{mft} without a constant represents the percentage of the variance of ϵ_{mft} explained by that fixed effect. In a workhorse model of multi-product firms, all products face the same demand and competition. This means that the contribution of the product-year fixed effect FE_{mt} should be zero. Instead, the results shown in Table 2 reveal that the firm-year and product-year fixed effects have an equal explanatory power, as they both account for around 25% of the variance in firm-product sales, respectively. The remaining 50% of the variance is explained by time-varying shocks idiosyncratic to the particular firm-product fm.

The results of Table 2 indicate that product-specific shocks, such as changes in demand and competition, are just as crucial as common shocks occurring within the firm, such as firm-level productivity shocks. Our results show that not only product-level attributes are important in explaining the rankings of sales within a firm across products, but also in explaining the over-time change of firm-product sales. Hence, our model will take seriously these product-specific attributes and study their effect on firm sales and sorting across products.

Table 2: Variance Decomposition

	(Firm-Year Shocks)	(Product-Year Shocks)
Variance of Firm-Product Log Sales	0.247***	0.246***
	(0.002)	(0.002)
# Obs.	83236	83236

Results from variance decomposition of (3). Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

3 Model

There are I countries, which are indexed by i for origin and j for destination. In each country, there are L_j consumers with per capita income y_j. There are M differentiated products, which we refer to as product categories, and are indexed by $m = 1, ..., M$. A continuum of multi-product firms can produce a unique variety ω for each product m. To draw a parallel with our empirical analysis, product m is a CN eight-digit code, and the varieties ω within a given product m are the different varieties produced by different firms within the same CN eight-digit code.

Products differ in their exogenous product attractiveness, denoted by a_{mj}, which is modeled as a demand shifter common for all the varieties of a given product m. The parameter a_{mj} captures any difference in products that make some more attractive than others, either because of higher appeal, higher value added, or lower fixed costs of product development. We should note that in reality, products could differ both in appeal modeled as a demand shifter and in fixed costs of producing a variety. However, in this model, we abstract from heterogeneity in fixed costs due to the fact that models that feature both preferences with a bounded utility and the presence of a fixed cost of production are less analytically tractable than our current model. As a result, in our model, product attractiveness also incorporates fixed costs per product.

In addition to attractiveness, products differ in the endogenous level of competition, which is a function of the degree of product attractiveness itself. Higher product attractiveness a_{mj} promotes introduction of new varieties by firms. However, this also leads to tougher competition which may lead to tougher selection, depending on the extent of competition. This latter channel is going to be governed by a key parameter that we will estimate in the data.

As in a standard Melitz (2003) model, there is a mass of firms J_i that paid a fixed cost of entry f_E in domestic labor units and discovered their marginal cost draw c. Marginal costs are drawn from a Pareto distribution with shape parameter θ and shift parameter b_i. Each firm can produce in any of the product category $m = 1, ..., M$.

The only drivers of selection of firms into different products are the attractiveness a_{mj} and the competition in these products. This assumption may seem in stark contrast with the literature that assumes the presence of a core competence (Eckel and Neary, 2010; Mayer et al., 2014; Arkolakis et al., 2021). However, this is done to maintain the selection of firms into different products as tractable as possible. Introducing a core competence that varies across firms would introduce a sizeable complication, since the order in which firms enter products would then become firm-specific. Note that this problem does not arise in the literature aforementioned, since competition and attractiveness are identical across products. To address this issue, we introduce idiosyncratic demand-shocks that affect firm-product level sales and are realized upon consumption. The introduction of these demand shocks is quantitatively important since they capture any firm-product specific characteristics that affects sales independently of attractiveness and competition, including core competence that varies across firms.

There is an iceberg trade cost $\tau_{mij} \geq 1$ of exporting from i to j which is specific to product m. These iceberg trade costs vary by product to account for the possibility that trade barriers might differ by product (Boehm et al., 2023). This assumption leads to the emergence of importer-exporter-product specific expenditure shares. For example, the proportion of Danish imports from Germany relative to Denmark's total consumption can vary by product. If iceberg trade costs were uniform across all products, then the proportion of Danish imports from Germany would remain constant across different products. Workers are the only factor of production and gain a wage w_i. Free entry drives expected profits to zero and, as a result, per capita income and wages are equal.

3.1 Consumer's Demand

Demand for each product m in country j originates from the Generalized Translated Power (GTP) preferences proposed by Bertoletti and Etro (2020):[7] The utility function equals:

$$U_{mj} = \int_{\Omega_{mj}} \delta_{mj}(\omega) \left(a_{mj}\xi_{mj}q_{mj}(\omega) - \frac{(\xi_{mj}q_{mj}(\omega))^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) d\omega + \frac{\xi_{mj}^{-\eta} - 1}{\eta}, \tag{4}$$

where $q_{mj}(\omega)$ represents the quantity of variety ω in product m and country j. $\delta_{mj}(\omega)$ is a demand shock which is specific to the variety ω. We follow Arkolakis et al. (2021) and assume that such a shock is *i.i.d.*, is realized upon consumption, and its expected value is one.[8] We denote with σ the standard deviation of the demand shock. $a_{mj} > 0$ is a demand shifter that is common for any varieties of product m sold in country j and captures the attractiveness or appeal of product m in country j. $\gamma > 0$ is a parameter that controls the

[7] Fally (2022) describes the regularity conditions for these preferences. Macedoni and Weinberger (2022) uses these preferences to study how regulations reduce misallocation in Chile.

[8] Because of these assumptions, the shock does not affect the standard free entry condition in models with heterogeneous firms (Melitz, 2003).

curvature of the demand. ξ_{mj} is a quantity aggregator that is implicitly defined as:

$$\xi_{mj}^{-\eta} = \int \delta_{mj}(\omega) \left(a_{mj}\xi_{mj}q_{mj}(\omega) - (\xi_{mj}q_{mj}(\omega))^{1+\frac{1}{\gamma}} \right) d\omega. \tag{5}$$

The parameter $\eta \in [-1, \infty)$ is crucial in our analysis. In particular, it controls the elasticity of the cost cutoff (and markups) with respect to the degree of remoteness of a product-destination. As a result, it controls the selection patterns of firms with different marginal costs into different products. That these selection patterns are controlled by η is a key advantage of these preferences. In other words, depending on the value of η, our model can feature either an allocation in which only the most efficient firms are able to sell the products with the largest attractiveness, or an opposite allocation where all active firms sell the products with the largest attractiveness. The most common preferences used in the literature fix the elasticity of cost cutoff (and markups) with respect to a product's market remoteness and, as a result, only allow for one type of sorting. These common preferences are nested in our framework. In fact, for $\eta = -1$, preferences are indirectly additive (IA) as described by (Bertoletti et al., 2018). For $\eta = 0$, preferences become homothetic with a single aggregator, similar to Matsuyama and Ushchev (2017). For $\eta \to \infty$, preferences become directly additive (DA), and generalize the preferences used by Melitz and Ottaviano (2008).

We assume that consumers spend a constant share α of their income on each product m, so that $\alpha = 1/M$. This is a stark assumption that allows us to link product attractiveness of a product to the selection forces for that product and not those for the other products. In other words, given this assumption, whether the most productive firms produce a variety of a certain product only depends on the attractiveness and the level of competition for that product and not on the attractiveness and competition of the other products.[9]

The consumer's budget constraint is:

$$\int_{\Omega_{mj}} p_{mj}(\omega)q_{mj}(\omega)d\omega \leq \alpha y_j.$$

[9] Given this assumption, the model can also be used to identify firm selection into different broadly defined markets m. This assumption is also equivalent to assuming that there are different sets of consumers purchasing each product. Finally, note that using a Cobb-Douglas aggregation of U_{mj} would generate general equilibrium effects that nullify the selection forces we are attempting at modeling.

Solving the consumers' problem yields the following inverse demand function:

$$p_{mj}(\omega) = \alpha y_j \xi_{mj}^{1+\eta} \delta_{mj}(\omega) \left[a_{mj} - \left(\xi_{mj} q_{mj}(\omega) \right)^{\frac{1}{\gamma}} \right], \tag{6}$$

where $p_{mj}(\omega)$ is the price of the variety ω.[10]

3.2 Firm's Problem

Let us now turn to the firm's problem. As the demand shocks $\delta_{mj}(\omega)$ are realized upon consumption, and are *i.i.d.*, they do not affect entry and production decisions of firms. This formulation is also present in Arkolakis et al. (2021), and it implies that firms decide in which product categories to introduce a variety and what prices to charge before the realization of the shocks. Upon consumption, the demand shocks are realized and firms revenues and profits will be affected, but the scope and pricing decisions of firms are sunk. In our paper, these variety-specific demand shocks capture any other determinants of firm scope aside from attractiveness and competition, such as firms' core competence (Eckel and Neary, 2010; Arkolakis et al., 2021) and flexibility (Macedoni and Xu, 2022).

Profits of a firm with marginal cost draw c from i producing a variety in product m and selling it at country j are given by:

$$\pi_{mij}(c) = L_j \alpha y_j \xi_{mj}^{1+\eta} \left[a_{mj} - \left(\xi_{mj} q_{mij}(c) \right)^{\frac{1}{\gamma}} \right] q_{mij}(c) - L_j \tau_{mij} w_i c q_{mij}(c). \tag{7}$$

The first order condition is:

$$\alpha y_j \xi_{mj}^{1+\eta} \left[a_{mj} - \frac{1+\gamma}{\gamma} \left(\xi_{mj} q_{mij}(c) \right)^{\frac{1}{\gamma}} \right] - \tau_{mij} w_i c = 0.$$

The cutoff firm with marginal cost from i to j, c^*_{mij}, is indifferent between selling product m at country j or not. To find c^*_{mij}, we set $q_{mij}(c^*_{mij}) = 0$ in the first order condition, which yields:

$$c^*_{mij} = \frac{\alpha a_{mj} y_j \xi_{mj}^{1+\eta}}{\tau_{mij} w_i}. \tag{8}$$

[10] If $\eta \to \infty$, $\xi_{mj} = 1$ and, therefore, $p_{mj}(\omega) = y_j \delta_{mj}(\omega) \left[a_{mj} - q_{mj}(\omega)^{\frac{1}{\gamma}} \right]$. If also $\gamma = 1$, the demand becomes identical to the linear demand of Melitz and Ottaviano (2008) without the aggregator.

Thus, the cutoff, which controls the sorting of firms into the various products, increases with the demand shifter a_{mj} and declines with the iceberg trade cost. The cutoff also depends on ξ_{mj}, which is a general equilibrium object that controls the endogenous response of competition to the demand shifter. The parameter η controls the relationship between the cutoff and ξ_{mj}. In fact, let us compare the cost cutoff of products m and k in the same origin-destination pair ij:

$$\frac{c^*_{mij}}{c^*_{kij}} = \frac{a_{mj}\xi_{mj}^{1+\eta}\tau_{kij}}{a_{kj}\xi_{kj}^{1+\eta}\tau_{mij}}. \tag{9}$$

The ratio between cutoffs, which determines the sorting of firms in the two products, depends on the ratio of the product demand shifters, the ratio between the general equilibrium object ξ_{mj}, and the relative trade costs. Conditional on the same product-destination pair mj, the ratio between the export cost cutoff and the domestic cost cutoff can be expressed in the following way:

$$\frac{c^*_{mij}}{c^*_{mjj}} = \frac{\tau_{mjj}w_j}{\tau_{mij}w_i}. \tag{10}$$

Using the cutoff definition, we can write the performance variables of a firm as:

$$q_{mij}(c) = \left(\frac{\gamma}{1+\gamma}\right)^{\gamma} \frac{a_{mj}^{\gamma}}{\xi_{mj}(c^*_{mij})^{\gamma}}(c^*_{mij} - c)^{\gamma}, \tag{11}$$

$$p_{mij}(c) = \frac{w_i\tau_{mij}}{1+\gamma}(c^*_{mij} + \gamma c), \tag{12}$$

$$r_{mij}(c) = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a_{mj}^{\gamma}}{\xi_{mj}(c^*_{mij})^{\gamma}}(c^*_{mij} - c)^{\gamma}(c^*_{mij} + \gamma c), \tag{13}$$

$$\pi_{mij}(c) = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a_{mj}^{\gamma}}{\xi_{mj}(c^*_{mij})^{\gamma}}(c^*_{mij} - c)^{1+\gamma}. \tag{14}$$

where $r_{mij}(c)$ denotes the revenues of firm c in product m from i to j. Letting $n_{ij}(c)$ denote the firm c product scope, the scope equals to:

$$n_{ij}(c) = \sum_{m=1}^{N} \mathbb{1}_{c \le c^*_{mij}}. \tag{15}$$

Notice that the demand shock $\delta_{mj}(c)$ faced by firm c across product-destinations mj affects the realized revenues and profits. In fact, realized revenues equal $\tilde{r}_{mij}(c) =$

$\delta_{mj}(c)r_{mij}$ and realized profits equal $\tilde{\pi}_{mij}(c) = \delta_{mj}(c)\pi_{mij}$.

3.3 Sorting into Products

By exploiting the gravity formulation of the model, we can characterize the equilibrium using a system of equations represented by a parsimonious set of parameters and data. The system of equations that characterizes the equilibrium is similar to most of the literature of international trade with heterogeneous firms. For this reason, we leave the derivations of the equilibrium to the appendix, and focus in this section on the key innovation of the model, which is how firms sort into different products.

The parameter η significantly influences how firms sort into various products. To better understand the role of this parameter, let us examine the effects of a product-destination pair's "remoteness" on the cost cutoff of selling there. To do this, we calculate the elasticity of c^*_{mjj} with respect to the "multilateral resistance" term of product-destination mj, $\Phi_{mj} = \sum_i J_i(b_i\tau_{mij}w_i)^{-\theta}$, which relates negatively to the "remoteness" of product-destination mj.[11] An increase in Φ_{mj} indicates that consumers in j are more accessible to the suppliers of product m around the globe, generating a higher overall demand for product m:

$$\frac{\partial \ln c^*_{mjj}}{\partial \ln \Phi_{mj}} = \frac{1}{\frac{1}{1+\eta} - (1+\theta)}. \tag{16}$$

The value of η dictates the sign of the elasticity. On one hand, a reduction in the remoteness of a market raises the overall demand for product m and tends to increase the cost cutoff c^*_{mjj}, with the size of this anti-selection effect increasing in $\frac{1}{1+\eta}$. On the other hand, a reduction in the remoteness intensifies competition in that market and decreases markups, which tends to decrease the cost cutoff c^*_{mjj}, with the size of this selection effect increasing in $1+\theta$. The net effect of Φ_{mj} on c^*_{mjj} thus depends on the relative magnitudes of $\frac{1}{1+\eta}$ and $1+\theta$. In fact, if $\eta > -\frac{\theta}{\theta+1}$, the elasticity is negative as it is typically assumed, meaning that a less remote market is associated with tougher selection, a smaller number of active firms, a larger average firm size measured by quantity, and a lower average price. Otherwise, if $-1 \geq \eta < -\frac{\theta}{\theta+1}$, a less remote market instead features a less selective environment, a larger number of active firms, a smaller average firm size, and a higher average price.

[11] For the derivation, see equations (B.3) and (B.9) in the appendix.

For Indirectly Additive preferences where $\eta = -1$, the cost cutoff is unaffected by changes in the remoteness. For Directly Additive preferences where $\eta \to \infty$ and homothetic preferences where $\eta = 0$, the cost cutoff decreases as remoteness decreases, with the most substantial decline occurring under homothetic preferences.

We can also calculate the elasticity of firm-product-level markup, represented as $\frac{p_{mjj}(c)}{c}$, with respect to the inverse of the remoteness measure:

$$\frac{\partial \ln\left(p_{mjj}(c)/c\right)}{\partial \ln \Phi_{mj}} = \frac{1}{c^*_{mjj} + \gamma c} \times \frac{1}{\frac{1}{1+\eta} - (1+\theta)}, \tag{17}$$

whose sign also depends on the relative magnitudes of $\frac{1}{1+\eta}$ and $1+\theta$.[12]

Having discussed the role of η, we can now focus on the sorting of firms from j in their domestic economy j. Solving the model, we can express the relative cutoff (9)$\frac{c^*_{mjj}}{c^*_{kjj}}$ as follows:

$$\frac{c^*_{mjj}}{c^*_{kjj}} = \left(\frac{\lambda_{mjj}}{\lambda_{kjj}}\right)^{-\frac{1}{\frac{1}{1+\eta}-(1+\theta)}} \left(\frac{a_{mj}}{a_{kj}}\right)^{\frac{\gamma+\frac{1}{1+\eta}}{\frac{1}{1+\eta}-(1+\theta)}}, \tag{18}$$

where λ_{mjj} is the domestic expenditure share of product m in destination j.[13] The higher the ratio, the larger the number of firms that produce a variety of product m. If the ratio is greater than one, more firms produce product m than product k.

Similar to the effect of the remoteness measure, the market cutoff can increase or decrease (with different elasticities) with the product attractiveness depending on the parameter η. In particular, consider the elasticity of the relative cutoff with respect to the relative product attractiveness, holding constant the relative trade shares $\frac{\lambda_{mjj}}{\lambda_{kjj}}$,

$$\frac{\partial \ln\left(c^*_{mjj}/c^*_{kjj}\right)}{\partial \ln\left(a_{mj}/a_{kj}\right)} = \frac{\gamma + \frac{1}{1+\eta}}{\frac{1}{1+\eta} - (1+\theta)}. \tag{19}$$

When the cutoff is increasing with product attractiveness ($-1 \geq \eta < -\frac{\theta}{\theta+1}$), both large and small firms would sell the most attractive products, despite the higher competition,

[12] Manova and Zhang (2012) and Harrigan et al. (2015) both document that within the same product, individual exporters charge higher prices at less remote markets, indicating that $-1 \geq \eta < -\frac{\theta}{\theta+1}$. The estimated parameters in the later section is indeed consistent with this finding.

[13] We use the relationship that $\Phi_{mj} = \frac{J_j (b_j \tau_{mjj} w_j)^{-\theta}}{\lambda_{mjj}}$. The ratio of cutoffs has a third component $\left(\frac{\tau_{kjj}}{\tau_{mjj}}\right)$ which is normalized to one by assuming that $\tau_{mjj} = \tau_{kjj} = 1$.

and only the most productive firms sell the least attractive products.[14] This is, for instance, the case of Indirectly Additive preferences, in which $\eta = -1$ is within the range for which the elasticity of the cutoff with respect to attractiveness is positive.[15]

In contrast, when the cutoff is decreasing with product attractiveness, the opposite sorting pattern occurs: the intense competition at the most attractive products dominates and only the most productive firms can survive making these products. For instance, this is the case under Directly Additive ($\eta \to \infty$) and homothetic preferences ($\eta = 0$). As firms are attracted by high-attractiveness products, the higher level of competition will reduce markups. The reduction in markups force less productive firms to exit. The resulting sorting is similar to that outlined by Nocke (2006), in which the product with the largest attractiveness features the strongest level of competition and, thus, only the most productive firms are active in such product category. Products with lower attractiveness are less competitive, and thus less productive firms are also able to participate.

We can also study the relationship between the relative cutoff and the relative trade openness of a country:

$$\frac{\partial \ln \left(c^*_{mjj}/c^*_{kjj}\right)}{\partial \ln \left(\lambda_{mjj}/\lambda_{kjj}\right)} = -\frac{1}{\frac{1}{1+\eta} - (1+\theta)}. \tag{20}$$

Note that $\lambda_{mjj}/\lambda_{kjj} = (\Phi_{mj}/\Phi_{kj})^{-1}$, so we can interpret an increase in $\lambda_{mjj}/\lambda_{kjj}$ as an increase in the remoteness of product market mj relative to product market kj for firms from j, or a decrease in import competition in market mj relative to kj. If the country j becomes more remote in product m and hence decreases its imports of m from abroad (that is, the relative domestic expenditure share λ_{mjj} rises), the number of domestic firms that sell product m increases only if $\eta > -\frac{\theta}{\theta+1}$, a pattern occurring when only the most productive firms produce the most attractive products.

Optimal Allocation. Market allocations in models with variable markups generally lead to inefficiencies, as demonstrated by Dhingra and Morrow (2019). These inefficiencies arise because markups distort the relationship between relative prices and relative marginal

[14] The correct lower bound for η such that $\frac{\partial(c^*_{mjj}/c^*_{kjj})}{\partial(a_{mj}/a_{kj})} > 0$ is $\eta > -1 - \frac{1}{\gamma}$. However, we assume that $\eta \geq -1$ and, therefore, $-1 - \frac{1}{\gamma}$ is outside the allowed values for η.

[15] When $\eta = -1$, the elasticity of $\frac{c^*_{mjj}}{c^*_{kjj}}$ with respect to $\frac{a_{mj}}{a_{kj}}$ converges to 1.

costs, causing high-markup firms to under-produce and low-markup firms to over-produce. As a result, the selection in market allocations may either exceed or fall short of the optimal level. Bertoletti and Etro (2020) and Macedoni and Weinberger (2022) have identified the range of parameters under which under- or over-selection occur, given our GTP preferences. In Appendix B.3, we examine the optimal sorting of firms into different products. Our findings indicate that the relationship between relative cutoffs and relative product attractiveness in the market allocation mirrors that of the planner's allocation. Hence, we conclude that while the absolute level of selection remains inefficient, the relative selection across products is efficient. In practical terms, if the parameters indicate an insufficient level of selection, this inefficiency is uniformly observed across all products, and the relative selection between products remains efficient.

4 Calibration

In this section, we describe the calibration procedure for the four key parameters of the model: the shape parameter of the marginal cost distribution θ, the demand curvature γ, the standard deviation of the demand shock σ, and the parameter that controls the sorting patterns η. To estimate the first three parameters, we use an exact identification strategy, while for the parameter η, we adopt an instrumental variable (IV) approach.

4.1 Exactly Identified Parameters: θ, γ, and σ

4.1.1 Moments

Shape Parameter of the Distribution of Firms' Marginal Costs θ. To calibrate θ, we target the trade elasticity, i.e., the elasticity of trade flows with respect to trade costs τ. We use the gravity equation to derive the trade elasticity, which equals the value of θ:[16]

$$\lambda_{mij} = \frac{L_i(b_i w_i \tau_{mij})^{-\theta}}{\sum_{v=1}^{I} L_v(b_v w_v \tau_{mvj})^{-\theta}} \quad \text{for } i, j = 1, ..., I. \tag{21}$$

[16] Our model assumes that there is a common trade elasticity across all products, which is a reasonable assumption for a single-sector model. This assumption is widely used in the literature, as evidenced by studies such as Arkolakis et al. (2012, 2019); Bertoletti et al. (2018).

This moment is widely used in the literature (Arkolakis et al., 2012, 2019; Bertoletti et al., 2018). For instance, Arkolakis et al. (2012) and Arkolakis et al. (2019) use the trade elasticity as the moment to keep constant across models in welfare comparisons. In the literature, the absolute value of the trade elasticity has been estimated to be between 4 and 8, as reported in studies such as Eaton and Kortum (2002), Simonovska and Waugh (2014), and Head and Mayer (2014). However, recent estimates that rely on time-series data and assume trade costs can be product and country-pair specific report even smaller estimates, ranging from 1.75 to 2.25 (Boehm et al., 2023). In our baseline results, we assume a trade elasticity of 4. In an extension, we also consider a trade elasticity of 2 to assess the robustness of our calibration.

Demand Curvature Parameter γ. To capture the demand curvature parameter γ, we use another widely used moment in the literature, the exporter sales advantage, as in Bernard et al. (2003), Jung et al. (2019), and Bertoletti et al. (2018). The exporter sales advantage is defined as the ratio of the average domestic sales of exporters to the average domestic sales of non-exporters. The parameter γ controls the difference in sales values between small and large firms by modulating the demand elasticity at different quantities. A higher γ value amplifies the differences between large and small firms, leading to an increase in the exporters' sales advantage.

Our model provides a closed-form expression for the exporters' sales advantage in each product m, allowing us to estimate the value of γ:

$$M_{mj}^{sadv} = \frac{\left[\left(\frac{c_{mjj}^*}{c_{ex}^*}\right)^{\theta} - 1\right]\left[B\left(\frac{c_{ex}^*}{c_{mjj}^*};\theta;\gamma+1\right) + \gamma B\left(\frac{c_{ex}^*}{c_{mjj}^*};\theta+1;\gamma+1\right)\right]}{B(\theta,\gamma+1) + \gamma B(\theta+1,\gamma+1) - B\left(\frac{c_{ex}^*}{c_{mjj}^*};\theta;\gamma+1\right) - \gamma B\left(\frac{c_{ex}^*}{c_{mjj}^*};\theta+1;\gamma+1\right)}, \tag{22}$$

where $c_{ex}^* = \max_{v\neq j} c_{mjj}^*$ is the highest export cutoff, which is the cutoff to export to the destination with the lowest export hurdle. $B(z,h)$ is the Euler Beta Function and $B(u;z,h)$ is the incomplete Euler Beta Function.[17] Detailed derivations for the expression can be found in the appendix.

[17] The Euler Beta Function is given by the following integral $B(z,h) = \int_0^1 t^{z-1}(1-t)^{h-1}\,dt$, while the incomplete Euler Beta Function is given by: $B(u;z,h) = \int_0^u t^{z-1}(1-t)^{h-1}\,dt$.

The cutoff ratio $\frac{c^*_{mjj}}{c^*_{ex}}$ in (22) can be computed given θ and the value for export participation: $\left(\frac{c^*_{mjj}}{c^*_{ex}}\right)^{\theta} = N_{mjj}/N^{ex}_{mj}$, where N^{ex}_{mj} is the number of exporters. To compute the export sales advantage we proceed as follows. First, for each eight-digit CN good and year, we calculate the firm-product level domestic sales by taking the difference between the total production value reported in the production statistics and the export value reported in the customs data.[18] Next, we compute firm-level domestic sales for each firm-year pair, and calculate the average domestic sales of both exporters and non-exporters for each year. Finally, to calculate the export participation, we divide the total number of exporters in our production survey by the total number of firms in the survey.

For our baseline year, 2000, the exporter sales advantage in our model equals 2.37, while the export participation is 0.55. These values are different than those documented for the US, as reported in Bernard et al. (2003): the exporter sales advantage is smaller in our case, while the export participation is larger. One possible reason for this difference could be that our production data only includes firms with at least 10 employees, which leads to an overestimation of the export participation (as most small firms do not export and they are not included in the survey) and an underestimation of the exporter sales advantage. We observe that the value of the sales advantage remains relatively stable in the first decade of the 2000s, around 2.5-2.6, but from 2009 onwards, it increases and takes on values greater than 3.5. The export participation also increases over time, rising from 0.55 in 2000 to 0.62 in 2015, as presented in Table C.1. To examine the robustness of our results, we also consider the moments documented for the US, where the exporter sales advantage is estimated to be 4.8 and the export participation is 0.18.

Standard Deviation of the Firm-Product Shock σ. To calibrate σ, we target the coefficient of variation of domestic sales in a product. This coefficient of variation is constant across products and is defined as the ratio between the standard deviation of sales across firms and the average sales across firms. Our assumption of *i.i.d* shocks and a Pareto distribution of unit costs allows us to derive the following closed-form expression for the coefficient of variation:

$$CV_{mj} = \frac{\left(T_3(1+\sigma) - \theta T_1^2\right)^{1/2}}{\theta^{1/2} T_1}. \tag{23}$$

[18] We discard any firm-products with a negative value of imputed domestic sales, which can arise due to carry-along trade.

where

$$T_1 = B(\theta, \gamma + 1) + \gamma B(\theta + 1, \gamma + 1),$$
$$T_3 = B(\theta, 2\gamma + 1) + 2\gamma B(\theta + 1, 2\gamma + 1) + \gamma^2 B(\theta + 2, 2\gamma + 1).$$

Given θ and γ, the coefficient of variation is positively related to the value of σ. Using our production survey data, we calculate the coefficient of variation of domestic sales for each CN eight-digit code. We then compute the simple average across all products, which is equal to 1.69 for the year 2000. We find that the coefficient of variation tends to increase over time, with values rising to around 1.8-1.9 in later years (see Table C.1). Previous research has estimated the standard deviation of ex-post demand shocks using an overidentified approach, which employs moments from the sales distribution and the scope distribution across firms, as documented in Arkolakis et al. (2021) and Macedoni and Xu (2022). Using the coefficient of variation to estimate σ is a faster approach and yields parameter estimates that are consistent with the literature.

Standard Errors. We compute the standard errors of γ and σ using a bootstrap method. The two moments for the estimation of these two parameters are the exporters domestic sales advantage and the coefficient of variation of domestic sales. These moments are computed based on a sample of $\mathcal{N}$ firm-product observations for a given year. We compute 100 bootstrapped samples of firm-product observations, by redrawing with replacement $\mathcal{N}$ firm-product observations to compute the two moments, along with the export participation rate that we need to compute the exporters domestic sales advantage. Then, we apply our algorithm to calibrate γ and σ, for which we then have 100 observations that we can use to compute the standard error. Notice that since the trade elasticity is taken from the literature, we cannot apply this procedure to the calibration of θ.

4.1.2 Results

Table 3 reports the results of the calibration using data from 2000. The value of the trade elasticity parameter θ is 4, which is consistent with the literature. The demand curvature parameter γ takes on a value of 0.59 with a standard error of 0.07, which is smaller than previous estimates from the literature (Bertoletti et al., 2018; Macedoni and Weinberger,

2022) due to the relatively low exporter sales advantage value. The standard deviation of the demand shock σ is estimated to be 2.18 with a standard error of 0.14, which is in line with previous estimates using Brazilian data (Arkolakis et al., 2021) and Chinese data (Macedoni and Xu, 2022). Overall, our calibration results are broadly consistent with previous estimates from the literature.

Table 3: Moments and Parameters

Moments	
Trade Elasticity	4
Exporters Sales Advantage	2.37
Export Participation	0.55
Coefficient of Variation of Sales	1.69
Parameters	
θ	4
γ	0.59
	(0.07)
σ	2.18
	(0.14)

Moments computed using data from the year 2000. Bootstrap standard errors in parenthesis.

As a robustness check, we conduct the calibration using the moments computed for each year between 2000 and 2015. The results are presented in Table C.2 in the appendix. We observe that a higher exporter sales advantage is associated with higher values of γ, with estimates between 0.82 and 1.02 from 2009 to 2015. The standard deviation of the demand shock ranges between 1.89 and 2.53 across the years. Changes in the value of σ are mainly driven by variations in the value of γ, as higher values of γ tend to increase the standard deviation of revenues (as it magnifies productivity differences across firms). Thus, the portion of the coefficient of variation that is left to be explained by σ becomes smaller. Overall, our calibration results remain robust across the years.

As an additional robustness check, we consider the case of a lower trade elasticity, equal to 2, and report the results in Table C.3 in the appendix. The resulting estimate of γ is higher than the baseline case, while the value of σ barely changes. A lower value of θ is associated with a lower coefficient of variation and a lower exporter sales advantage, but the increase in γ offsets these two effects. In another robustness check, we consider the case of using the US exporter sales advantage and the US export participation values, and report the results in Table C.4 in the appendix. In this case, the estimate of γ is significantly higher, at 1.6, while the value of σ is lower, at 0.86.

4.2 The Sorting Parameter: η

4.2.1 Moment

We exploit a novel prediction of our model to calibrate the parameter η that governs the sorting pattern in different products. Combining (18) with the Pareto distribution of marginal cost c, we can rewrite the relative number of firms selling product m as

$$\frac{N_{mjj}}{N_{kjj}} = \left(\frac{\lambda_{mjj}}{\lambda_{kjj}}\right)^{-\frac{\theta}{\frac{1}{1+\eta}-(1+\theta)}} \left(\frac{a_{mj}}{a_{kj}}\right)^{\frac{(\gamma+\frac{1}{1+\eta})\theta}{\frac{1}{1+\eta}-(1+\theta)}}. \tag{24}$$

The relative number of producers of m depends on the relative domestic expenditure share and the relative product attractiveness of product m. Inspecting the elasticity of (N_{mjj}/N_{kjj}) with respect to $(\lambda_{mjj}/\lambda_{kjj})$, we find that the sign of the elasticity is also informative about the sorting pattern:

$$\frac{\partial \ln\left(N_{mjj}/N_{kjj}\right)}{\partial \ln\left(\lambda_{mjj}/\lambda_{kjj}\right)} = -\frac{\theta}{\frac{1}{1+\eta} - (1+\theta)}. \tag{25}$$

As the domestic expenditure share of product m increases so that the product-level import competition lessens and the product-level remoteness rises, the number of domestic firms producing product m decreases when $-1 \geq \eta < -\frac{\theta}{\theta+1}$. In this case, the effect of declining overall demand for product m due to reduced trade openness (increased trade remoteness) dominates. Hence, the market becomes more selective and fewer domestic firms producing m remain active. In contrast, when $\eta > -\frac{\theta}{\theta+1}$, the effect of reduced competition due to lowered trade openness dominates for product m, leading to a less selective market in which a larger number of domestic firms can survive.

Fixing country j, we can further express the number of domestic firms producing m as follows:

$$\ln N_{mjj} = -\frac{\theta}{\frac{1}{1+\eta} - (1+\theta)} \cdot \ln \lambda_{mjj} + \frac{(\gamma + \frac{1}{1+\eta})\theta}{\frac{1}{1+\eta} - (1+\theta)} \cdot \ln a_{mj} + \Gamma_j, \tag{26}$$

where the parameter Γ_j is a collection of variables that only depend on j.

Let j be Denmark, so we can ignore the j subscript to simplify the notations. We compute the number of active Danish firms that sell product m in year t, N_{mt}, and Denmark's domestic expenditure share of product m in year t, λ_{mt}. Taking the first-difference form of (26), we obtain the key regression specification:

$$\Delta \ln N_{mt} = \kappa \Delta \ln \lambda_{mt} + \epsilon_{mt}, \tag{27}$$

where $\kappa = -\frac{\theta}{\frac{1}{1+\eta}-(1+\theta)}$ and the error term $\epsilon_{mt} = \frac{(\gamma+\frac{1}{1+\eta})\theta}{\frac{1}{1+\eta}-(1+\theta)}\Delta \ln a_{mt} + \Delta\Gamma_t$ contains the overtime changes in product attractiveness a_{mt} in Denmark and other variables Γ_t that are specific to Denmark but (assumed to be) constant across different products.[19]

Ideally, if we can consistently estimate the value of κ in (27), we can compute the value of the sorting parameter η given the calibrated value of θ. In fact,

$$\eta = \frac{\theta(\frac{1}{\hat{\kappa}} - 1)}{1 - \theta(\frac{1}{\hat{\kappa}} - 1)}. \tag{28}$$

To obtain a standard error for the parameter η we proceed as follows. Specifically, we draw 100 realizations of a normally distributed random variables with mean $\hat{\kappa}$ and with standard deviation the estimated standard deviation of $\hat{\kappa}$. We compute the corresponding value of η for each draw and compute the resulting standard error.

4.2.2 Identification

Estimating (27) using OLS may yield an inconsistent estimate of κ because of omitted variable bias, as in the error term we have the level of product attractiveness a_{mt}, which is unobservable to the researcher. In fact, our model predicts that an increase in the product attractiveness of m in Denmark in the error term, $\Delta \ln a_{mt} > 0$, has an effect on the number of firms selling to that country and, simultaneously, on Denmark's domestic expenditure share of m, $\Delta \ln \lambda_{mt}$, through general equilibrium effects driven by changes in the level of competition. To tackle this endogeneity issue, we adopt an instrumental variable approach to estimate κ, making sure that the variation of $\Delta \ln \lambda_{mt}$ used in the estimation is plausibly orthogonal to $\Delta \ln a_{mt}$ and other product-specific factors in Denmark.

We construct the following instrument for $\Delta \ln \lambda_{mt}$ by purging the supply-side factors

[19] Appendix C.2 describes how we calculate λ_{mt} using the Danish data.

in other countries that affect λ_{mt}:

$$\Delta \ln \tilde{\lambda}_{mt} = -\ln\left(\lambda_{m\text{DNK},t-1} + (1-\lambda_{m\text{DNK},t-1}) \sum_{i\neq \text{DNK}} Imp\_Share_{mi,t-1} \times \frac{EX^{\text{high-income}}_{mi,t}}{EX^{\text{high-income}}_{mi,t-1}}\right). \tag{29}$$

In equation (29), $\lambda_{m\text{DNK},t-1}$ is Denmark's domestic expenditure share of product m in year $t-1$, and $Imp\_Share_{mi,t-1}$ is the share of Denmark's imports from i on product m over Denmark's total imports on product m. These trade shares at $t-1$ are considered predetermined for the subsequent changes from $t-1$ to t. The variables $EX^{\text{high-income}}_{mi,t}$ and $EX^{\text{high-income}}_{mi,t-1}$ are country i's total exports of product m to a set of other high-income countries other than Denmark in years t and $t-1$, respectively. The ratio $(EX^{\text{high-income}}_{mi,t}/EX^{\text{high-income}}_{mi,t-1})$ aims to capture the product-level export growth driven by pure supply-side factors of country i (e.g., productivity growth), and thus is presumably orthogonal to the change in product attractiveness $\Delta \ln a_{mt}$. Autor et al. (2013) and Hummels et al. (2014) use similar assumptions and strategies to construct instruments for increases in import competition and offshoring that are orthogonal to the demand-side factors, respectively. Intuitively, an increase in country i's supply-side export capability at product m should result in an increase in rising export from i to Denmark, and hence a decrease in Denmark's domestic expenditure share of product m, λ_{mt}. Recall that an increase in domestic expenditure share corresponds to an increase in remoteness. So our instrument aims to capture the variations in remoteness that are caused by "foreign shocks". Therefore, the use of instrument $\Delta \ln \tilde{\lambda}_{mt}$ should consistently recover the parameter of interest κ.

For robustness, we experiment with a few different choices of the set of high-income countries when constructing the instrument. The first set follows Autor et al. (2013) and contains Australia, Finland, Germany, Japan, New Zealand, Spain, Switzerland, and USA. We refer to this country set as the "ADH set". The second set is the EU member states before the 2004 EU enlargement, including Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, and Sweden. We refer to this country set as the "EU set". We focus on a twenty-year period from 1996 to 2016.

4.2.3 Results

We report the results of estimating (27) in Table 4. To mitigate any potential time-varying confounding factors common to all products, we also control for year-specific fixed effects in columns 2, 4 and 6. Columns 1-2 present the ordinary least squares (OLS) estimation result. The OLS estimator yields an estimated κ of almost zero.

However, the OLS estimator may be subject to omitted variable bias, so we use $\Delta \ln \tilde{\lambda}_{mt}$ as the instrument for $\Delta \ln \lambda_{mt}$ to obtain a more consistent estimate of κ. Columns 3-6 present the IV regression results using the "ADH set" and the "EU set" to construct our instrument. The Kleibergen–Paap (K-P) LM statistics are all statistically significant at the 1% level and the K-P F statistics are all larger than 10, so under-identification or weak instrument does not seem a major concern. The IV estimator yields an estimate of κ ranging from -0.096 to -0.161. Therefore, an increase in domestic expenditure share by 1% drives down the number of domestic firms by about $0.10 - 0.16\%$.

Table 4: IV Regression: Baseline

	Dependent Variable: Log change in number of firms					
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	OLS	IV	IV	IV	IV
Log change in domestic	0.004	0.007	-0.115*	-0.161***	-0.096*	-0.153**
expenditure share	(0.005)	(0.005)	(0.069)	(0.074)	(0.054)	(0.061)
Year FE	No	Yes	No	Yes	No	Yes
Country set for IV	.	.	ADH	ADH	EU	EU
Kleibergen-Paap LM stat.	.	.	12.819***	13.506***	13.785***	14.818***
Kleibergen-Paap F stat.	.	.	14.650	15.501	15.658	16.869
# Obs.	13,845	13,845	13,845	13,845	13,870	13,870

Estimation results of (27). Standard errors clustered at the six-digit level of the CN classification are shown in parentheses. *, **, and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels.

Our estimated elasticities of the number of firms with respect to the domestic expenditure share range between 0.003 (OLS specification) to -0.161 (one of the IV specifications). Despite the change in sign, these elasticities give a similar value of η between -1, with standard error equal to 0.001, and -0.96, with standard error equal to 0.013. The result of $\hat{\eta} \approx -1$ indicates the presence of indirectly additive preferences, and a sorting pattern in which a more attractive product market hosts a larger number of firms despite tougher competition in that market.[20]

[20] In Appendix C.4, we replicate the results of Table 4 by re-estimating the regression outlined in (27), with the sample divided by sector. Our analysis does not reveal any significant sector-specific differences. This is likely due to the insufficient variation in the sector-level data, which hinders the detection of statistical

5 Quantification

Using the calibrated parameters discussed in the previous section, this section presents the results of two quantification exercises. First, we quantify the importance of heterogeneous product attractiveness for firm-product-level sales. Our model predicts that firm-sales within a certain product category are determined by two crucial factors: the product's attractiveness (which determines the endogenous level of competition) and the firm-product specific demand shock that accounts for any other factors not related to product attractiveness or competition. Second, we infer the level of product attractiveness for each eight-digit CN good in our sample and explore its correlations with observable product-level characteristics. This analysis will help us understand how product features affect product attractiveness and can potentially provide insights into strategies that firms can use to improve their sales.

5.1 The Role of the Cost Cutoff in Explaining Sales Variations

To measure the impact of heterogeneity in product attractiveness and competition on firm-level sales, we undertake two exercises. First, we use the revenue predictions from our model and conduct a variance decomposition analysis to determine the proportion of total variance that each component of firm-level sales explains. This model-based decomposition provides insight into the relative importance of product attractiveness and endogenous competition on firm sales.

Second, we compare our model to a standard model used in the literature that does not account for product heterogeneity. By quantifying the differences between the two models, we can assess the importance of incorporating product heterogeneity into the analysis. This exercise will help us understand how much of the variance in firm sales can be explained by product heterogeneity and the potential limitations of using a model that ignores this heterogeneity.

significance. Additionally, our instruments demonstrate greater efficacy at an aggregate level rather than at the sector level.

5.1.1 Decomposing Firm-Level Sales

We can rewrite the revenue equation for a firm with unit cost c producing product m and selling it from j to j (as shown in equation (13) in the model section) as follows:

$$\ln(r(c))_m = FE_m + \ln\left[(c_m^* - c)^\gamma (c_m^* + \gamma c)\right] + \ln \delta_m(c), \tag{30}$$

where FE_m is a product fixed effect, and we drop origin and destination subscript since we only focus on domestic sales of Danish firms. By using data generated by the model, as we describe below, such a regression generates an R^2 of one. To assess the significance of the second component in (30), which represents the heterogeneity in cutoffs across products, we estimate the following regression:

$$\ln(r(c))_m = FE_m + \ln \delta_m(c). \tag{31}$$

We can then compare the R^2 of this regression to the R^2 of equation (30). The difference between the two R^2 values indicates the explanatory power of the term $\ln\left[(c_m^* - c)^\gamma (c_m^* + \gamma c)\right]$.

Apart from assessing the importance of heterogeneity in product attractiveness and competition, we are also interested in quantifying the significance of the demand shock term $\delta_m(c)$. In our model, the demand shock term captures all factors that may cause a firm to have different sales across products, aside from the product-specific attractiveness and competition (such as differences in productivity). To achieve this, we estimate the following regression:

$$\ln(r(c))_m = FE_m. \tag{32}$$

We can calculate the quantitative significance of the demand shock by comparing the R^2 value of equation (31) to that of equation (32).

Simulation Algorithm. To conduct the counterfactual analysis, we need to simulate the sales of firms. Here is how we proceed. To ensure that we have the maximum number of observations available for our algorithm, we simulate the firms' sales conditional on them being active in the most popular product R, i.e., the product with the largest number of active firms N_R. We can normalize the cutoff c_R^* for this product to one, without loss of generality.

Next, we derive the cost cutoffs for all other products as follows:

$$c_m^* = \left(\frac{N_m}{N_R}\right)^{\frac{1}{\theta}},$$

where we use the result that $N_m = J(c_m^*/b)^{\theta}$.

For each Danish CN two-digit industry, we consider the 20 most successful eight-digit CN codes based on the number of firms producing them, denoted by $m = 1, ..., 20$.[21] We then compute $\frac{N_m}{N_R}$ for each product in each industry and take the average across industries. Using these average relative number of firms, we can determine the cost cutoffs c_m^* for each product.

We simulate the draws of marginal cost c_f for $10,000$ hypothetical firms. Subscript f denotes a firm level variable in our simulation. We first draw realizations of the uniform distribution u_f. Then, we compute the marginal cost draws as:

$$c_f = u_f^{\frac{1}{\theta}}.$$

If $c_f < c_m^*$, we compute the revenues of the firm f in product m as:

$$\tilde{r}_{fm} = (c_m^* - c_f)^{\gamma}(c_m^* + \gamma c_f)\delta_{fm},$$

where δ_{fm} is drawn from a log-normal distribution for each firm-product combination fm with mean one and standard deviation σ. To simplify our analysis, we can drop the multiplicative component of revenues from our model, as it will be captured by the product fixed effect. This assumption does reduce the explanatory power of the product-specific fixed effects, but that is not the main focus of our counterfactual analysis

Results. After simulating the revenues for our hypothetical firms, we can estimate the regression equations (30), (31), and (32). Results are in Table 5. The difference in R^2 between equation (30) and equation (31) is 26%. This indicates that the presence of product-specific cutoffs that depend on the product-specific attractiveness and competition can explain 26% of the variance in sales across firm-product pairs. The remaining variance is largely due to the demand shocks, which account for 72% of the variance (i.e., in R^2

[21] We consider only eight-digit CN products with at least 5 firms.

between (31) and (32)). We can achieve similar results using the parameter estimated under a lower trade elasticity assumption. Moreover, using the higher US sales advantage moment yields the largest explanatory power for the heterogeneity in cutoffs of 82%.

We also replicate our analysis using the demeaned revenues per firm as the dependent variable (see Table D.1 in the appendix). Specifically, we first regress the logarithm of firm revenues on firm fixed effects and then use the residual of this regression as the dependent variable in equations (30), (31), and (32). We find that the results are similar to those obtained using the original revenues as the dependent variable. The explanatory power of the product-level cutoffs ranges from 20-50%, while that of the demand shock ranges from 20-70%.

Table 5: Variance of Sales Across Firms

	Product-Level Cutoffs	Demand Shock
Baseline Estimates	26	72
Low Trade Elasticity	28	64
Higher Sales Advantage	82	16

The first column reports the difference in R^2 between (30) and (31). The second column reports the difference in R^2 between (31) and (32). Different rows use different parameters for the simulation of firm sales: the Baseline Estimates are reported in Table 3, the Low Trade Elasticity estimates are reported in Table C.3, and the Higher Sales Advantage estimates are reported in Table C.4. All values are multiplied by 100.

5.1.2 Model Comparison

Next, we consider a variance decomposition of the following regression:

$$\ln(r)_{cm} = FE_m + FE_c + \epsilon_{cm}, \tag{33}$$

where FE_m is a product fixed effect, FE_c is a firm fixed effect, and ϵ_{cm} is the residual. In order to decompose the variance of log revenues across different components, we follow the approach used by Hottman et al. (2016) and Bernard et al. (2022). Specifically, we regress the estimated fixed effects on the logarithm of revenues without a constant, and use the resulting coefficient as a measure of the percentage of the variance in log revenues explained by the fixed effects.[22]

We consider two values for the logarithm of revenues. First, we use the logarithm

[22] To make full use of the Stata command *reghdfe*, we first regress $\ln(r)_{cm}$ on a constant, and then apply the decomposition outlined. This initial step is required to fully account for the variance with the fixed effects outlined: without this initial step, part of the variance would be captured by the constant of the regression.

of revenues generated by our baseline model. In this case, the variance in the residual captures both the demand shock and the interaction between the firm fixed effect and product fixed effect due to cutoff heterogeneity across products. Second, we consider the logarithm of revenues generated by a model in which the cutoffs across products are identical (and equal to one). In this case, the variance in the residual is due to the demand shock only. We use the same simulation algorithm used in the previous section. Results are in Table 6.

Table 6: Variance Decomposition - Baseline Parameters

	(Product FE)	(Firm FE)	(Residual)
Baseline Model	0.076***	0.201***	0.723***
	(0.001)	(0.002)	(0.002)
Observations	74,898	74,898	74,898

	(Product FE)	(Firm FE)	(Residual)
No Cutoff Heterogeneity	0.000***	0.305***	0.695***
	(0.000)	(0.001)	(0.001)
Observations	200,000	200,000	200,000

Results from the variance decomposition of log revenues on product fixed effect, firm fixed effect, and residual. Log revenues are simulated using our baseline model in the first row and using a model in which all product cutoffs are identical in the second part of the table. The revenues are simulated using the baseline parameters are reported in Table 3.

Our results show that relative to the model with no cutoff heterogeneity, our baseline model features a smaller explanatory power of the firm fixed effect and a larger explanatory power of the residual and the product fixed effect. This suggests that when we examine the determinants of firm-product revenues and ignore the product-specific levels of attractiveness and competition that affect firm's choices, we tend to overestimate the explanatory power of firms and underestimate the explanatory power of the residual and of product characteristics.

We find similar results when we use alternative parameters to simulate revenues. Specifically, we use a lower trade elasticity and a higher sales advantage moment. In both cases, our results are consistent with those obtained using the baseline parameter values. However, we find that the differences in explanatory power between the models with and without cutoff heterogeneity are magnified when we use a higher value for the sales advantage moment (see Tables D.2 and D.3 in the appendix).

5.2 Product Attractiveness

In this section, we calculate the level of attractiveness for each specific product, denoted as a_{mjt}, where we add a subscript t for year relative to the expression in the model to allow for variations over time. In this analysis, we will examine the product attractiveness of eight-digit CN codes in Denmark. By using equation (18), we can express the relative product attractiveness of product m compared to k as follows:

$$\frac{a_{mjt}}{a_{kjt}} = \left(\frac{\lambda_{mjjt}}{\lambda_{kjjt}}\right)^{-\frac{1+\eta}{1+\gamma(\eta+1)}} \left(\frac{c^*_{mjjt}}{c^*_{kjjt}}\right)^{-\frac{\theta+\eta+\theta\eta}{1+\gamma(\eta+1)}}. \tag{34}$$

We define product k for each CN two-digit sector and year as the product with the largest number of firms producing a variety of that particular product. Using this definition, we can calculate the relative cutoffs as $\frac{c^*_{mjjt}}{c^*_{kjjt}} = \left(\frac{N_{mjt}}{N_{kjt}}\right)^{\frac{1}{\theta}}$. To compute the domestic expenditure shares λ_{mjjt}, we follow the same procedure outlined in Section 4.2, but this time we apply it to eight-digit CN products. With the estimated values for the parameters η, γ, and θ, we can then calculate the relative product attractiveness. Our baseline values are $\gamma = 0.59$ and $\theta = 4$. We test two different values for η: the estimated value of -1 using OLS, and the value of -0.97 estimated using the IV strategy. As there is not a significant difference between the two estimates, it is not surprising that the results do not vary significantly.

Next, we regress the log of $\frac{a_{mj}}{a_{kj}}$ on product-year specific variables, including sector-year fixed effects, where each sector is defined by a two-digit CN code.

In Table 7, we consider the relationship between product attractiveness and the following variables: the log of total production and total domestic sales, the log of the number of firms, the log of exports and imports, and the export participation rate. Export participation rate is defined as the ratio of the number of firms exporting product m relative to the total number of firms producing product m. The most attractive products are typically characterized by the largest total production, total domestic sales, and number of firms. Export levels also tend to be higher in the most attractive products, although this effect becomes less significant when we control for total production. Moreover, we find that export participation rates tend to be lower in more attractive products. This result is likely driven by selection bias, as the larger number of firms making the most attractive products means that a smaller proportion of them are engaged in exporting. Finally, we observe that more attractive products also tend to have higher levels of imports.[23]

Table 7: Product Attractiveness and Product Characteristics

	Dependent Variable: Product Attractiveness							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Total Production	0.054***					0.066***	0.045***	
	(0.000)					(0.001)	(0.001)	
Log Domestic Sales		0.052***						
		(0.000)						
Log # Firms			0.254***					
			(0.000)					
Log Exports				0.030***		-0.006***		
				(0.000)		(0.001)		
Log Imports					0.047***		0.020***	
					(0.001)		(0.001)	
Export Participation								-0.058***
								(0.003)
CN2-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.71	0.71	1.00	0.61	0.65	0.69	0.72	0.57
# Obs.	29763	29763	29763	23733	29460	23733	29460	29763

Results from OLS of the estimated log of $\frac{a_{mj}}{a_{kj}}$ on the product-time characteristics described in the rows. $\frac{a_{mj}}{a_{kj}}$ is estimated using the OLS estimate of η. Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

Table 8 explores additional product-level features and their relationship with product attractiveness.[24] Specifically, we examine two measures of product differentiation: the average demand elasticity, sourced from Broda and Weinstein (2006), and a dummy variable for product differentiation, based on Rauch (1999). Our analysis reveals that products with higher attractiveness are typically more differentiated. This is evidenced by a negative and statistically significant coefficient for average demand elasticity—suggesting that more attractive products are less price-sensitive—and a positive and statistically significant coefficient for the product differentiation indicator.

Next, we analyze the relationship between product attractiveness and product quality through two distinct measures. First, we use the quality ladder concept introduced by Khandelwal (2010), which gauges the potential for quality differentiation within products. Our findings indicate that higher product attractiveness is correlated with a lesser degree of quality differentiation, suggesting a more uniform quality level among varieties

[23] While Table 7 presents results using the OLS estimate of the parameter η, which is used to calculate product attractiveness, Table D.4 in the appendix provides findings using the IV estimate of η, yielding similar outcomes.

[24] Table 8 presents results using the OLS estimate of η and Table D.5 in the appendix provides findings using the IV estimate of η.

of highly attractive products. Second, we explore how attractiveness relates to the average unit value, a proxy for quality. In our model, differences in average prices across products are only driven by selection effects. In particular, given our estimates of η, the weaker selection in highly attractive products lead to a larger presence of low-productivity firms and, hence, higher prices. Our empirical results contradict this, as there is a negative correlation between average price and product attractiveness. This result indicate the possible presence of quality differentiation across firms, with low-productivity firms offering lower quality goods that consequently involve lower marginal costs.[25]

Table 8: Product Attractiveness and Product Characteristics

	Dependent Variable: Product Attractiveness				
	(1)	(2)	(3)	(4)	(5)
Avg. Demand Elasticity	-0.001**				
	(0.000)				
Differentiated Product		0.064***			
		(0.007)			
Quality Ladder			-0.012***		
			(0.002)		
Average Log Unit Price				-0.001*	
				(0.001)	
Average Capital Intensity					0.022***
					(0.001)
CN2-year FE	Yes	Yes	Yes	Yes	Yes
R^2	0.58	0.57	0.60	0.56	0.57
# Obs.	23559	29734	17074	24618	29734

Results from OLS of the estimated log of $\frac{a_{mj}}{a_{kj}}$ on the product-time characteristics described in the rows. $\frac{a_{mj}}{a_{kj}}$ is estimated using the OLS estimate of η. Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

Finally, we investigate the relationship between attractiveness and capital intensity. Due to the challenges of attributing specific capital and labor inputs to individual eight-digit CN products in multi-product firms, we calculate the average capital intensity for each product based on the average across all firms producing that product. Our findings indicate a positive association between product attractiveness and average capital intensity. This suggests that more attractive products typically require higher levels of fixed investments, reflecting greater capital deployment in their production processes.

[25] Notice that our predictions on markups are analogous to a model with quality differentiation. See for instance, Macedoni and Weinberger (2022).

6 Conclusions

We presented a novel model of multi-product firms in international trade that takes into account product-specific characteristics and their impact on firms' product mix decisions. Using Danish manufacturing data, we provide empirical evidence supporting the importance of product-specific factors in understanding firm behavior. Our model diverges from standard multi-product firm models by incorporating a discrete set of products, with firms offering unique varieties within each product category. We use a general demand function that allows for different sorting patterns of firms into product categories based on their attractiveness and level of competition. Our quantitative analysis demonstrates the significance of accounting for product-level attributes and the sorting patterns that emerge as a result. We find that incorporating product-specific levels of attractiveness and competition explains a quarter of the sales variance across firms.

The results of our estimation indicate that the increase in competition associated with higher product attractiveness does not sufficiently drive out the least productive firms from the markets of the most attractive products. Hence, the most attractive products tend to be produced by all firms, while the least attractive products are made only by the most productive firms. Our measure of product attractiveness encapsulates both demand and supply features: on the demand side, attractive products tend to be differentiated, with limited scope for quality differentiation. On the supply side, attractiveness correlates with increased capital intensity, indicating higher investment requirements. Furthermore, product attractiveness is linked to the internationalization of firms, suggesting that more attractive products are more likely to be involved in global trade.

Our model can be used for alternative applications in which the characteristics of a certain market generate endogenous levels of competition that affect the sorting of firms into these markets. For instance, the model can be applied in the context of economic geography in which firms sort in different regions and the most attractive regions feature the toughest competition. Our model also provides a foundation for analyzing the effects of trade policy changes on firm product-mix, particularly in the context of product-specific tariffs and regulations.

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A Data

A.1 Summary Statistics

Table A.1: Scope Distribution (2000)

Average Scope	2.96
Std. Dev. Scope	4.92
25 Perc.	1
50 Perc.	2
75 Perc.	3
90 Perc.	5
95 Perc.	8

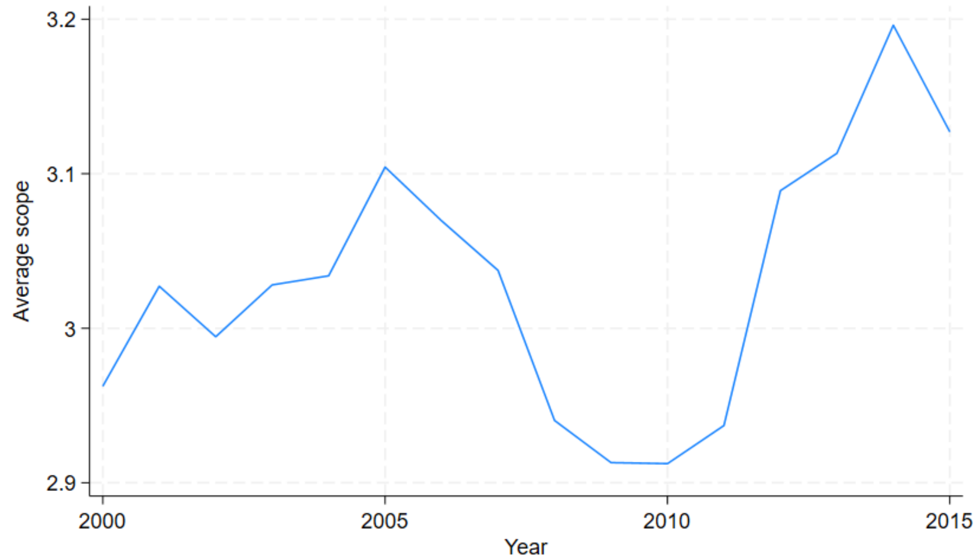
The table reports the average and standard deviation of scope across firms in 2000. We also provide information on the scope at various percentiles of the distribution. For confidentiality reasons, we divide firm-level scope observations in 100 bins, and report the average scope per bin.

Table A.2: Average Scope by Sector

Sector	Average Scope
Animal Products	5.21
Chemicals	2.77
Foodstuffs	4.13
Machinery and Electrical	1.97
Metals	1.61
Miscellaneous	1.82
Plastics and Rubbers	1.66
Stone and Glass	1.98
Textiles	6.51
Transportation	1.35
Wood Products	2.10

The table reports the average scope across firms in 2000 by sector.

Figure A.1: Average Scope over Time



A.2 Robustness Table for Stylized Facts

Table A.3: Within Firm Ranking - Year=2000

	Dependent Variable: Within firm rank of product sales			
	(1)	(2)	(3)	(4)
Log Number of Firms	-7.105***	16.059***	7.258***	6.794***
	(0.742)	(1.061)	(1.084)	(1.097)
Log Domestic Sales		-13.537***	-16.649***	-15.859***
		(0.466)	(0.466)	(0.551)
Product Market Share			-0.688***	-0.690***
			(0.029)	(0.029)
Log Import Competition				1.203***
				(0.446)
Firm FE	Yes	Yes	Yes	Yes
R^2	0.73	0.76	0.78	0.78
# Obs.	8696	8696	8696	8696

Results from OLS of (1). Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%. We restrict the sample to the year 2000.

B Model

In this appendix, we characterize the open economy equilibrium with multiple countries and multiple products and investigate several important comparative statics of the model.

B.1 The Open Economy Equilibrium

We begin with solving for the average revenues conditional on firms being active in product-origin-destination mij:

$$
\begin{aligned}
\bar{r}_{mij} &= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right)\frac{L_j w_i \tau_{mij} a_{mj}^{\gamma}}{\xi_{mj}(c^*_{mij})^{\gamma}}\int_0^{c^*_{mij}}(c^*_{mij}-c)^{\gamma}(c^*_{mij}+\gamma c)\frac{\theta c^{\theta-1}}{(c^*_{mij})^{\theta}}dc = \\
&= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right)\frac{L_j w_i \tau_{mij} a_{mj}^{\gamma} c^*_{mij}}{\xi_{mj}}\int_0^{c^*_{mij}}(1-c/c^*_{mij})^{\gamma}(1+\gamma c/c^*_{mij})\frac{\theta c^{\theta-1}}{(c^*_{mij})^{\theta}}dc = \\
&= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right)\frac{L_j w_i \tau_{mij} a_{mj}^{\gamma} c^*_{mij}}{\xi_{mj}}\int_0^{1}(1-t)^{\gamma}(1+\gamma t)\theta t^{\theta-1}dt = \\
&= \left(\frac{\tilde{T}_1\theta\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right)\frac{L_j w_i \tau_{mij} a_{mj}^{\gamma} c^*_{mij}}{\xi_{mj}} = \\
&= \left(\frac{\tilde{T}_1\theta\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right)L_j a_{mj}^{\gamma+\frac{1}{1+\eta}}\left(w_i\tau_{mij}c^*_{mij}\right)^{1-\frac{1}{1+\eta}}\left(\alpha y_j\right)^{\frac{1}{1+\eta}},
\end{aligned}
$$

where we substituted $\xi_{mj} = \left(\frac{\tau_{mij}w_i c^*_{mij}}{a_{mj}\alpha y_j}\right)^{\frac{1}{1+\eta}}$, used the change of variable technique ($t = c/c^*_{mij}$), and where

$$
\tilde{T}_1 = \int_0^1 (1-t)^{\gamma}(1+\gamma t)t^{\theta-1} = \frac{\Gamma(\theta+2)\Gamma(\gamma+2)}{\Gamma(\theta+\gamma+2)}.
$$

Aggregate revenues in product-origin-destination mij are then given by:

$$
\begin{aligned}
R_{mij} = J_i\left(\frac{c^*_{mij}}{b_i}\right)^{\theta}\bar{r}_{mij} &= \left(\frac{\tilde{T}_1\theta\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right)\frac{J_i L_j w_i \tau_{mij} a_{mj}^{\gamma}(c^*_{mij})^{\theta+1}}{\xi_{mj} b_i^{\theta}} = \\
&= \left(\frac{\tilde{T}_1\theta\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right)L_j a_{mj}^{\gamma+\frac{1}{1+\eta}} J_i(b_i w_i \tau_{mij})^{-\theta}(w_j c^*_{mjj})^{\theta+1-\frac{1}{1+\eta}}\left(\alpha y_j\right)^{\frac{1}{1+\eta}}.
\end{aligned}
\tag{B.1}
$$

We can define a the expenditure share of country i in country j for product m as follows:

$$\lambda_{mij} = \frac{R_{mij}}{\sum_{v=1}^{I} R_{mvj}} = \frac{J_i(b_i \tau_{mij} w_i)^{-\theta}}{\sum_{v=1}^{I} J_v(b_v \tau_{mvj} w_v)^{-\theta}}. \tag{B.2}$$

Note that the property $c^*_{mij} = c^*_{mjj} \frac{\tau_{mjj} w_j}{\tau_{mij} w_i}$ simplifies the expression.

Let us consider the aggregate revenues generated in product-destination mj, R_{mj}. Using (10) and (8), we obtain:

$$\begin{aligned}
R_{mj} = \sum_{i=1}^{I} R_{mij} &= \left(\frac{\tilde{T}_1 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}} \right) \frac{L_j a_{mj}^\gamma}{\xi_{mj}} \sum_{i=1}^{I} J_i b_i^{-\theta} w_i \tau_{mij} (c^*_{mij})^{\theta+1} = \\
&= \left(\frac{\tilde{T}_1 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}} \right) \frac{L_j a_{mj}^\gamma (c^*_{mjj} w_j \tau_{mjj})^{\theta+1}}{\xi_{mj}} \sum_{i=1}^{I} J_i (b_i w_i \tau_{mij})^{-\theta} = \\
&= \left(\frac{\tilde{T}_1 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}} \right) \frac{L_j a_{mj}^\gamma (a_{mj} \alpha y_j \xi_{mj}^{1+\eta})^{\theta+1}}{\xi_{mj}} \sum_{i=1}^{I} J_i (b_i w_i \tau_{mij})^{-\theta} = \\
&= \left(\frac{\tilde{T}_1 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}} \right) L_j a_{mj}^{\theta+\gamma+1} (\alpha y_j)^{\theta+1} \xi_{mj}^{(1+\eta)(\theta+1)-1} \Phi_{mj} = \\
&= \left(\frac{\tilde{T}_1 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}} \right) L_j a_{mj}^{\gamma+\frac{1}{1+\eta}} (w_j c^*_{mjj})^{\theta+1-\frac{1}{1+\eta}} (\alpha y_j)^{\frac{1}{1+\eta}} \Phi_{mj}.
\end{aligned}$$

To streamline the notation, we denote $\Phi_{mj} = \sum_{i=1}^{I} J_i (b_i w_i \tau_{mij})^{-\theta}$ as the "multilateral resistance term". A higher level of Φ_{mj} means that country j is less remote and more accessible to the suppliers of product m around the world, and thus should generate a higher overall demand for product m.

By market clearing $R_{mj} = L_j \alpha y_j$. Hence,

$$c^*_{mjj} = \left[\left(\frac{\tilde{T}_1 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}} \right) a_{mj}^{\gamma+\frac{1}{1+\eta}} \Phi_{mj} (\alpha y_j)^{-\frac{\eta}{1+\eta}} \right]^{\frac{1}{\frac{1}{1+\eta}-(1+\theta)}} (w_j)^{-1}. \tag{B.3}$$

Since $c^*_{mij} = c^*_{mjj} \frac{\tau_{mjj} w_j}{\tau_{mij} w_i}$, we obtain:

$$c^*_{mij} = \left[\left(\frac{\tilde{T}_1 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}} \right) a_{mj}^{\gamma+\frac{1}{1+\eta}} \Phi_{mj} (\alpha y_j)^{-\frac{\eta}{1+\eta}} \right]^{\frac{1}{\frac{1}{1+\eta}-(1+\theta)}} (\tau_{mij} w_i)^{-1}. \tag{B.4}$$

Therefore, the ratio of cutoffs from i to j across different products, not only depends on the demand shifters, but also on the product specific trade costs and on the product specific trade shares:

$$\frac{c^*_{mij}}{c^*_{kij}} = \left(\frac{a_{mj}}{a_{kj}}\right)^{\frac{\gamma+\frac{1}{1+\eta}}{\frac{1}{1+\eta}-(1+\theta)}} \left(\frac{\Phi_{mj}}{\Phi_{kj}}\right)^{\frac{1}{\frac{1}{1+\eta}-(1+\theta)}} \left(\frac{\tau_{mij}}{\tau_{kij}}\right)^{-1} =$$

$$= \left(\frac{a_{mj}}{a_{kj}}\right)^{\frac{\gamma+\frac{1}{1+\eta}}{\frac{1}{1+\eta}-(1+\theta)}} \left(\frac{\lambda_{mjj}}{\lambda_{kjj}}\right)^{-\frac{1}{\frac{1}{1+\eta}-(1+\theta)}} \left(\frac{\tau_{mij}}{\tau_{kij}}\right)^{-1},$$

where $\lambda_{mjj} = J_j(b_j w_j)^{-\theta}/\Phi_{mj}$.

Let us now consider the free entry condition. First, we need to derive the average profits of active firms from i to j in product m conditional on $c \leq c^*_{mij}$:

$$\begin{aligned}
\bar{\pi}_{mij} &= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a^\gamma_{mj}}{\xi_{mj}(c^*_{mij})^\gamma} \int_0^{c^*_{mij}} (c^*_{mij} - c)^{1+\gamma} \frac{\theta c^{\theta-1}}{(c^*_{mij})^\theta} dc = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a^\gamma_{mj} c^*_{mij}}{\xi_{mj}} \int_0^1 (1-t)^{1+\gamma} \theta t^{\theta-1} dt = \\
&= \left(\frac{\tilde{T}_2 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a^\gamma_{mj} c^*_{mij}}{\xi_{mj}} = \\
&= \left(\frac{\tilde{T}_2 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) L_j (w_i \tau_{mij})^{1-\frac{1}{1+\eta}} a_{mj}^{\gamma+\frac{1}{1+\eta}} {c^*_{mij}}^{1-\frac{1}{1+\eta}} (\alpha y_j)^{\frac{1}{1+\eta}},
\end{aligned}$$

where we used the same change of variable technique we used before and where

$$\tilde{T}_2 = \int_0^1 (1-t)^{1+\gamma} t^{\theta-1} dt = \frac{\Gamma(\theta+1)\Gamma(\gamma+2)}{\Gamma(\theta+\gamma+2)}.$$

The expected profits of producing for product-origin-destination mij, taking into account the probability of $c \leq c^*_{mij}$, are given by:

$$\begin{aligned}
E[\pi_{mij}] &= \left(\frac{c^*_{mij}}{b_i}\right)^\theta \bar{\pi}_{mij} = \left(\frac{\tilde{T}_2 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{mij} a^\gamma_{mj} (c^*_{mij})^{\theta+1}}{b_i^\theta \xi_{mj}} = \\
&= \left(\frac{\tilde{T}_2 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) L_j (w_i \tau_{mij})^{1-\frac{1}{1+\eta}} a_{mj}^{\gamma+\frac{1}{1+\eta}} {c^*_{mij}}^{(\theta+1)-\frac{1}{1+\eta}} (\alpha y_j)^{\frac{1}{1+\eta}} b_i^{-\theta} = \\
&= \left(\frac{\tilde{T}_2 \theta \gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) L_j a_{mj}^{\gamma+\frac{1}{1+\eta}} (b_i w_i \tau_{mij})^{-\theta} (w_j c^*_{mjj})^{(\theta+1)-\frac{1}{1+\eta}} (\alpha y_j)^{\frac{1}{1+\eta}}
\end{aligned}$$

By using the definition of aggregate revenues in a product-origin-destination (B.1), we obtain:

$$E[\pi_{mij}] = \frac{R_{mij}\tilde{T}_2}{J_i\tilde{T}_1} = \frac{R_{mij}}{J_i(\theta+1)}.$$

We assume that an entrant pays its entry cost using the labor in its origin country i. We find the mass of entrants by using the free entry condition, namely, the expected profit of an entrant equals the entry cost:

$$\sum_{j=1}^{I}\sum_{m=1}^{M} E[\pi_{mij}] = f_E w_i \Rightarrow \frac{\sum_{j=1}^{I}\sum_{m=1}^{M} R_{mij}}{J_i(\theta+1)} = f_E w_i \Rightarrow \frac{\sum_{j=1}^{I}\sum_{m=1}^{M} R_{mji}}{J_i(\theta+1)} = f_E w_i.$$

Using the trade balance condition $\sum_{m=1}^{M}\sum_{j=1}^{I} R_{mij} = \sum_{m=1}^{M}\sum_{j=1}^{I} R_{mji}$, the market clearing condition $\sum_{m=1}^{M}\sum_{j=1}^{I} R_{mji} = L_i y_i$ and $y_i = w_i$, we solve for the mass of entrants hosted by country i:

$$J_i = \frac{L_i}{f_E(\theta+1)}. \tag{B.5}$$

The multilateral resistance of country j in product m is thus:

$$\Phi_{mj} = \frac{\sum_{i=1}^{I} L_i(b_i w_i \tau_{mij})^{-\theta}}{f_E(\theta+1)}. \tag{B.6}$$

This implies that the equilibrium cutoff is:

$$\begin{aligned} c^*_{mjj} &= \left[\left(\frac{\tilde{T}_1\theta\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right)\frac{a_{mj}^{\gamma+\frac{1}{1+\eta}}\Phi_{mj}}{(\alpha w_j)^{\frac{\eta}{1+\eta}}}\right]^{\frac{1}{\frac{1}{1+\eta}-(1+\theta)}}(w_j)^{-1} = \\ &= \left[\left(\frac{\tilde{T}_1\theta\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}(\theta+1)}\right)\frac{L_j a_{mj}^{\gamma+\frac{1}{1+\eta}}}{\lambda_{mjj} f_E(b_j w_j)^{\theta}(\alpha w_j)^{\frac{\eta}{1+\eta}}}\right]^{\frac{1}{\frac{1}{1+\eta}-(1+\theta)}}(w_j)^{-1}. \end{aligned} \tag{B.7}$$

Therefore, any effects of the changes in international trade costs τ_{mij} and foreign supply-side conditions, e.g., b_i, w_i and L_i, on the domestic cost cutoff c^*_{mjj} are summarized by the change in domestic trade share and the elasticity $-\frac{1}{\frac{1}{1+\eta}-(1+\theta)}$:

$$\hat{c}^*_{mjj} = \hat{\lambda}_{mjj}^{-\frac{1}{\frac{1}{1+\eta}-(1+\theta)}}. \tag{B.8}$$

B.2 Comparative Statics

The comparative statics focus on the effects of product appeal a_{mj} and multilateral resistance Φ_{mj} on the domestic cutoff c^*_{mjj}. Increases in a_{mj} and Φ_{mj} raise the overall demand for product m in country j. In the equilibrium, such increases must be balanced by a change in the competitive environment in mj that keeps the expected profit unchanged.

The responses of the competitive environment may exhibit two opposite forces. First, an increase in the overall demand tends to intensify competition and depress markups, leading to a selection effect in the product-destination mj that drives down the cost cutoff c^*_{mjj}. This change decreases prices (markups) and the number of active domestic firms, but increases average firm size measured by quantity. The size of this effect is increasing in $1+\theta$. Second, an increase in the overall demand may instead induce an anti-selection by increasing markups and raising the the cost cutoff c^*_{mjj}. In this case, such a change increases prices (markups) and number of active domestic firms, but decreases average firm size. This effect is increasing in $\frac{1}{1+\eta}$.

The net effects of an increases in a_{mj} or Φ_{mj} thus depend on the relative magnitude of $1/(1+\eta)$ and $1+\theta$ as follows:

$$\frac{\partial \ln c^*_{mjj}}{\partial \ln \Phi_{mj}} = \frac{1}{\frac{1}{1+\eta} - (1+\theta)}, \tag{B.9}$$

$$\frac{\partial \ln c^*_{mjj}}{\partial \ln a_{mj}} = \frac{\gamma + \frac{1}{1+\eta}}{\frac{1}{1+\eta} - (1+\theta)}. \tag{B.10}$$

If $-1 \geq \eta < -\frac{\theta}{1+\theta}$, the anti-selection dominates and results in a less selective product market, so that less productive firms can survive. Otherwise, the selection effect dominates and results in a more competitive market environment, so that only the most productive firms can survive.

B.3 Planner's Allocation

In this section, we explore the optimal sorting pattern across products by solving the social planner's problem. It is well established that the variable markups in our model lead to market inefficiencies. Dhingra and Morrow [2019] demonstrate that in a setting characterized by markup heterogeneity across firms and a broad range of preferences, firms with higher markups tend to under-produce, whereas those with lower markups over-produce. This misalignment stems from the efficient allocation principle, where relative prices should mirror relative marginal costs. However, in a monopolistically competitive market, such alignment is only achieved when markups are constant across firms. Despite the optimal entry of firms, dictated by the Pareto distribution of marginal costs, variable markups may result in either excessively weak or overly stringent selection [Dhingra and Morrow, 2019].

Bertoletti and Etro [2020] explore the optimality of market allocation within a single-sector model employing the GTP preferences utilized in our analysis, and Macedoni and Weinberger [2022] extend this model to address quality heterogeneity. These studies have not examined scenarios where firms sort into different products. Therefore, our goal is to determine whether there is an excessive or insufficient response to competition among products of varying attractiveness. Specifically, we investigate how the relative cost cutoff between products is influenced by their relative attractiveness and how this compares to the market allocation. However, we do not delve into whether the cutoff level is too high or too low, as this aspect has already been thoroughly examined in Bertoletti and Etro [2020] and Macedoni and Weinberger [2022].

To solve the planner's problem, we focus on a closed economy, thereby omitting country-specific subscripts. The planner's objective is to maximize the utility derived from the consumption of each product m, constrained by the resources of the economy. In our baseline model, consumers allocate a constant share of their income to each product m, with the utility from consuming the varieties within product m defined by equation (4). For the planner's problem, we adopt a similar approach. The planner aims to maximize utility U_m for each product market while also allocating a constant proportion α_p of the labor force to the production of the mass of varieties of product m.

The planner's maximizes the following utility function:

$$U_m = \int_{\Omega_m} \delta_m(\omega) \left(a_m \xi_m q_m(\omega) - \frac{(\xi_m q_m(\omega))^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) d\omega + \frac{\xi_m^{-\eta} - 1}{\eta}, \tag{B.11}$$

where

$$\xi_m^{-\eta} = \int_{\Omega_m} \delta_m(\omega) \left(a_m \xi_m q_m(\omega) - (\xi_m q_m(\omega))^{1+\frac{1}{\gamma}} \right) d\omega. \tag{B.12}$$

For market m, the planner faces the following resource constraint:

$$\int_{\Omega_m} Lc(\omega) q_m(\omega) d\omega \leq \alpha_p L. \tag{B.13}$$

As our primary concern is the relative cutoffs across products, we can set aside the decision regarding the optimal mass of entrants, J. Notice that due to the Pareto distribution of marginal costs, the mass of entrants is efficient in the market allocation [Macedoni and Weinberger, 2024]. To align our approach with the baseline model, the planner's weights, α_p, for each product will remain exogenous.[26]

Demand shocks occur upon consumption after the allocation has been set. Hence, in the remainder we assume that the demand shocks take their expected value of one. The first order condition for quantity in the planner's problem is expressed as:

$$a_m \xi_m - \xi_m^{1+\frac{1}{\gamma}} q_m(\omega)^{\frac{1}{\gamma}} = \lambda_m Lc(\omega), \tag{B.14}$$

where λ_m is the Langrangian multiplier of the planner's problem. Multiplying both sides by $q_m(\omega)$ and integrating in $d\omega$ yields:

$$\int_{\Omega_m} \left(a_m \xi_m q_m(\omega) - \xi_m^{1+\frac{1}{\gamma}} q_m(\omega)^{1+\frac{1}{\gamma}} \right) d\omega = \lambda_m \int_{\Omega_m} Lc(\omega) q_m(\omega) d\omega \tag{B.15}$$

$$\xi_m^{-\eta} = \lambda_m \alpha_p L. \tag{B.16}$$

Hence, substituting for λ_m in (B.14), the first order condition becomes:

$$a_m \xi_m - \xi_m^{1+\frac{1}{\gamma}} q_m(\omega)^{\frac{1}{\gamma}} = \xi_m^{-\eta} c(\omega)/\alpha_p. \tag{B.17}$$

Setting quantity to zero in equation (B.17) leads to the cutoff marginal costs chosen by the

[26] Additionally, the planner must adhere to an overarching resource constraint that combines labor allocated to production and labor used for covering fixed entry costs. This constraint is formulated as $Jf_E + M\alpha_p L \leq L$. Here, Jf_E represents the labor dedicated to the fixed cost of entry across all firms, and $M\alpha_p L$ signifies the total labor allocated to the production of output across all products. The total labor used in both these activities must not exceed the total available labor, L.

planner for product m:

$$c_m^* = \alpha_p a_m \xi_m^{1+\eta}. \tag{B.18}$$

The ratio of the cutoffs between any two products, m and k, is given by.

$$\frac{c_m^*}{c_k^*} = \frac{a_m \xi_m^{1+\eta}}{a_k \xi_k^{1+\eta}}. \tag{B.19}$$

Substituting the cutoff defined in equation (B.18) in the first order condition (B.17) yields the following expression for the optimal quantity:

$$q_m(c) = \frac{a_m^\gamma}{\xi_m}\left(1 - \frac{c}{c_m^*}\right)^\gamma. \tag{B.20}$$

To obtain ξ_m, we need to solve two integrals, applying the change of variable technique (i.e., $t = c/c^*m$). In particular,

$$\begin{aligned} J(c_m^*)^\theta \int_0^{c_m^*} a_m \xi_m q_m(c) \theta \frac{c^{\theta-1}}{(c_m^*)^\theta} dc &= J(c_m^*)^\theta \theta a_m^{1+\gamma} \int_0^{c_m^*} \left(1 - \frac{c}{c_m^*}\right)^\gamma \frac{c^{\theta-1}}{(c_m^*)^\theta} dc = \\ = J\theta a_m^{1+\gamma} (c_m^*)^\theta \int_0^1 (1-t)^\gamma t^{\theta-1} dt &= J\theta a_m^{1+\gamma} (c_m^*)^\theta \frac{\Gamma(\theta+1)\Gamma(\gamma+1)}{\Gamma(\theta+\gamma+1)}, \end{aligned} \tag{B.21}$$

$$\begin{aligned} J(c_m^*)^\theta \int_0^{c_m^*} (\xi_m q_m(c))^{1+\frac{1}{\gamma}} \theta \frac{c^{\theta-1}}{(c_m^*)^\theta} dc &= J(c_m^*)^\theta \theta a_m^{1+\gamma} \int_0^{c_m^*} \left(1 - \frac{c}{c_m^*}\right)^{1+\gamma} \frac{c^{\theta-1}}{(c_m^*)^\theta} dc = \\ = J\theta a_m^{1+\gamma} (c_m^*)^\theta \int_0^1 (1-t)^{1+\gamma} t^{\theta-1} dt &= J\theta a_m^{1+\gamma} (c_m^*)^\theta \frac{\Gamma(\theta+1)\Gamma(\gamma+2)}{\Gamma(\theta+\gamma+2)}. \end{aligned} \tag{B.22}$$

Hence, the aggregator ξ_m is given by the difference between (B.21) and (B.22):

$$\xi_m^{-\eta} = J\theta a_m^{1+\gamma} (c_m^*)^\theta \left[\frac{\Gamma(\theta+1)\Gamma(\gamma+1)}{\Gamma(\theta+\gamma+1)} - \frac{\Gamma(\theta+1)\Gamma(\gamma+2)}{\Gamma(\theta+\gamma+2)}\right]. \tag{B.23}$$

The ratio between ξ_m and ξ_k equals:

$$\frac{\xi_m^{-\eta}}{\xi_k^{-\eta}} = \frac{a_m^{1+\gamma}(c_m^*)^\theta}{a_k^{1+\gamma}(c_k^*)^\theta}$$

$$\frac{\xi_m}{\xi_k} = \left(\frac{a_m}{a_k}\right)^{-\frac{1+\gamma}{\eta}} \left(\frac{c_m^*}{c_k^*}\right)^{-\frac{\theta}{\eta}}. \tag{B.24}$$

Substituting (B.24) in (B.19) yields:

$$\begin{aligned}
\frac{c_m^*}{c_k^*} &= \left(\frac{a_m}{a_k}\right)\left(\frac{\xi_m}{\xi_k}\right)^{1+\eta} \\
&= \left(\frac{a_m}{a_k}\right)^{1-\frac{(1+\gamma)(1+\eta)}{\eta}} \left(\frac{c_m^*}{c_k^*}\right)^{-\frac{\theta(1+\eta)}{\eta}} \\
\left(\frac{c_m^*}{c_k^*}\right)^{1+\frac{\theta(1+\eta)}{\eta}} &= \left(\frac{a_m}{a_k}\right)^{1-\frac{(1+\gamma)(1+\eta)}{\eta}} \\
\frac{c_m^*}{c_k^*} &= \left(\frac{a_m}{a_k}\right)^{\frac{1-\frac{(1+\gamma)(1+\eta)}{\eta}}{1+\frac{\theta(1+\eta)}{\eta}}} \\
\frac{c_m^*}{c_k^*} &= \left(\frac{a_m}{a_k}\right)^{\frac{\gamma+\frac{1}{1+\eta}}{\frac{1}{1+\eta}-(1+\theta)}},
\end{aligned}$$

which is the same expression of the baseline model. This suggests that the relative selection across products is efficient. However, as Bertoletti and Etro [2020] have established, the absolute selection –specifically, the cutoff level for any given product–is inefficient. In fact, Bertoletti and Etro [2020] show that an insufficient level of selection occurs when:

$$\frac{1}{1+\eta} < \frac{\gamma \ln\left(1+\frac{1}{\gamma}\right)}{\ln\left(1+\frac{1}{\kappa}\right)}. \tag{B.25}$$

The condition is satisfied for DA and IA preferences, and in the case of homothetic preferences when κ is sufficiently large. Therefore, if the parameters indicate an insufficient level of selection, this under-selection will apply uniformly across all products, yet the relative selection between markets remains efficient.

C Calibration

C.1 Derivations

Demand Curvature Parameter γ. Recall that the revenues of firm c in product m from j to j equal:

$$r_{mjj}(c) = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_j \tau_{jj} a_{mj}^{\gamma}}{\xi_{mj}(c^*_{mjj})^{\gamma}} (c^*_{mjj} - c)^{\gamma} (c^*_{mjj} + \gamma c) \delta_{mjj}(c). \tag{C.1}$$

Revenues can be re-written as:

$$r_{mjj}(c) = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^{\gamma} c^*_{mjj}}{\xi_{mj}} \left(1 - \frac{c}{c^*_{mjj}}\right)^{\gamma} \left(1 + \gamma \frac{c}{c^*_{mjj}}\right) \delta_{mjj}(c).$$

The average revenues equal (note that the shock δ is *i.i.d.* with mean one)

$$\begin{aligned}
\bar{r} &= \int_0^{c^*_{mjj}} r_{mjj}(c) \frac{\theta c^{\theta-1}}{(c^*_{mjj})^{\theta}} dc = \\
&= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^{\gamma} c^*_{mjj}}{\xi_{mj}} \int_0^{c^*_{mjj}} \left(1 - \frac{c}{c^*_{mjj}}\right)^{\gamma} \left(1 + \gamma \frac{c}{c^*_{mjj}}\right) \frac{\theta c^{\theta-1}}{(c^*_{mjj})^{\theta}} dc.
\end{aligned}$$

We apply the change of variable technique and substitute $t = c/c^*_{ex}$.

$$\begin{aligned}
\bar{r} &= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^{\gamma} c^*_{mjj}}{\xi_{mj}} \theta \int_0^1 (1-t)^{\gamma} (1+\gamma t) t^{\theta-1} dt = \\
&= \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^{\gamma} c^*_{mjj}}{\xi_{mj}} \theta T_1,
\end{aligned}$$

where T_1 is a constant that equals:[27]

$$\begin{aligned}
T_1 &= \int_0^1 (1-t)^{\gamma} (1+\gamma t) t^{\theta-1} dt = \\
&= \int_0^1 (1-t)^{\gamma} t^{\theta-1} dt + \gamma \int_0^1 (1-t)^{\gamma} t^{\theta} dt \\
&= B(\theta, \gamma+1) + \gamma B(\theta+1, \gamma+1),
\end{aligned}$$

[27] $T_1 = \tilde{T}_1$, but is written in a different way here for convenience.

where $B(z,h)$ is the Euler Beta Function:[28]

$$B(z,h) = \int_0^1 (1-t)^{h-1} t^{z-1} dt.$$

Let $c_{ex}^* = \max_{v \neq j}\{c_{mjv}^*\}$ denote the cutoff for exporters: the smallest cost-cutoff across destination markets. The average domestic revenues for exporters equal:

$$\begin{aligned}
\bar{r}_{ex} &= \int_0^{c_{ex}^*} r_{mjj}(c) \frac{\theta c^{\theta-1}}{(c_{ex}^*)^\theta} dc = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma c_{mjj}^*}{\xi_{mj}} \int_0^{c_{ex}^*} \left(1 - \frac{c}{c_{mjj}^*}\right)^\gamma \left(1 + \gamma \frac{c}{c_{mjj}^*}\right) \frac{\theta c^{\theta-1}}{(c_{ex}^*)^\theta} dc = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj} (c_{ex}^*)^\theta} \theta \int_0^{\frac{c_{ex}^*}{c_{mjj}^*}} (1-t)^\gamma (1+\gamma t) t^{\theta-1} dt = \\
&= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj} (c_{ex}^*)^\theta} \theta T_2,
\end{aligned}$$

where we applied the change of variable techique and where T_2 is a constant that equals:

$$\begin{aligned}
T_2 &= \int_0^{\frac{c_{ex}^*}{c_{mjj}^*}} (1-t)^\gamma (1+\gamma t) t^{\theta-1} dt = \\
&= \int_0^{\frac{c_{ex}^*}{c_{mjj}^*}} (1-t)^\gamma t^{\theta-1} dt + \gamma \int_0^{\frac{c_{ex}^*}{c_{mjj}^*}} (1-t)^\gamma t^\theta dt = \\
&= B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta; \gamma+1\right) + \gamma B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta+1; \gamma+1\right),
\end{aligned}$$

where $B(u;z,h)$ is the incomplete Euler Beta function:[29]

$$B(u;z,h) = \int_0^u (1-t)^{h-1} t^{z-1} dt.$$

Let us now consider the average revenues of non-exporters:

$$\bar{r}_{dom} = \int_{c_{ex}^*}^{c_{mjj}^*} r_{mjj}(c) \frac{\theta c^{\theta-1}}{(c_{mjj}^*)^\theta - (c_{ex}^*)^\theta} dc =$$

[28] In Matlab, this is computed with the function *beta(z,h)*.
[29] In Matlab, this is computed as *betainc(u,z,h)*beta(z,h)*.

$$= \int_{c_{ex}^*}^{c_{mjj}^*} \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma c_{mjj}^*}{\xi_{mj}} \left(1-\frac{c}{c_{mjj}^*}\right)^\gamma \left(1+\gamma\frac{c}{c_{mjj}^*}\right) \frac{\theta c^{\theta-1}}{(c_{mjj}^*)^\theta - (c_{ex}^*)^\theta} dc =$$

$$= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj}((c_{mjj}^*)^\theta - (c_{ex}^*)^\theta)} \theta \int_{\frac{c_{ex}^*}{c_{mjj}^*}}^{1} (1-t)^\gamma (1+\gamma t)\, t^{\theta-1} dt =$$

$$= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj}((c_{mjj}^*)^\theta - (c_{ex}^*)^\theta)} \theta \left[\int_0^1 (1-t)^\gamma (1+\gamma t)\, t^{\theta-1} dt\right] -$$

$$\left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj}((c_{mjj}^*)^\theta - (c_{ex}^*)^\theta)} \theta \left[\int_0^{\frac{c_{ex}^*}{c_{mjj}^*}} (1-t)^\gamma (1+\gamma t)\, t^{\theta-1} dt\right] =$$

$$= \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma (c_{mjj}^*)^{\theta+1}}{\xi_{mj}((c_{mjj}^*)^\theta - (c_{ex}^*)^\theta)} \theta (T_1 - T_2).$$

The sales advantage moment M_{mj}^{sadv} equals:

$$M_{mj}^{sadv} = \frac{\bar{r}_{ex}}{\bar{r}_{dom}} = \left[\left(\frac{c_{mjj}^*}{c_{ex}^*}\right)^\theta - 1\right] \frac{T_2}{T_1 - T_2} = \tag{C.2}$$

$$= \left[\left(\frac{c_{mjj}^*}{c_{ex}^*}\right)^\theta - 1\right] \frac{B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta; \gamma+1\right) + \gamma B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta+1; \gamma+1\right)}{B(\theta, \gamma+1) + \gamma B(\theta+1, \gamma+1) - B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta; \gamma+1\right) - \gamma B\left(\frac{c_{ex}^*}{c_{mjj}^*}; \theta+1; \gamma+1\right)}. \tag{C.3}$$

where $\left(\frac{c_{mjj}^*}{c_{ex}^*}\right)^\theta = N_{mjj}/N_{ex}$, where $N_e x$ is the number of exporters. The ratio is the reciprocal of export participation (i.e., the ratio of exporters to the total number of firms).

Standard Deviation of the Firm-Product Shock σ. Let us re-write revenues in the following way:

$$r_{mjj}(c, \delta) = f_{mjj}(c)\delta, \tag{C.4}$$

where

$$f_{mjj}(c) = \left(\frac{\gamma^\gamma}{(1+\gamma)^{1+\gamma}}\right) \frac{L_j w_i \tau_{jj} a_{mj}^\gamma c_{mjj}^*}{\xi_{mj}} \left(1-\frac{c}{c_{mjj}^*}\right)^\gamma \left(1+\gamma\frac{c}{c_{mjj}^*}\right) = r_{0mj} \left(1-\frac{c}{c_{mjj}^*}\right)^\gamma \left(1+\gamma\frac{c}{c_{mjj}^*}\right), \tag{C.5}$$

and

$$r_{0mj} = \left(\frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}} \right) \frac{L_j w_i \tau_{jj} a_{mj}^{\gamma} c_{mjj}^{*}}{\xi_{mj}}. \tag{C.6}$$

The variance of the product of two random variables equals:

$$Var(XY) = Var(X)Var(Y) + Var(X)E(Y)^2 + Var(Y)E(X)^2. \tag{C.7}$$

Hence, the variance of firms' revenues equals

$$Var(f_{mjj}(c)\delta) = Var(f_{mjj}(c))Var(\delta) + Var(f_{mjj}(c))E(\delta)^2 + Var(\delta)E(f_{mjj}(c))^2, \tag{C.8}$$

since the two variables are independent. The variance of δ is σ and the expected value of δ is 1. The expected value of $f_{mjj}(c)$ equals:

$$E(f_{mjj}(c)) = r_{0mj}\theta T_1. \tag{C.9}$$

We now have to compute the variance of $f_{mjj}(c)$. First, we compute the expected value of the square of $f_{mjj}(c)$.

$$\begin{aligned}
E(f_{mjj}(c)^2) &= \int_0^{c_{mjj}^*} f_{mjj}(c)^2 \frac{\theta c^{\theta-1}}{(c_{ex}^*)^{\theta}} dc = \\
&= r_{0mj}^2 \int_0^{c_{mjj}^*} \left(1 - \frac{c}{c_{mjj}^*}\right)^{2\gamma} \left(1 + \gamma \frac{c}{c_{mjj}^*}\right)^2 \frac{\theta c^{\theta-1}}{(c_{mjj}^*)^{\theta}} dc = \\
&= r_{0mj}^2 \int_0^1 (1-t)^{2\gamma} (1+\gamma t)^2 \theta t^{\theta-1} dt = \\
&= r_{0mj}^2 \int_0^1 (1-t)^{2\gamma} \left(1 + 2\gamma t + \gamma^2 t^2\right) \theta t^{\theta-1} dt = \\
&= r_{0mj}^2 \theta \left[B(\theta, 2\gamma+1) + 2\gamma B(\theta+1, 2\gamma+1) + \gamma^2 B(\theta+2, 2\gamma+1) \right] = \\
&= r_{0mj}^2 \theta T_3.
\end{aligned}$$

Hence, the variance of $f_{mjj}(c)$ equals:

$$Var(f_{mjj}(c)) = E(f_{mjj}(c)^2) - E(f_{mjj}(c))^2 = r_{0mj}^2 \theta \left(T_3 - \theta T_1^2 \right). \tag{C.10}$$

We then obtain that:

$$\begin{aligned}Var(f_{mjj}(c)\delta) &= r_{0mj}^2\theta\left(T_3-\theta T_1^2\right)\sigma + r_{0mj}^2\theta\left(T_3-\theta T_1^2\right) + \sigma r_{0mj}^2\theta^2 T_1^2 = \\ &= r_{0mj}^2\theta\left(T_3(1+\sigma)-\theta T_1^2\right).\end{aligned}$$

As a result, the coefficient of variation is given by:

$$CV_{mj} = \frac{Var(f_{mjj}(c)\delta)^{0.5}}{E(f_{mjj}(c)\delta} = \frac{\theta^{0.5}\left(T_3(1+\sigma)-\theta T_1^2\right)^{0.5}}{\theta T_1}. \tag{C.11}$$

C.2 Measuring the Domestic Expenditure Share

The formula to calculate the domestic expenditure share in Denmark for product m and year t (21) is:

$$\lambda_{mt} = \frac{Output_{mt}}{Output_{mt} + Imports_{mt} - Exports_{mt}}, \tag{C.12}$$

where $Output_{mt}$ is the value of manufacturing production in Denmark for product m at time t, $Imports_{mt}$ is the value of imports to Denmark, and $Exports_{mt}$ is the value of exports from Denmark. As our instrumental variable relies on export data at the six-digit level of the Harmonized System (HS) classification from BACI, we define a product as a six-digit HS code, which is analogous to the CN six-digit classification.

While data on total imports and exports is available, data on the value of manufacturing output at the product level is accessible through the dataset VARS. As mentioned in the main text, VARS is a survey that covers all manufacturing firms that have at least 10 employees. Consequently, using the value of output from VARS can introduce bias in the calculation of domestic expenditure shares due to the under-representation of smaller firms. This underestimation of manufacturing output implies that the denominator of (C.12) can be negative. Given that we take logarithms in our calculations, this underestimation also results in a reduction in the number of observations.

To address this issue, we employ the following modified formula for the domestic expenditure share:

$$\lambda_{mt} = \frac{\frac{Output_{mt}}{Exports_{mt}}}{\frac{Output_{mt}}{Exports_{mt}} + \frac{Imports_{mt}}{Exports_{mt}} - 1}, \tag{C.13}$$

where $\frac{Output_{mt}}{Exports_{mt}}$ represents the ratio of output production to exports and is calculated solely using the exports of manufacturing firms included in VARS. Conversely, $\frac{Imports_{mt}}{Exports_{mt}}$ represents the ratio of imports to exports and is computed using aggregate imports and exports from the Danish customs data (UHDI). This method implicitly assumes that the ratio of output to exports for the firms sampled in VARS is representative of the ratio for the entire population of firms, which is a necessary approximation under the given circumstances. Using this approach, we significantly reduce the number of product-years where the numerator in the domestic expenditure share formula is negative.

C.3 Results

Table C.1: Moments Across Years

Year	Trade Elast.	Sales Adv.	Export Part.	CV of Sales
2000	4	2.37	0.55	1.69
2001	4	2.46	0.55	1.75
2002	4	2.63	0.56	1.77
2003	4	2.57	0.58	1.84
2004	4	2.68	0.57	1.78
2005	4	2.99	0.57	1.90
2006	4	2.64	0.57	1.85
2007	4	2.59	0.59	1.73
2008	4	1.99	0.58	1.69
2009	4	3.69	0.56	1.78
2010	4	4.32	0.57	1.84
2011	4	3.52	0.59	1.80
2012	4	3.87	0.63	1.77
2013	4	3.67	0.62	1.86
2014	4	3.59	0.62	1.89
2015	4	3.68	0.62	1.87

Moments computed across years.

Table C.2: Parameters Across Years

Year	θ	γ	σ
2000	4	0.59	2.18
		(0.07)	(0.14)
2001	4	0.62	2.29
		(0.09)	(0.16)
2002	4	0.66	2.30
		(0.08)	(0.16)
2003	4	0.63	2.53
		(0.07)	(0.16)
2004	4	0.66	2.31
		(0.06)	(0.13)
2005	4	0.74	2.52
		(0.07)	(0.19)
2006	4	0.65	2.53
		(0.08)	(0.17)
2007	4	0.63	2.22
		(0.11)	(0.22)
2008	4	0.45	2.38
		(0.17)	(0.24)
2009	4	0.91	1.92
		(0.10)	(0.20)
2010	4	1.02	1.89
		(0.14)	(0.24)
2011	4	0.84	2.08
		(0.19)	(0.26)
2012	4	0.87	1.95
		(0.26)	(0.37)
2013	4	0.84	2.24
		(0.22)	(0.41)
2014	4	0.82	2.35
		(0.33)	(0.48)
2015	4	0.84	2.27
		(0.37)	(0.48)

Parameters estimated using the moments shown in Table C.1. Bootstrap standard errors in parenthesis.

Table C.3: Moments and Parameters - Low Trade Elasticity

Moments	
Trade Elasticity	2
Exporters Sales Advantage	2.37
Export Participation	0.55
Coefficient of Variation of Sales	1.69
Parameters	
θ	2
γ	0.72
	(0.09)
σ	2.21
	(0.13)

Results from estimating the parameters using a lower trade elasticity. All other moments are as in (C.1). Bootstrap standard errors in parenthesis.

Table C.4: Moments and Parameters - US Moments

Moments	
Trade Elasticity	4
Exporters Sales Advantage	2.37
Export Participation	0.55
Coefficient of Variation of Sales	1.69
Parameters	
θ	4
γ	1.60
σ	0.86
	(0.06)

Results from estimating the parameters using the exporters sales advantage and the export participation from Bernard et al. [2003]. All other moments are as in (C.1). Bootstrap standard errors in parenthesis. Since the exporters domestic sales advantage for the US is taken from the literature, we cannot compute a standard error for γ.

C.4 Results at Sector Level

In this section, we replicate the results of Table 4 by re-estimating the regression outlined in (27), with the sample divided by sector. For sector classification, we adopt two distinct approaches: one using the Combined Nomenclature (CN) classification and the other employing the Broad Economic Category (BEC) classification.

The outcomes derived from six CN sectors are displayed in Tables C.5 and C.6. Predominantly, these results are not statistically significant. Specifically, in the OLS regressions, the coefficients are observed to be nearly zero. In the IV regressions, the instruments appear weak. This weakness is attributed to the limited variation in the sector-level data.

The results pertaining to the BEC classification, which involves three sectors, are detailed in Tables C.7 and C.8. In the OLS regressions, the coefficients are approximately zero. However, a notable exception is the positive and statistically significant coefficient for Capital goods (0.032). Interestingly, this significance dissipates in the IV regression. In contrast, both Consumption goods and Intermediate goods exhibit negative and statistically significant coefficients in the IV regression. The magnitude of these coefficients aligns with our baseline findings in Table 4, which suggests that these goods are driving the results of the baseline regression. Nonetheless, the similarity of the coefficients for Consumption goods and Intermediate goods indicates no substantial difference in their parameters.

In summary, our analysis does not reveal any significant sector-based differences. This lack of differentiation is primarily attributed to the insufficient variation in the sector-level data, which hinders the detection of statistical significance. Additionally, our instruments demonstrate greater efficacy at an aggregate level rather than at the sector level.

Table C.5: OLS Regression: Sectors Based on CN Classification

	Dependent Variable: Log change in number of firms					
	(1)	(2)	(3)	(4)	(5)	(6)
Log change in domestic	0.006	0.003	-0.011	0.029**	0.016	-0.007
expenditure share	(0.014)	(0.012)	(0.009)	(0.014)	(0.010)	(0.019)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
# Obs.	2238	2202	2193	1855	3432	2162
R^2	0.10	0.10	0.10	0.08	0.06	0.09

Estimation results of (27). Standard errors clustered at the six-digit level of the CN classification are shown in parentheses. *, **, and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels. Each column corresponds to a different sector: (1) Animal products, Vegetable products, and Foodstuffs (CN two-digit 01-24); (2) Mineral products, Chemical products, and Plastic and rubber (CN two-digit 25-40); (3) Leather and Fur, Textiles, and Footwear and headgear (CN two-digit 41-43 and 50-67); (4) Stone and glass and Metals (CN two-digit 68-83); (5) Machinery and electrical and Transportation (CN two-digit 85-89); (6) Wood products and Miscellaneous (CN two-digit 44-49 and 90-97).

Table C.6: IV Regression: Sectors Based on CN Classification

	Dependent Variable: Log change in number of firms					
	(1)	(2)	(3)	(4)	(5)	(6)
Log change in domestic expenditure share	-0.217	0.036	-0.263	0.012	-0.658	-0.270
	(0.215)	(0.071)	(0.191)	(0.040)	(0.762)	(0.224)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
# Obs.	2238	2202	2193	1855	3432	2162
F-Stat	2.08	4.45	3.48	8.10	0.83	3.56
KP LM Stat	1.82	4.55	3.06	5.22	0.81	2.34
KP LM p-value	0.18	0.03	0.08	0.02	0.37	0.13

Estimation results of (27). Standard errors clustered at the six-digit level of the CN classification are shown in parentheses. *, **, and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels. Each column corresponds to a different sector: (1) Animal products, Vegetable products, and Foodstuffs (CN two-digit 01-24); (2) Mineral products, Chemical products, and Plastic and rubber (CN two-digit 25-40); (3) Leather and Fur, Textiles, and Footwear and headgear (CN two-digit 41-43 and 50-67); (4) Stone and glass and Metals (CN two-digit 68-83); (5) Machinery and electrical and Transportation (CN two-digit 85-89); (6) Wood products and Miscellaneous (CN two-digit 44-49 and 90-97).

Table C.7: OLS Regression: Sectors Based on BEC Classification

	Dependent Variable: Log change in number of firms		
	(Capital)	(Consumption)	(Intermediate)
Log change in domestic expenditure share	0.032**	0.009	-0.005
	(0.016)	(0.008)	(0.007)
Year FE	Yes	Yes	Yes
# Obs.	2765	4845	6386
R^2	0.06	0.09	0.07

Estimation results of (27). Standard errors clustered at the six-digit level of the CN classification are shown in parentheses. *, **, and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels. Each column corresponds to a different sector: Capital goods, Consumption goods, and Intermediate goods.

Table C.8: IV Regression: Sectors Based on BEC Classification

	Dependent Variable: Log change in number of firms		
	(Capital)	(Consumption)	(Intermediate)
Log change in domestic expenditure share	-0.562	-0.152*	-0.123*
	(1.240)	(0.087)	(0.072)
Year FE	Yes	Yes	Yes
# Obs.	2765	4845	6386
F-Stat	0.24	10.75	9.49
KP LM Stat	0.24	7.65	9.38
KP LM p-value	0.63	0.01	0.00

Estimation results of (27). Standard errors clustered at the six-digit level of the CN classification are shown in parentheses. *, **, and *** indicate significance at the 10 percent, 5 percent, and 1 percent levels. Each column corresponds to a different sector: Capital goods, Consumption goods, and Intermediate goods.

D Quantification

Table D.1: Variance of Sales Within Firms

	Product-Level Cutoffs	Demand Shock
Baseline Estimates	20	70
Low Trade Elasticity	24	60
Higher Sales Advantage	50	20

The first column reports the difference in R^2 between (30) and (31). The second column reports the difference in R^2 between (31) and (32). The dependent variables in these regression is the residual of a regression of the log of firm revenues on firm fixed effects. Different rows use different parameters for the simulation of firm sales: the Baseline Estimates are reported in Table 3, the Low Trade Elasticity estimates are reported in Table C.3, and the Higher Sales Advantage estimates are reported in Table C.4. All values are multiplied by 100.

Table D.2: Variance Decomposition - Low Trade Elasticity

	(Product FE)	(Firm FE)	(Residual)
Baseline Model	0.177***	0.155***	0.668***
	(0.002)	(0.002)	(0.002)
Observations	74,898	74,898	74,898

	(Product FE)	(Firm FE)	(Residual)
No Cutoff Heterogeneity	0.000***	0.338***	0.662***
	(0.000)	(0.001)	(0.001)
Observations	200,000	200,000	200,000

Results from the variance decomposition of log revenues on product fixed effect, firm fixed effect, and residual. Log revenues are simulated using our baseline model in the first row and using a model in which all product cutoffs are identical in the second part of the table. The revenues are simulated using the parameters that are reported in Table C.3.

Table D.3: Variance Decomposition - US Moments

	(Product FE)	(Firm FE)	(Residual)
Baseline Model	0.149***	0.482***	0.368***
	(0.002)	(0.003)	(0.002)
Observations	74,898	74,898	74,898

	(Product FE)	(Firm FE)	(Residual)
No Cutoff Heterogeneity	0.000	0.850***	0.150***
	(0.000)	(0.001)	(0.001)
Observations	200,000	200,000	200,000

Results from the variance decomposition of log revenues on product fixed effect, firm fixed effect, and residual. Log revenues are simulated using our baseline model in the first row and using a model in which all product cutoffs are identical in the second part of the table. The revenues are simulated using the parameters that are reported in Table C.4.

Table D.4: Product Attractiveness and Product Characteristics

	Dependent Variable: Product Attractiveness							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log Total Production	0.046***					0.059***	0.040***	
	(0.000)					(0.001)	(0.000)	
Log Domestic Sales		0.045***						
		(0.000)						
Log # Firms			0.206***					
			(0.000)					
Log Exports				0.024***		-0.008***		
				(0.000)		(0.000)		
Log Imports					0.037***		0.013***	
					(0.000)		(0.000)	
Export Participation								-0.056***
								(0.003)
CN2-year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.72	0.72	0.99	0.60	0.64	0.70	0.72	0.57
# Obs.	29734	29734	29734	23714	29431	23714	29431	29734

Results from OLS of the estimated log of $\frac{a_{mj}}{a_{kj}}$ on the product-time characteristics described in the rows. $\frac{a_{mj}}{a_{kj}}$ is estimated using the IV estimate of η. Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

Table D.5: Product Attractiveness and Product Characteristics

	Dependent Variable: Product Attractiveness				
	(1)	(2)	(3)	(4)	(5)
Avg. Demand Elasticity	-0.000**				
	(0.000)				
Differentiated Product		0.053***			
		(0.006)			
Quality Ladder			-0.009***		
			(0.002)		
Average Log Unit Price				-0.001	
				(0.001)	
Average Capital Intensity					0.017***
					(0.001)
CN2-year FE	Yes	Yes	Yes	Yes	Yes
R^2	0.58	0.56	0.59	0.55	0.56
# Obs.	23559	29734	17074	24618	29734

Results from OLS of the estimated log of $\frac{a_{mj}}{a_{kj}}$ on the product-time characteristics described in the rows. $\frac{a_{mj}}{a_{kj}}$ is estimated using the IV estimate of η. Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.