Mechanics of Spatial Growth

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Introduction

- In recent decades (90s-20), we have seen developing countries growing, at the same time increase in trade openness, increase in internal migration, large aggregate TFP growths
  - Examples: Vietnam, Laos, Philippines, China and more
- Part of aggregate economic development is shaped by the dynamics of the distribution of economic activity in space
  - Regions are different: exposed to trade, internal migration, local characteristics
- In this study we want to understand/quantify the role of spatial growth on aggregate economic activity

This paper

- Develop a tractable dynamic spatial growth model to understand the role of trade, internal migration and factor accumulation for the process of economic development
  - The best global and local ideas contribute to the local stock of knowledge
  - Global ideas diffuse more to locations that are relatively more exposed to international trade
  - Workers learn about local ideas and diffuse these ideas across space when they migrate
- Embed diffusion of ideas process into a spatial framework with trade, forward looking migration and endogenous capital accumulation
  - Local stock of ideas characterized by a tractable system of difference equations
  - Existence and uniqueness of the balanced growth path
- Application: Study China's aggregate and spatial development over the 90s and 00s
  - Quantify the importance of our mechanisms for China's growth
  - Provide empirical evidence on ideas diffusion thru trade and migration

Related literature

Quantitative spatial economics: Redding and Rossi-Hansberg (2017), Caliendo et al. (2018), Fajgelbaum et al. (2019b), Fajgelbaum and Redding (2021)

Spatial dynamics

- labor market dynamics and trade: Artuc et al. (2010); Dix-Carneiro (2014); Dix Carneiro and Kovak (2017), Caliendo, Dvorkin, Parro (2019)
- capital accumulation: Caliendo and Parro (2020), Kleinman et al. (2021)
- innovation, local diffusion of technology and growth: Desmet and Rossi-Hansberg (2014), Desmet et al. (2018), Rossi-Hansberg and Cruz (2022)

Trade and growth: Buera and Oberfield (2020), Cai, Li and Santacreu (2022)

- semi-endogenous growth: Jones (1995), Kortum (1997), Atkeson and Burstein (2010)

Evidence on migration and ideas: Buchardi et al. (2021), Sequeira et al. (2020), Hunt and Gauthier-Loiselle (2010), Stuen et al. (2012), Ackcigit et al. (2017), Moser et al. (2014), Lewis (2011), among others

- internal migration in China: Imbert et al. (2022), Facchini et al. (2019), Combes et al. (2017), Meng and Zhang (2010)

On the origins of the China Shock: Tombe and Zhu (2020), Caliendo and Parro (2022) for a review

Roadmap

- Model
  - Start describing the process of innovation and diffusion
  - Production, factor demand, supply and trade
  - Evolution to the local stock of knowledge with trade and migration
  - Characterize the equilibrium properties of the model
- Application
  - Data
  - Results
  - Empirical evidence on ideas diffusion

Ideas diffusion

- Single economy, continuum of varieties produced
- For each variety, a large set of potential producers
  - Producers have heterogeneous ideas/productivities to produce
  - Denote by q the productivity of an idea to produce a variety
- Between time t and $t+1$, producers interact with other agents and get exposed to new ideas
  - The number of new ideas exposed and the productivity of them are stochastic (randomness in usage of new ideas)
- The productivity of a new idea is zq'^{ρ}
  - z is the original component; draw from exogenous distribution $H(z)$
  - q' is an insight drawn from external source distribution $G_t(q')$
  - ρ captures the contribution of the quality of insights from others to the productivity of new ideas

Ideas diffusion

Assumption 1
a) Internal distribution $H(z)$ of original ideas is Pareto

$$H(z) = 1 - \left(\frac{z}{\bar{z}}\right)^{-\theta},$$

$\bar{z}$ is the lower bound of the support, $\theta > 1$ is the shape parameter

b) Strength of ideas diffusion, $\rho \in [0, 1)$

c) Number of new ideas arriving between t and $t+1$ follows a Poisson distribution with mean $\Lambda_t = \alpha_t \bar{z}^{-\theta}$

d) Source distribution has sufficiently thin tail

Let $F_t(q)$ denote the fraction of varieties whose best producer has productivity no greater than q

- *Frontier of knowledge*

Ideas diffusion

- Under Assumption 1, one can show (Proposition 1) that $F_t(\cdot)$ is Fréchet at any t,

$$F_t(q) = \exp\left(-A_t q^{-\theta}\right)$$

  Scale parameter θ, location parameter A_t

- The **law of motion for the stock of knowledge** is given by

$$A_{t+1} = A_t + \alpha_t \int_0^{\infty} x^{\rho\theta} dG_t(x)$$

- Faster growth with arrival of new ideas α_t and better insights (from source distribution)
  - Importantly, no need to specify the external source distribution yet

Production and trade structure

We now consider an economy with N locations indexed by i, and n

- At i, distribution of productivity is Fréchet $F_{i,t}(q) = e^{-A_{i,t}q^{-\theta}}$
  - Use results from Eaton and Kortum (2002)
- Production of an intermediate variety with efficiency q

$$y_{i,t} = q_{i,t}\left(L_{i,t}^{\xi}K_{i,t}^{1-\xi}\right)^{\gamma}M_{i,t}^{1-\gamma},$$

$L_{i,t}$, $K_{i,t}$, and $M_{i,t}$ are the input demands γ is the input share, ξ labor shares

- Unit price of input bundle is

$$x_{i,t} = B\left(w_{i,t}^{\xi}r_{i,t}^{1-\xi}\right)^{\gamma}P_{i,t}^{1-\gamma}$$

  - Trade is costly, κ_{in}
    - Cost of purchasing a variety with efficiency q from n given by

$$\kappa_{in}x_{n,t}/q$$

Production and trade structure

- Searching for the lowest cost supplier we obtain fraction of goods purchased in location i from location n

$$\lambda_{in,t} = \frac{A_{n,t}\left(\kappa_{in} x_{n,t}\right)^{-\theta}}{\sum_{h=1}^{N} A_{h,t}\left(\kappa_{ih} x_{h,t}\right)^{-\theta}}$$

- Price index is given by

$$P_{i,t} = \left(\sum_{n=1}^{N} A_{n,t}\left(\kappa_{in} x_{n,t}\right)^{-\theta}\right)^{-1/\theta}$$

Goods and factor demand

- Total expenditure on goods at i

$$X_{i,t} = (1-\gamma)\sum_{n=1}^{N}\lambda_{ni,t}X_{n,t} + w_{i,t}L_{i,t} + r_{i,t}K_{i,t}$$

- Labor demand

$$w_{i,t}L_{i,t} = \xi\gamma\sum_{n=1}^{N}\lambda_{ni,t}X_{n,t},$$

- Capital demand

$$r_{i,t}K_{i,t} = (1-\xi)\gamma\sum_{n=1}^{N}\lambda_{ni,t}X_{n,t},$$

Factor supply - capital accumulation

- Landowners, log utility, see Kleinman, Liu and Redding (2021), Moll (2014) have access to an investment technology in local capital (capital structures)
  - In each location, atomistic landowners with log utility

$$\max_{\{C_{i,t},K_{i,t+1}\}_{t=0}^{\infty}} U = \sum_{t=0}^{\infty} \beta^t \log\left(C_{i,t}\right),$$
$$\text{s.t. } r_{i,t}K_{i,t} = P_{i,t}\left(C_{i,t} + K_{i,t+1} - (1-\delta)\,K_{i,t}\right),$$

  - Policy functions given by

$$\begin{aligned} C_{i,t} &= (1-\beta)\left(r_{i,t}/P_{i,t} + (1-\delta)\right)K_{i,t}, \\ K_{i,t+1} &= \beta\left(r_{i,t}/P_{i,t} + (1-\delta)\right)K_{i,t}, \\ K_{i,0} &= given. \end{aligned}$$

- Capital accumulation shapes part of the spatial distribution of economic activity

Households' dynamic labor supply decision

- Value of household at time t is given by

$$v_{i,t} = log(w_{i,t}/P_{i,t}) + \max_{\{n\}_{n=1}^{N}} \{\beta E_t[v_{n,t+1}] - m_{in} + \nu \epsilon_{n,t}\}$$

  - m_{in} mobility costs, $\varepsilon_{n,t}$ amenity shocks Type I EV with dispersion ν
- Aggregation: $V_{i,t+1} \equiv E_t[v_{i,t+1}]$, expected value of the HH (over $\epsilon_{i,t}$)

$$V_{i,t} = log\left(w_{i,t}/P_{i,t}\right) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta V_{n,t+1} - m_{in}\right)^{1/\nu}\right)$$

$$\mu_{in,t} = \frac{exp\left(\beta V_{n,t+1} - m_{in}\right)^{1/\nu}}{\sum_{l=1}^{N} exp\left(\beta V_{l,t+1} - m_{il}\right)^{1/\nu}}$$

- Evolution of the distribution of labor across markets

$$L_{i,t+1} = \sum_{n=1}^{N} \mu_{ni,t} L_{n,t}$$

Trade and migration with ideas

- Consider the case where producers in location i obtain new insights from two sources

$$q = z q_{\ell}^{\rho_{\ell}} q_{m}^{\rho_{m}},$$

- They represent two different sources of (independent) ideas
  - Sellers - ideas from producers in other locations are embedded in imported intermediate varieties
  - Migrants - carry insights around as they arrive in a new location
- ρ_{ℓ}, ρ_{m} capture the learning intensity from both types of insights; with $\rho_{\ell} + \rho_{m} < 1$

We now can obtain that the local stock of knowledge evolves over time as

$$A_{n,t+1} - A_{n,t} = \alpha_t \int_0^{\infty} \int_0^{\infty} \left(q_{\ell}^{\rho_{\ell}} q_{m}^{\rho_{m}} \right)^{\theta} dG_{n,t}(q_{\ell}, q_{m})$$

Trade and migration with ideas

- Assume q_ℓ insights from people, q_m insights from goods (Buera and Oberfield, 2020)
- At the end of t, when a household from i decides to move to n, she carries an insight q'_ℓ
  - a draw from frontier distribution of i, $F_{i,t}(q_\ell)$
- At time t, in n, when a new idea arrive, producers randomly meet others currently living in n, and the insight from this person is the insight component of the new idea. Hence,

$$G_{n,t}(q'_\ell) = \sum_{i=1}^{N} s_{in,t} F_{i,t}(q'_\ell)$$

where $s_{in,t} = \frac{\mu_{in,t} L_{i,t}}{\sum_{h=1}^{N} \mu_{hn,t} L_{h,t}}$ is the share of workers in location n that arrived from i at the end of period t

Trade and migration with ideas

- The evolution of the stock of knowledge at each location is given by

$$A_{n,t+1} - A_{n,t} = \alpha_t \Gamma_\rho \underbrace{\left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t} \right)^{\rho_\ell} \right]}_{people} \underbrace{\left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}} \right)^{\rho_m} \right]}_{goods}.$$

where Γ_ρ is a constant

Assumption 3 *Assume α_t has constant growth rate g_α, namely,*

$$\alpha_t = \alpha_0 (1 + g_\alpha)^t.$$

- We now define the equilibrium of the dynamic spatial growth model

Equilibrium

Equilibrium *Given an initial distribution of the stock of knowledge* $\{A_{i,0}\}_{i=1}^{N}$*, factor endowments* $\{L_{i,0}, K_{i,0}\}_{i=1}^{N}$*, fundamentals* $\{\alpha_0, m_{n,}, \kappa_{in}\}_{i=1,n=1}^{N,N}$ *, parameters and elasticities* $(\rho_\ell, \rho_m, \theta, \nu, \gamma, \xi, \beta)$*, the sequential competitive equilibrium of the dynamic multi-country trade growth model is characterized by a sequence of values, factor prices, goods prices, labor allocations, capital stocks, and stock of knowledge,* $\left\{V_{i,t}, w_{i,t}, r_{i,t}, P_{i,t}, L_{i,t}, K_{i,t}, A_{i,t}\right\}_{i=1,t=0}^{N,\infty}$*, that satisfy equilibrium conditions (1-7)*

Equilibrium

Balanced Growth Path (BGP). *Along the balanced growth path all equilibrium variables growth at a constant rate, the stock of knowledge* $1 + g_A = (1 + g_\alpha)^{\frac{1}{(1-\rho_l-\rho_m)}}$, *capital stock* $1 + g_k = (1 + g_A)^{\frac{1}{\theta\xi\gamma}}$, *and values* $1 + g_v = (1 + g_A)^{\frac{1}{\theta\xi\gamma(1-\beta)}}$.

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Proposition 2: *Existence and uniqueness of BGP (up to scale)*

- Sketch of proof. Use results from Allen and Arkolakis (2021) and Kleinman et. al. (2022), show that the spectral radius of the matrix of elasticities of the nonlinear system at the BGP is equal to 1

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- 6 Eigenvalues

$$Eigenvalues = \left(1, 1, \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0, \rho_\ell + \rho_m\right)$$

where $a = \beta + \nu + \theta\gamma\nu\xi,\ b = -\nu(1 + \beta - \gamma\xi(1 + \theta(1 - \beta))),\ c = \beta(\nu - 1 - \gamma\xi\nu(1 + \theta))$

Quantitative application: Sources of the China shock

Sources of the China shock

During the 1990s and far into the 2000s, China underwent a considerable economic transformation

- During this time, China experienced fast economic growth, sustained capital accumulation, shifts in the spatial relocation of inputs and production, an increasing role in product markets, and trade openness
- Recent literature has studied the effects of the China shock on economic outcomes across the world
  - Caliendo and Parro (2022) - forthcoming ARE
- How important was international trade (access to cheaper intermediate goods) for China's development?
- How important was internal migration for China's development?

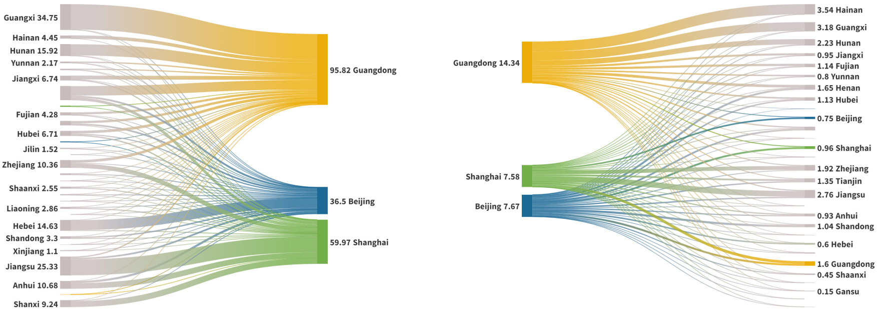
Use our spatial growth model to study: How China would have developed absent of the changes to trade and migration that occurred after the 90's

Data source

- Model for the year 1990, with 30 provinces and one rest-of the world
  - Migration: five-years mobility rates across provinces in China from the 1 percent sample of the 1990 Census from IPUMS
    - Condition on Hukou status Model with Hukou
  - Exports and imports between provinces and RoW, and production and employment from the China compendium of statistics
- Elasticities/parameters: 5-year discount factor $\beta = 0.86$, trade elasticity $\theta = 4.55$, we set the dispersion of taste shocks $1/\nu = 0.15$, $g_{\alpha} = 0.013$ from the 90's (Great moderation) in the U.S., $\gamma = 0.38$ and $\xi = 0.54$, values from the year 1990 EORA world's aggregates

Internal migration in China

Figure: Mobility across provinces in China (1985-1990)

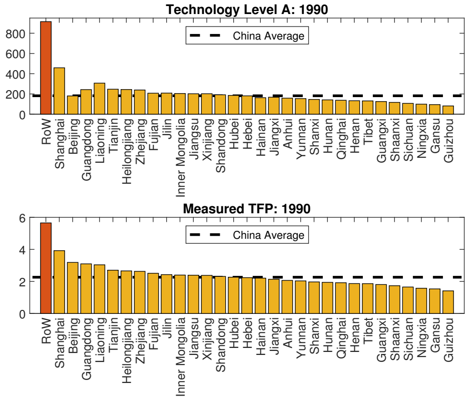


Note: 1990 Census from IPUMS. The left axis show the origin and the right axis show the destination provinces. Scale 1 = 10,000.

Stock of knowledge

- We obtain the initial province level stock of knowledge as

$$A_{n,1990} = B\lambda_{nn,1990}\left(\frac{realGDP_{n,1990}}{L_{n,1990}^{\xi}K_{n,1990}^{1-\xi}}\right)^{\theta\gamma}$$



Elasticities/parameters

- Let $\Theta \equiv (\alpha_0, \rho_m, \rho_l)'$ set of parameters that we want to estimate
  - Moment conditions or orthogonality conditions as $g_N(\hat{\Theta}) \equiv (1/N)\, h(\varpi_t, \hat{\Theta})$
  - $h(\varpi_t, \hat{\Theta})$ is a vector of moment conditions and $\varpi_t \equiv (\lambda_{in,t}, s_{ni,t}, A_{i,t}, A_{i,t+1})'$

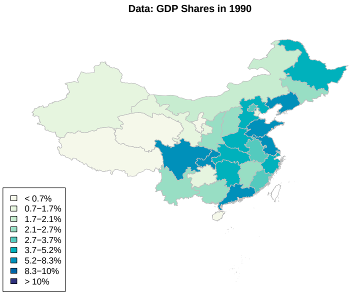
$$\hat{\Theta} = \arg\min : g_N(\hat{\Theta})' W g_N(\hat{\Theta})$$

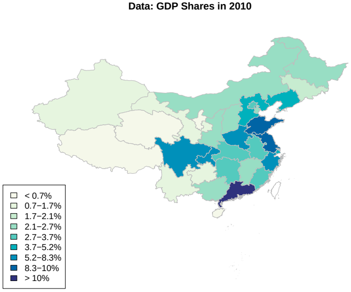
  - Estimates for the period 1990-2000, using five moment conditions : 1) average change in fundamental TFP, 2) average growth rate in fundamental TFP, 3) variance in the time changes in fundamental TFP, 4) covariance between the initial fundamental TFP and the change in fundamental TFP, 5) covariance between the initial fundamental TFP and the growth rate in fundamental TFP
  - Estimate $\rho_l = 0.2$, $\rho_m = 0.61$, and $\alpha_0 = 0.18$

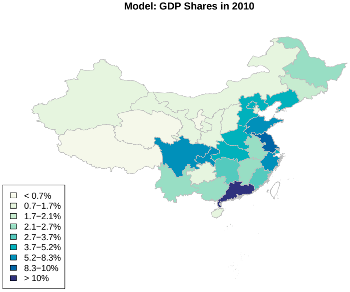
Quantifications

- Structural diff-in-diff - Dynamic Hat Algebra (CDP 2019)
- Initial year 1990
  - Baseline economy
    - Solve for evolution of the Chinese economy with 1990 fundamentals
    - Do not impose the Chinese economy is on a balance growth path
  - Counterfactual economy
    - Head-Ries index to infer changes in trade and internal migration costs
    - How observed changes in international trade costs from 1990-2010 contributed to growth
    - How observed changes in international migration costs from 1990-2000 contributed to growth

Spatial distribution of economic activity







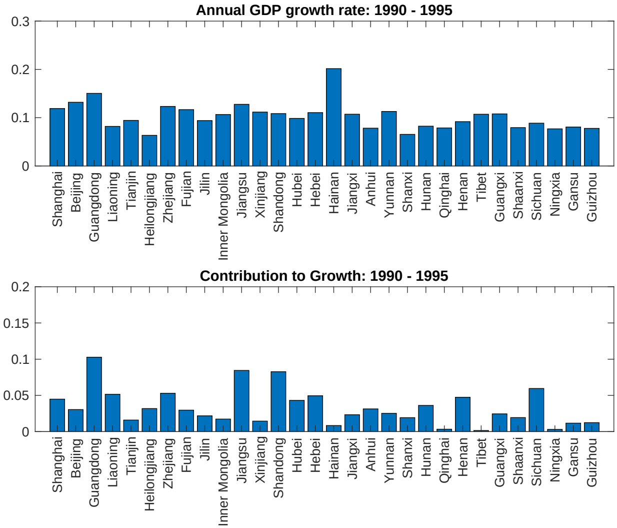
Aggregate growth (Baseline Economy)

Table: Annual GDP Growth Rate

	90-95	90-00	90-05	90-10	90-15	90-20
Actual GDP growth	12.3%	10.4%	10.2%	10.5%	9.9%	9.3%
With fundamentals at 1990	10.6%	10.1%	9.6%	9.2%	8.9%	8.6%
W/o ideas from people ($\rho_l = 0$)	8.7%	8.0%	7.4%	6.9%	6.5%	6.2%
W/o ideas from goods ($\rho_m = 0$)	6.4%	5.4%	4.7%	4.1%	3.7%	3.4%
W/o capital accumulation	5.0%	4.9%	4.7%	4.6%	4.5%	4.5%

Source: World Development Indicators and own calculations

Spatial growth and contribution



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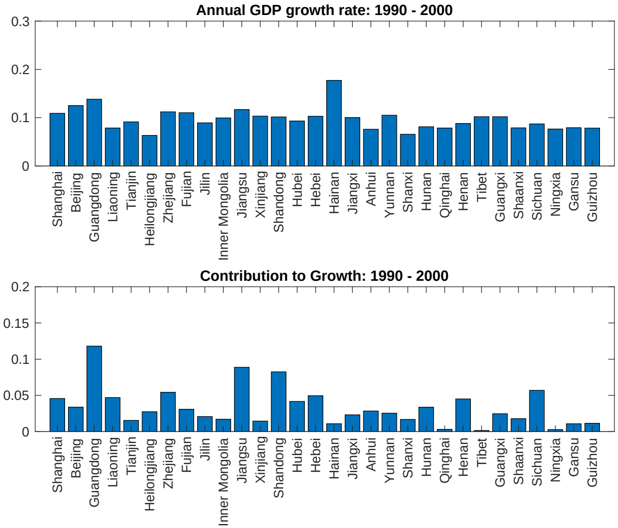

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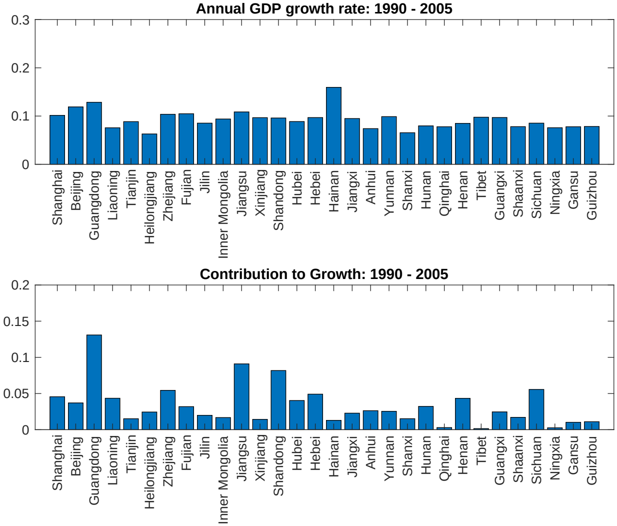

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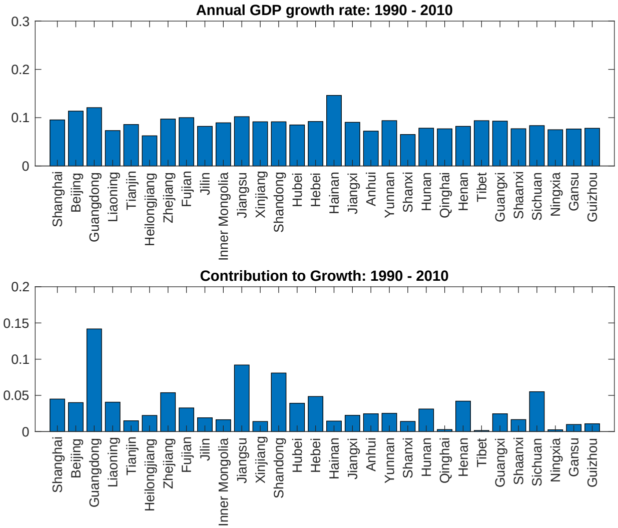

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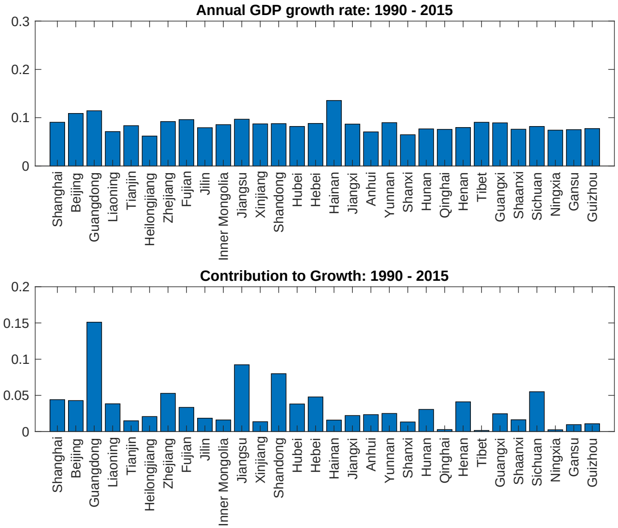

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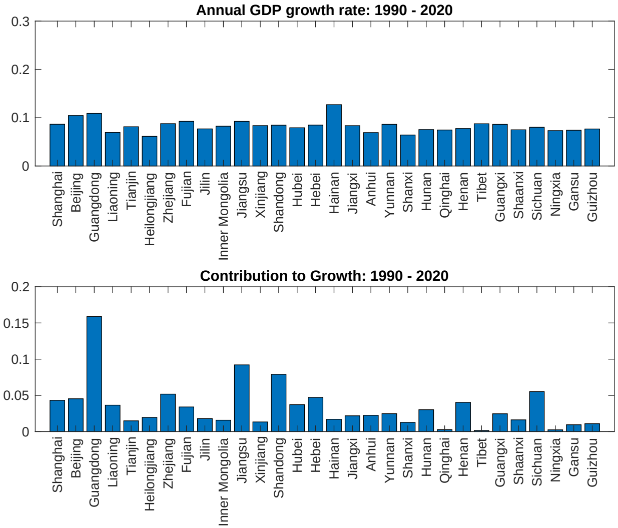
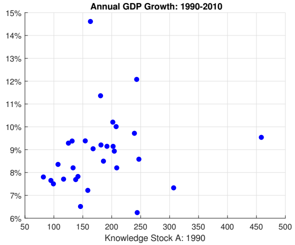
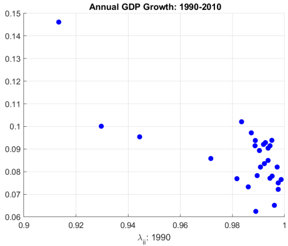


Real GDP growth versus initial conditions

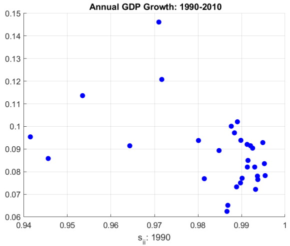
a) Real GDP growth versus initial stock of knowledge



b) Real GDP growth initial trade openness



c) Real GDP growth versus initial mobility

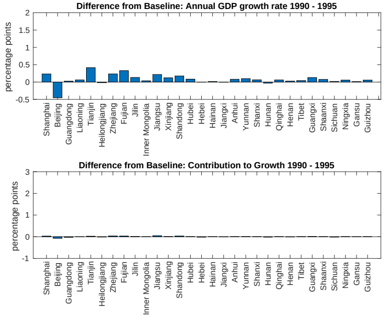


Findings and further quantifications

- Initial conditions seem to be important to understand and quantify the process of spatial development across economies
  - Trade, migration, and capital accumulation interact in non-obvious ways
- As China joins the WTO, how do provinces more exposed to trade developed relative to the less exposed?
  - **Include observed reduction in trade costs**
- Because of Hukou, return migration might foster ideas flows back
  - **Include observed reduction in internal migration costs ("Hukou")**

Effects relative to baseline

a) Effects of reduction in trade cost vs. baseline



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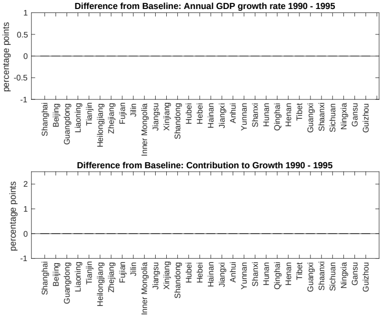

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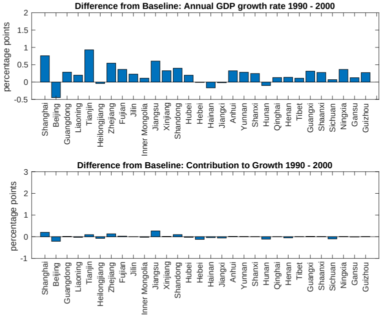

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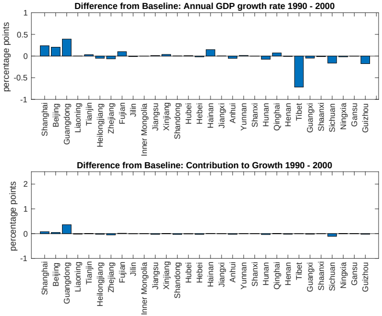

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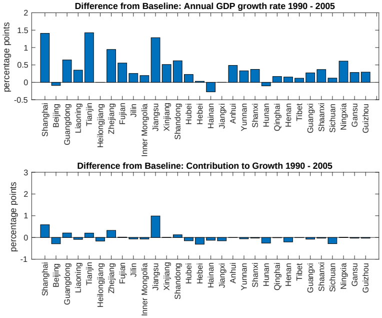

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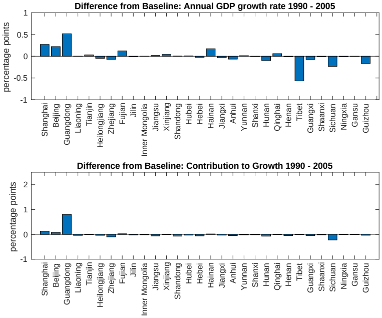

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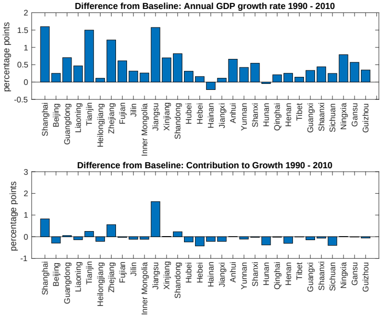

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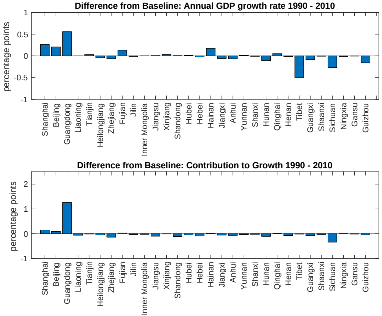

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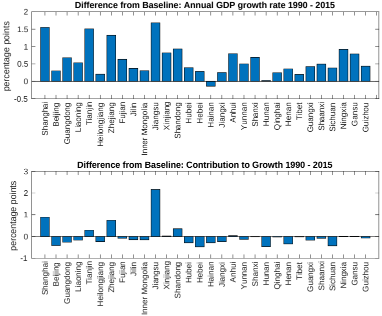

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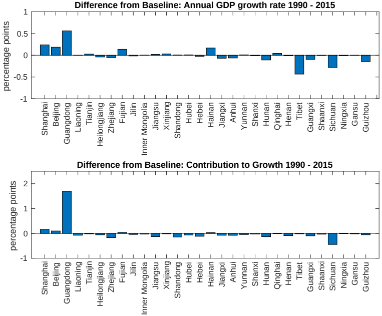
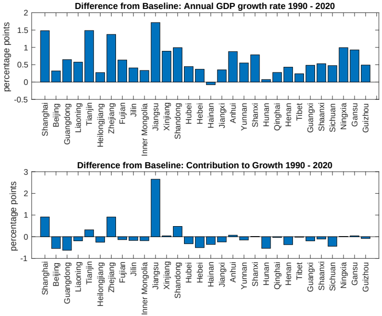
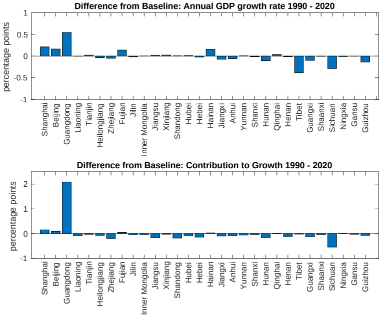


Effects relative to baseline

a) Effects of reduction in trade cost vs. baseline



b) Effects of reduction in Hukou vs. baseline



Empirical Evidence of Ideas Diffusion

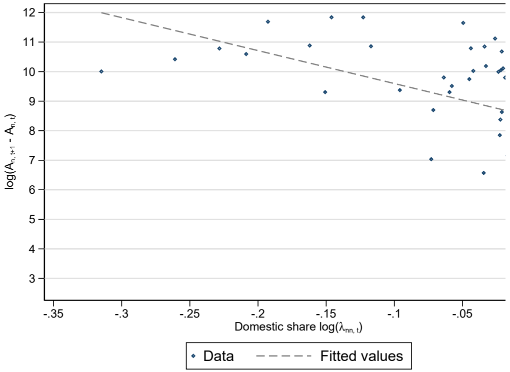
- Provide reduced-form evidence of ideas diffusion from trade and migration
- Use province-level patent data (China Statistics Yearbooks) as a proxy for the local stock of knowledge
- Measure of knowledge stock, $A_{n,t}$, cumulative approved patents at the province level for each year over the period 1995-2010
  - Compute the change in the stock of knowledge every five years from 1995 to 2010
  - For the ROW: Google Patent from 1995-2010
- Empirical specification:

$$\log\left(A_{n,t+1} - A_{n,t}\right) = \tau + \beta_m \log \lambda_{nn,t} + \beta_l \log\left(migration_{n,t}\right) + \tau_n + \tau_t + \epsilon_{n,t}.$$

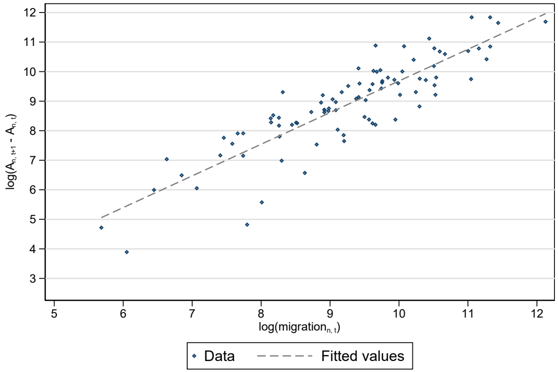
- Define $\log\left(migration_{n,t}\right) = \log\left[\sum_{i=1}^{N} s_{in,t} A_{i,t}\right]$
  - Expect $\beta_m < 0$, $\beta_l > 0$
- We then separate immigration from natives, $\log\left(immigration_{n,t}\right) = \log\left[\sum_{i \neq n}^{N} s_{in,t} A_{i,t}\right]$

Empirical Evidence of Ideas Diffusion

a) Trade openness



b) Internal migration



Empirical Evidence of Ideas Diffusion

Table: Estimates of knowledge diffusion through trade and migration

	$\log\left(A_{n,t+1} - A_{n,t}\right)$		
	(1)	(2)	(3)
$\log \lambda_{nn,t}$	-6.138***		-5.782**
	(2.222)		(2.296)
$\log\left(migration_{n,t}\right)$		0.353*	0.274*
		(0.201)	(0.153)
Constant	8.692***	5.448**	5.960***
	(0.113)	(2.003)	(1.535)
Observations	90	90	90
R-squared	0.544	0.473	0.564
Year FE	✓	✓	✓
Province FE	✓	✓	✓

Note: Robust standard errors in parentheses.

Empirical Evidence of Ideas Diffusion

Table: Diffusion through trade and local and non-local knowledge

	(1)	(2)	(3)	(4)
$\log \lambda_{nn,t}$		-5.234**		-5.200**
		(2.412)		(2.399)
$\log(immigration_{n,t})$			0.133*	0.128*
			(0.076)	(0.070)
$\log A_{n,t}$	0.655***	0.549***	0.703***	0.596***
	(0.211)	(0.185)	(0.218)	(0.189)
Constant	3.121*	3.825**	2.012	2.753
	(1.775)	(1.548)	(2.049)	(1.724)
Observations	90	90	90	90
R-squared	0.537	0.610	0.550	0.622
Year FE	✓	✓	✓	✓
Province FE	✓	✓	✓	✓

IV

Conclusion

- Understanding economic development requires understanding the process by which countries accumulate factors of production, and the process by which countries can increase the productivity of such factors
- Aggregate economic development is shaped by the dynamics of the distribution of economic activity in space
- We present a dynamic spatial growth model to study spatial development
  - Model can be used to study the effects of trade and migration on aggregate growth
  - Tractable and easy to take to the data

Ideas diffusion

Proposition 1 *Under Assumption 1, between t and $t+1$, the probability that the best new idea has productivity no greater than q, given by*

$$F_t^{best\ new}(q) = \exp\left(-\alpha_t q^{-\theta} \int_0^\infty x^{\rho\theta} dG_t(x)\right).$$

- Proposition 1 shows that the probability distribution of the best new ideas is Fréchet
  - Scale parameter θ, location parameter $\alpha_t \int_0^\infty x^{\rho\theta} dG_t(x)$
  - Importantly, no need to specify the external source distribution yet

Details

Probability that the best new idea has productivity no greater than q is

$$
\begin{aligned}
& F_{n,t}^{best\ new}(q) \\
& = \Pr\left[\text{all new ideas are no greater than } q\right] \\
& = \sum_{s=0}^{\infty} \Pr[\text{\# new ideas = s}] \cdot \Pr\left[\text{all new ideas are no greater than } q \,|\text{\# new ideas = s}\right] \\
& = \sum_{s=0}^{\infty} \frac{\left(\alpha_{n,t}\bar{z}^{-\theta}\right)^s e^{-\left(\alpha_{n,t}\bar{z}^{-\theta}\right)}}{s!} \cdot F_{n,t}^{new}(q)^s \\
& = \underbrace{\sum_{s=0}^{\infty} \frac{\left[\alpha_{n,t}\bar{z}^{-\theta} F_{n,t}^{new}(q)\right]^s \cdot e^{-\left(\alpha_{n,t}\bar{z}^{-\theta}\right)F_{n,t}^{new}(q)}}{s!}}_{=1} \cdot e^{-\left(\alpha_{n,t}\bar{z}^{-\theta}\right)\left(1-F_{n,t}^{new}(q)\right)} \\
& = e^{-\left(\alpha_{n,t}\bar{z}^{-\theta}\right)\left(1-F_{n,t}^{new}(q)\right)}.
\end{aligned}
$$

Hukou status

- Individuals with different Hukou status that we discipline in the data
  - Equilibrium conditions of the dynamic labor supply decisions of workers given by

$$V_{i,t}^{H} = log(\psi_i^H w_{i,t}/P_{i,t}) + \nu log\left(\sum_{n=1}^{N} exp\left(\beta V_{n,t+1}^{H} - m_{in,t}^{H}\right)^{1/\nu}\right),$$

$$\mu_{in,t}^{H} = \frac{exp\left(\beta V_{n,t+1}^{H} - m_{in,t}^{H}\right)^{1/\nu}}{\sum_{g=1}^{N} exp\left(\beta V_{g,t+1}^{H} - m_{ig,t}^{H}\right)^{1/\nu}},$$

$$L_{i,t+1} = \sum_{H}\sum_{n=1}^{N} \mu_{ni,t}^{H} L_{n,t}^{H}$$

back

Empirical Evidence of Ideas Diffusion

- Recall
$$A_{n,t+1} - A_{n,t} = \alpha_t \Gamma_{\rho_\ell,\rho_m} \left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t} \right)^{\rho_\ell} \right] \left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}} \right)^{\rho_m} \right].$$
- Taking logs on both sides, we obtain
$$\log \left(A_{n,t+1} - A_{n,t} \right) = \log \left(goods_{n,t} \right) + \log \left(people_{n,t} \right) + \log \alpha_t + \log \Gamma_{\rho_\ell,\rho_m},$$
where $\log \left(goods_{n,t} \right) = \log \left[\sum_{i=1}^{N} \lambda_{ni,t} \left(\frac{A_{i,t}}{\lambda_{ni,t}} \right)^{\rho_m} \right]$ and
$\log \left(people_{n,t} \right) = \log \left[\sum_{i=1}^{N} s_{in,t} \left(A_{i,t} \right)^{\rho_\ell} \right]$.
- Specification
$$\log \left(A_{n,t+1} - A_{n,t} \right) = \log \left(goods_{n,t} \right) + \log \left(people_{n,t} \right) + \tau_t + \tau + \epsilon_{n,t},$$

Empirical Evidence of Ideas Diffusion

- IV - fix the shares of trade and migration at the pre-period (1985)
- Now $\log(goods)$ is defined as

$$\log\left[\sum_{i=1}^{N}\lambda_{ni,1985}\left(\frac{A_{i,t}}{\lambda_{ni,1985}}\right)^{\rho_m}\right],$$

- Now $\log(people_{n,t})$ is defined as

$$\log\left[\sum_{i=1}^{N}s_{in,1985}\left(A_{i,t}\right)^{\rho_\ell}\right]$$

Empirical Evidence of Ideas Diffusion

Table: Diffusion through trade and migration on knowledge growth

Dept. Var: $\log\left(A_{n,t+1} - A_{n,t}\right)$	IV
$\log\left(goods_{n,t}\right)$	0.390**
	(0.162)
$\log\left(people_{n,t}\right)$	5.055***
	(0.300)
Constant	-4.157***
	(1.034)
Kleibergen-Paap rk Wald F statistic	87.54
Observations	87
R-squared	0.935
Year FE	✓

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The end

- Thanks!